Towards a provably resilient scheme for graph-based watermarking

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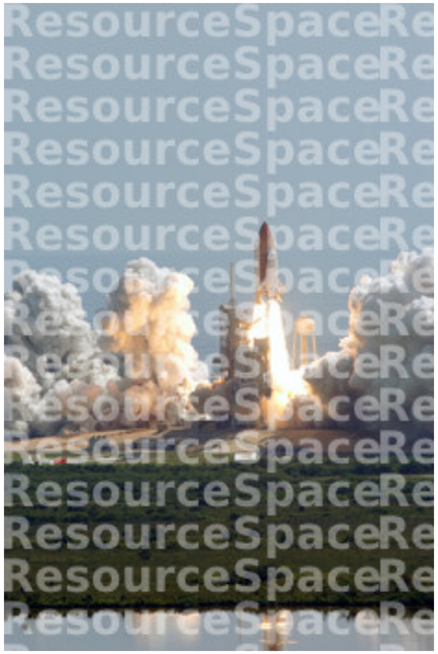
UNIVERSIDADE FEDERAL DO RIO DE JANEIRO
UFRJ

INMETRO

Watermarks

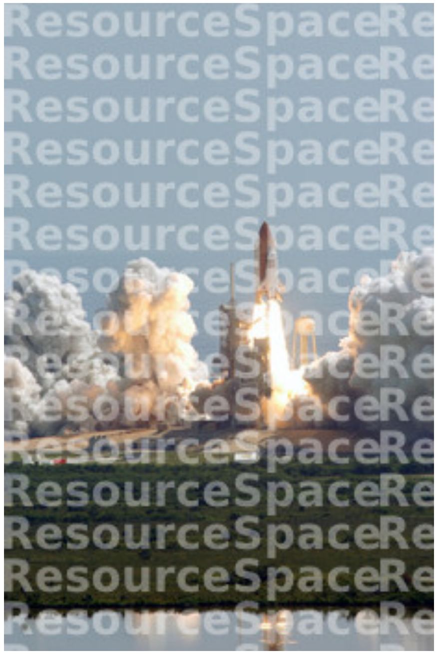

10
FEDERAL RESERVE NOTE
10
THE UNITED STATES OF AMERICA
GK 00000000 T
K11
SPECIMEN
We The People
SPECIMEN
GK 00000000 T
10
TEN
DOLLARS

Watermarks



Watermarks


10 FEDERAL RESERVE NOTE 10
THE UNITED STATES OF AMERICA
GK 00000000 T
K11
SPECIMEN
We the People
TEN DOLLARS


ResourceSpaceRe


®2011 Vinícius Pereira de Sá

Software watermarking

?!

What for?

How?

Software watermarking

```
int fibonnaci (int n) {
   int a = 1, b = 1;

   for (int i = 1; i < n; i++) {
      int sum = a + b;
      a = b;
      b = sum;
   }

   return b;
}
```

Software watermarking

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int fibonnaci (int n) {
    int a = 1, b = 1;

    for (int i = 1; i < n; i++) {
        int sum = a + b;
        a = b;
        b = sum;
    }
    // author: Vinícius
    return b;
}
```

Software watermarking

```
int fibonnaci (int n) {
    int a = 1, b = 1;
    string author = “Vinícius”;
    for (int i = 1; i < n; i++) {
        int sum = a + b;
        a = b;
        b = sum;
    }

    return b;
}
```

Graph-based software watermarking

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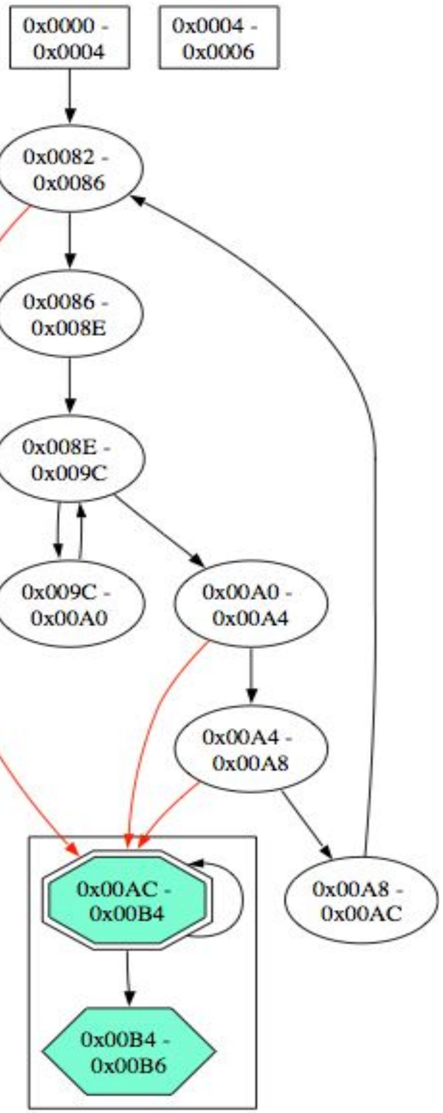
Davidson and Myhrvold (1996)
Venkatesan, Vazirani and Sinha (2001)
Collberg et al. (WG 2003)


Control flow graph

Graph-based software watermarking

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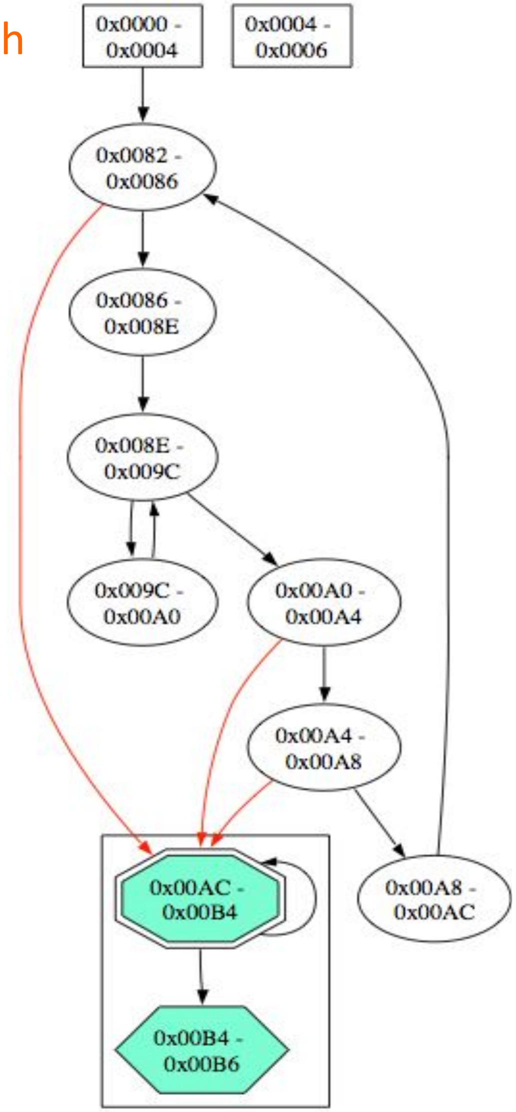
Control flow graph



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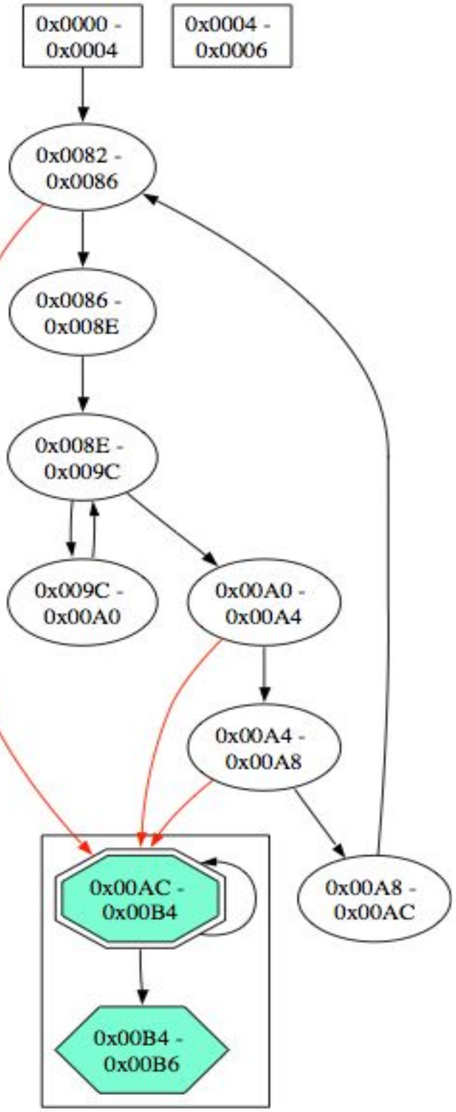
(10010101000111010101001101011)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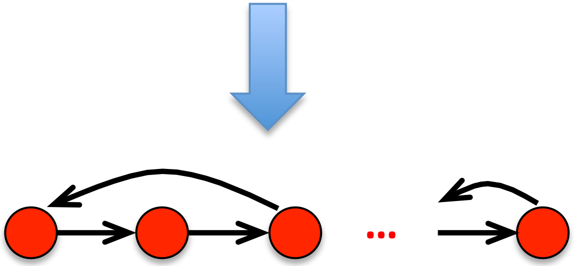
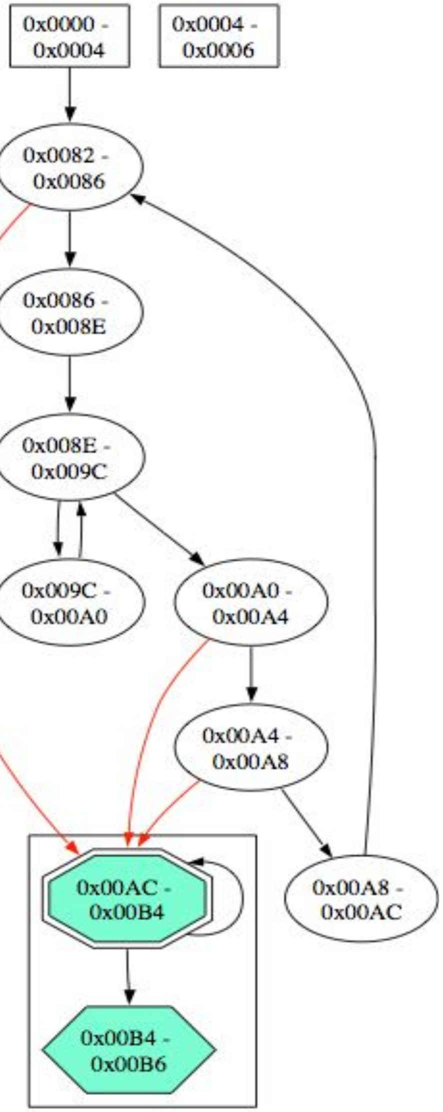


Control flow graph



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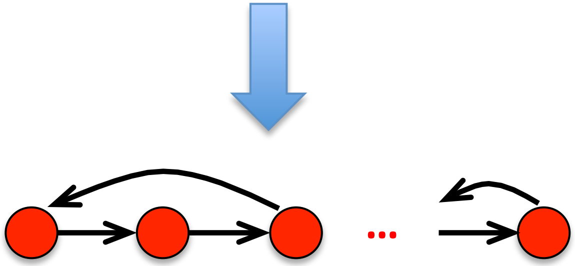

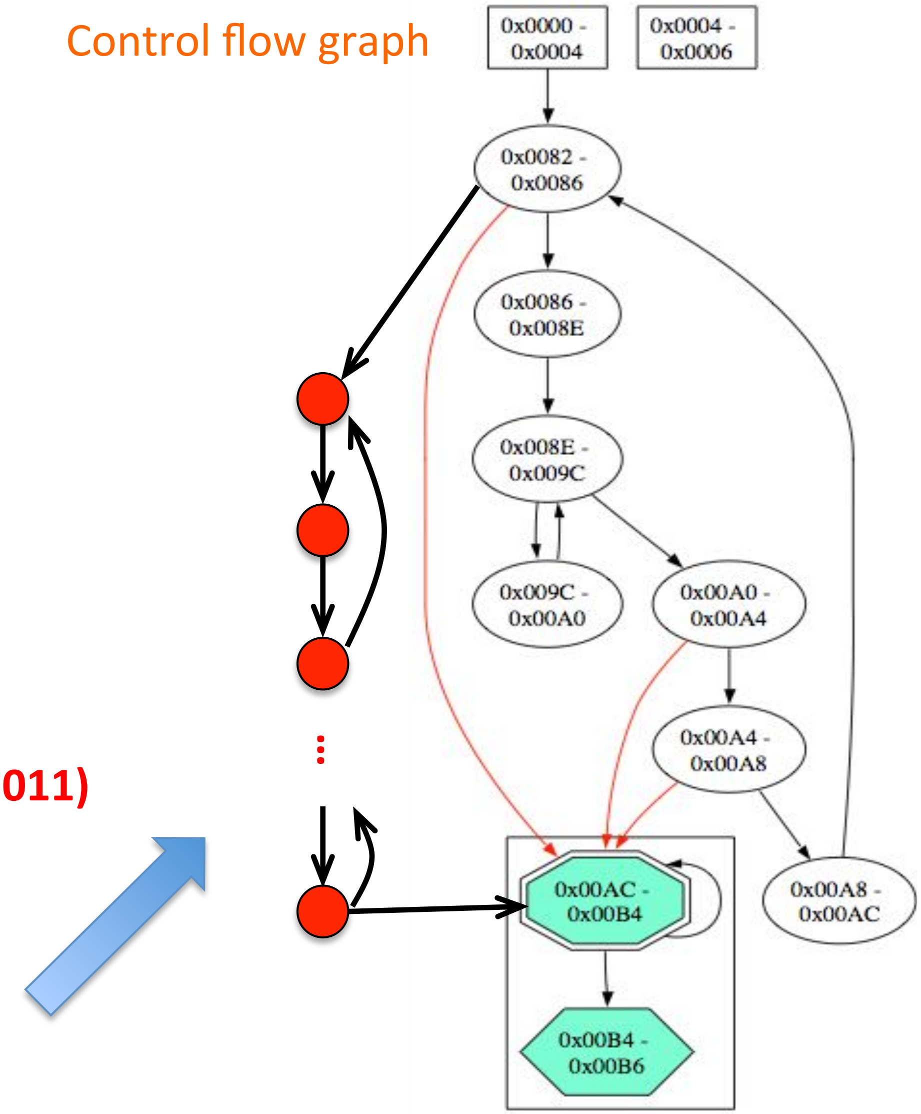

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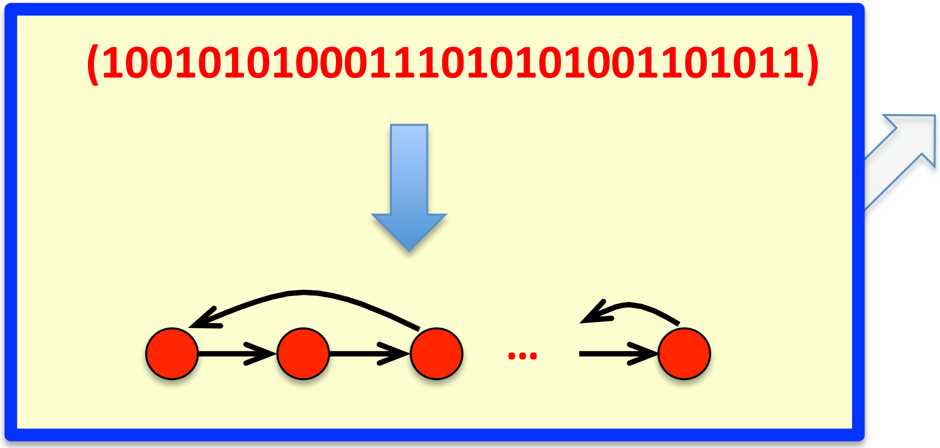

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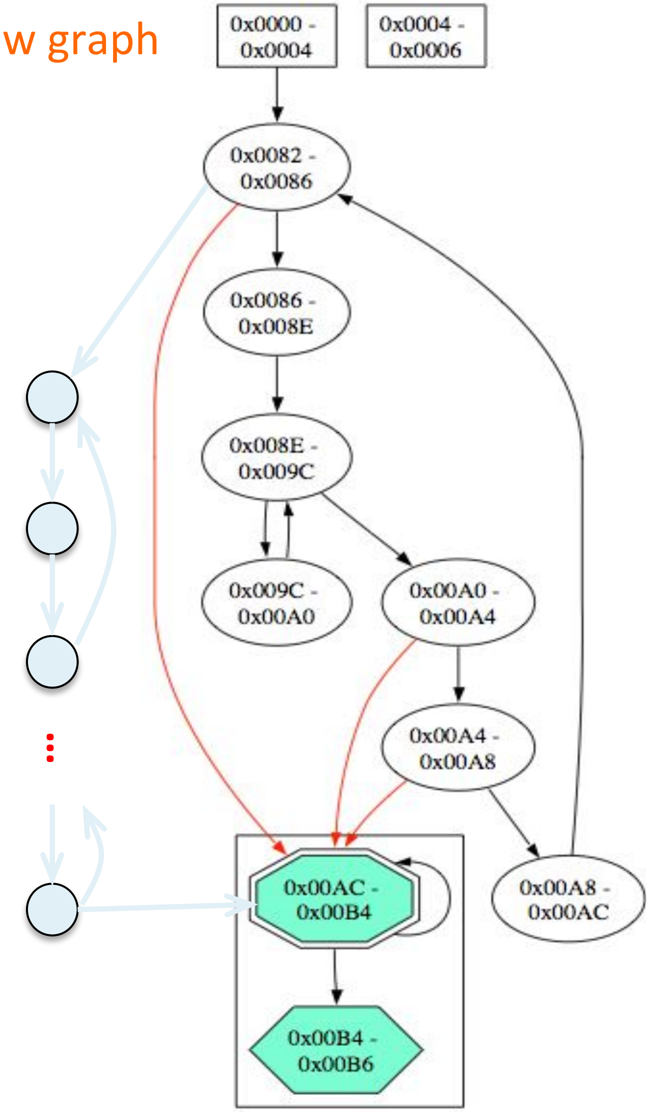
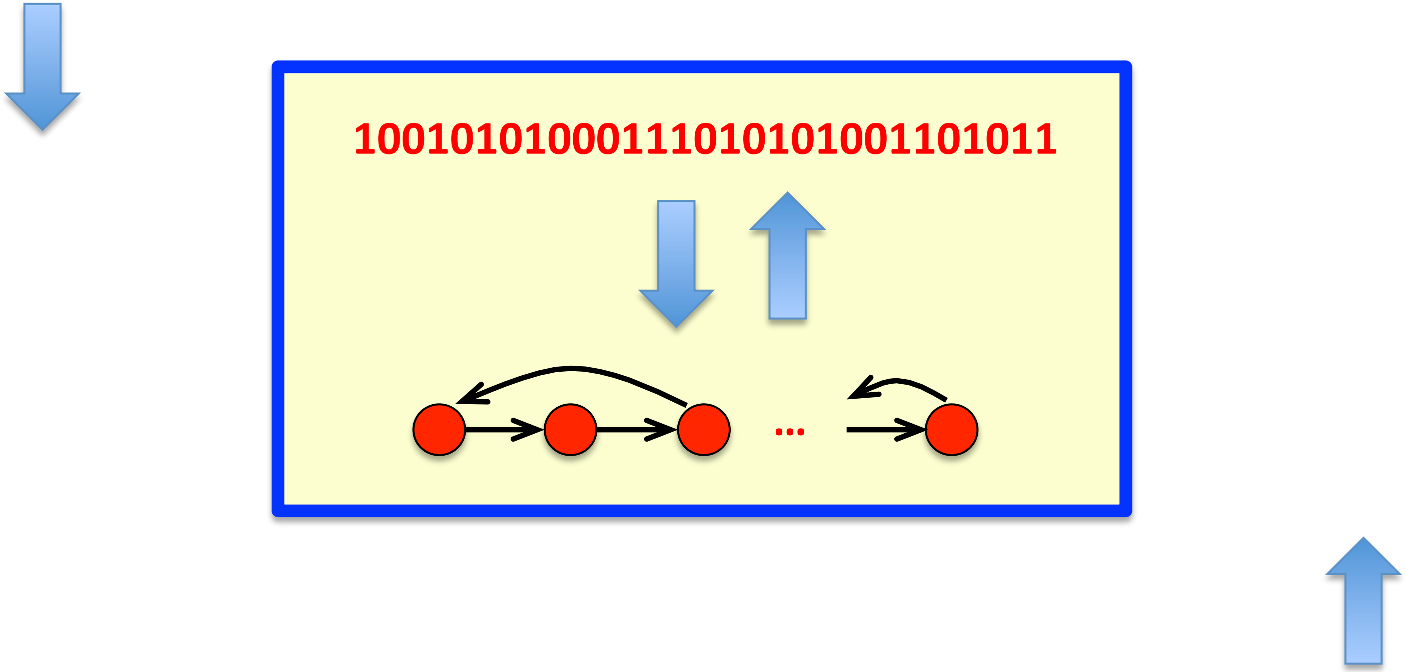
Control flow graph



Graph-based software watermarking

Chroni and Nikolopoulos (2011)

Encoding



10010101000111010101001101011

Decoding

The codec from Chroni and Nikolopoulos

Chroni and Nikolopoulos (2011)

key $\omega = 29$

The codec from Chroni and Nikolopoulos

Chroni and Nikolopoulos (2011)

key $\omega = 29$

B = 11101 $n = 5$

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$\bar{B} = \mathbf{00010}$

$B^* = \underbrace{11111}_{n \text{ 1's}}\ \mathbf{00010}\ 0$

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$Z_0 = 6, 7, 8, 10, 11$

$Z_1 = 1, 2, 3, 4, 5, 9$

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$Z_1 = 1, 2, 3, 4, 5, 9$

$P_b = \overbrace{\mathbf{6, 7, 8, 10, 11,}}^{Z_0}\ \overbrace{\mathbf{9, 5, 4, 3, 2, 1}}^{Z_1^R}$

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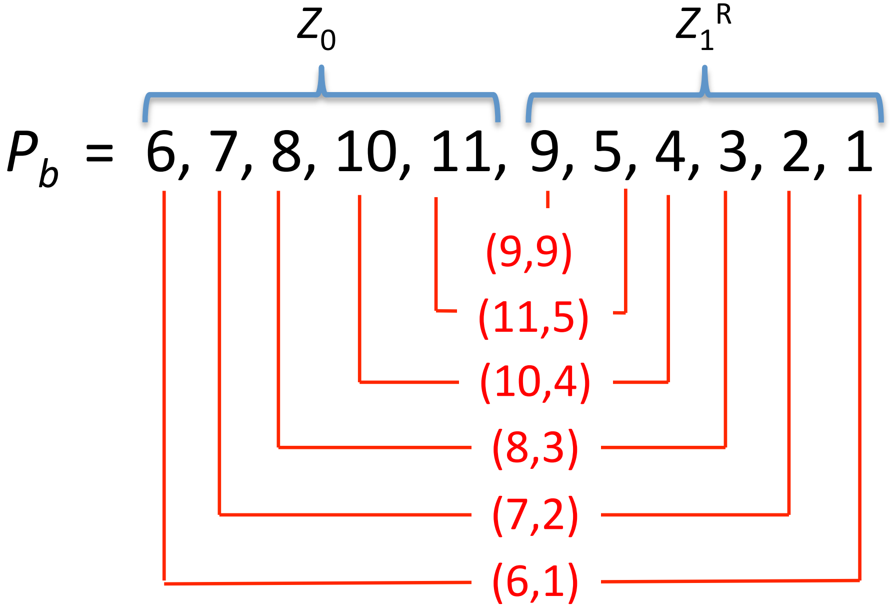

$$P_b = \overbrace{6, 7, 8, 10, 11}^{Z_0},\ \overbrace{9, 5, 4, 3, 2, 1}^{Z_1^R}$$

(9,9)

(11,5)

(10,4)

(8,3)

(7,2)

(6,1)

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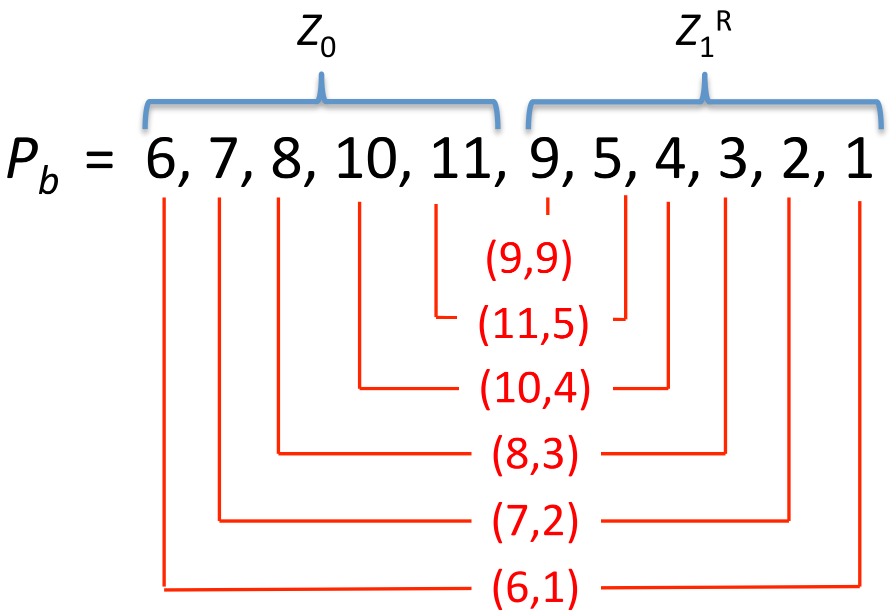

$$P_b = \overbrace{6, 7, 8, 10, 11}^{Z_0}, \overbrace{9, 5, 4, 3, 2, 1}^{Z_1^R}$$

$P_s = 6, 7, 8, 10, 11, 1, 2, 3, 9, 4, 5$

1 2 3 4 5 6 7 8 9 10 11

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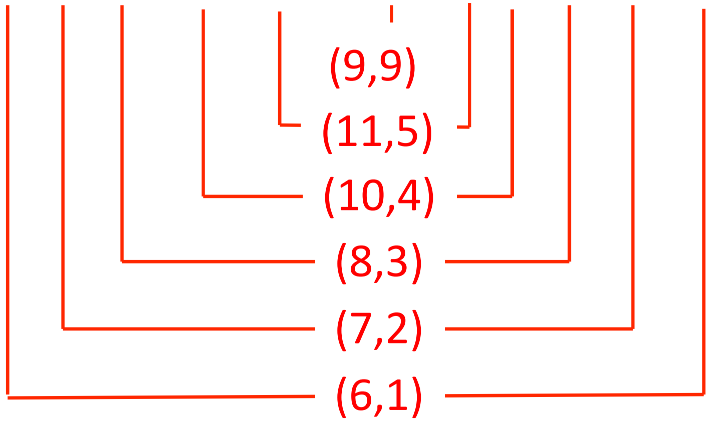

(9,9)
(11,5)
(10,4)
(8,3)
(7,2)
(6,1)

$$P_s = 6, 7, 8, 10, 11, 1, 2, 3, \boxed{9}, 4, 5$$

1 2 3 4 5 6 7 8 9 10 11

fixed element

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$P_s = 6, 7, 8, 10, 11, 1, 2, 3, 9, 4, 5$

12, 6, 7, 8, 10, 11, 1, 2, 3, 9, 4, 5

↑

2*n*+2

The codec from Chroni and Nikolopoulos

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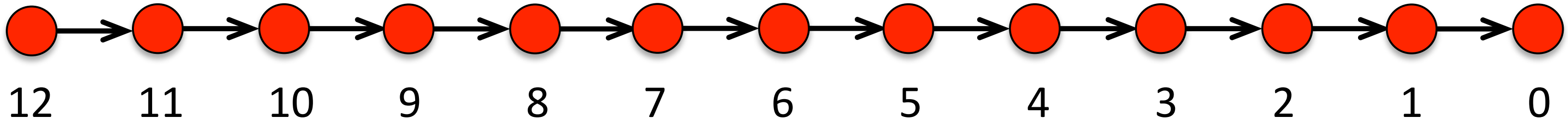
12, 6, 7, 8, 10, 11, 1, 2, 3, 9, 4, 5



The codec from Chroni and Nikolopoulos

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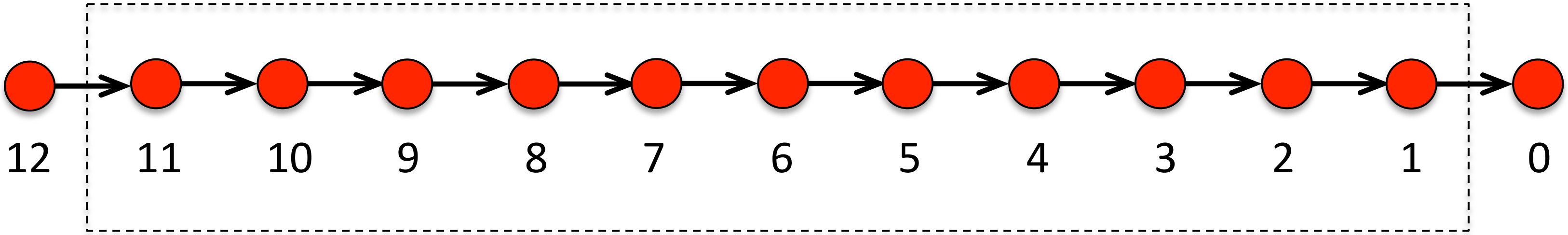

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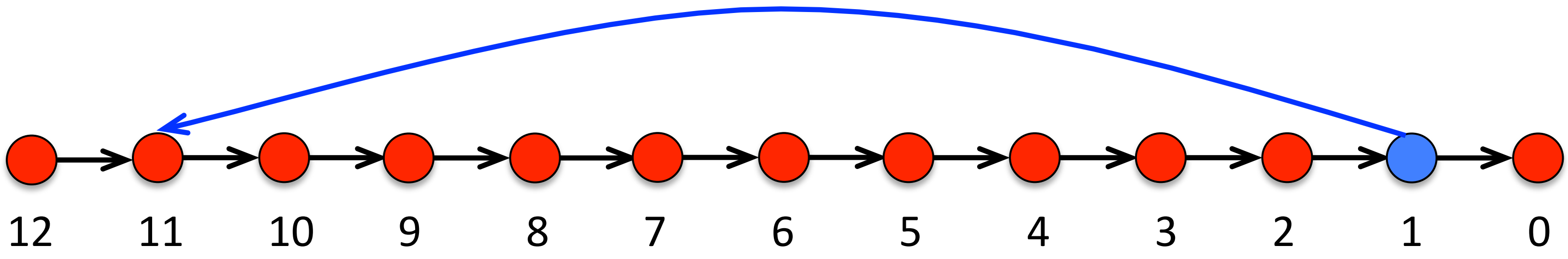
12, 6, 7, 8, 10, **11**, **1**, 2, 3, 9, 4, 5



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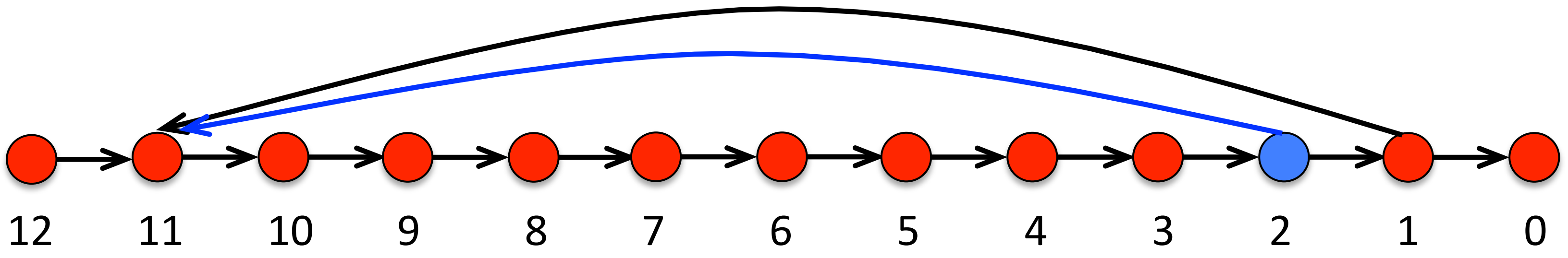
12, 6, 7, 8, 10, **11**, 1, **2**, 3, 9, 4, 5



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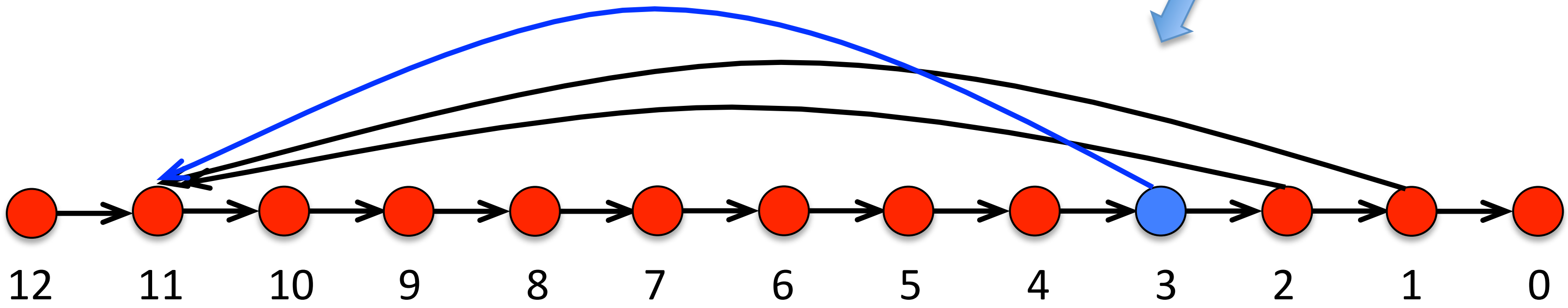
12, 6, 7, 8, 10, **11**, 1, 2, **3**, 9, 4, 5



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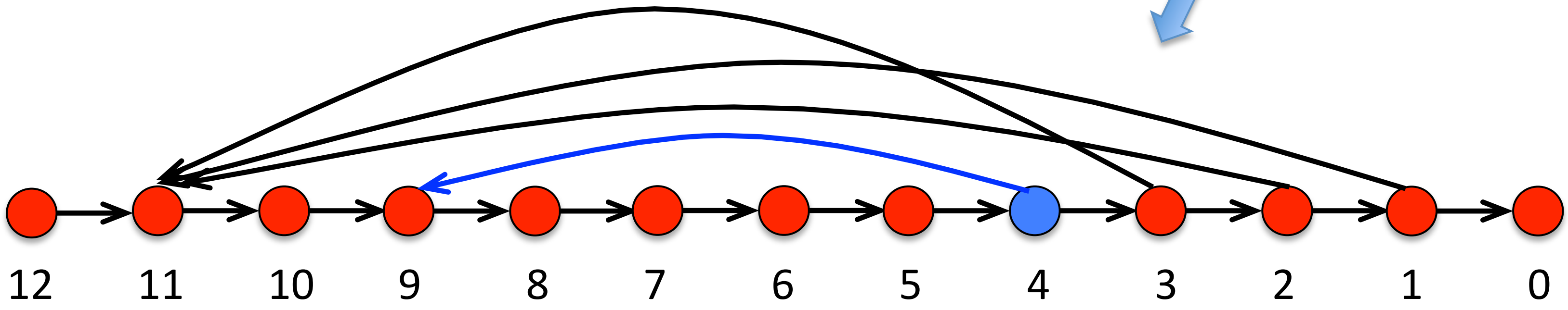
12, 6, 7, 8, 10, 11, 1, 2, 3, **9**, **4**, 5



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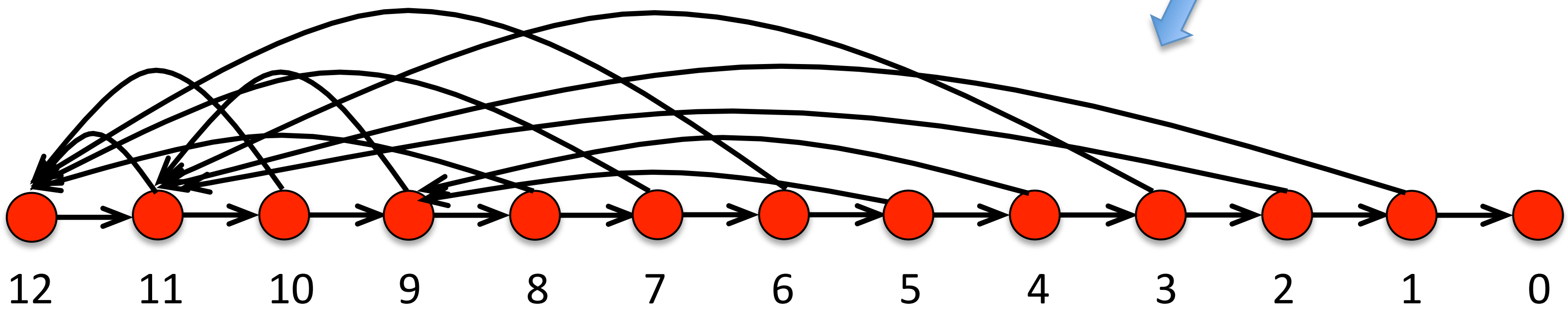
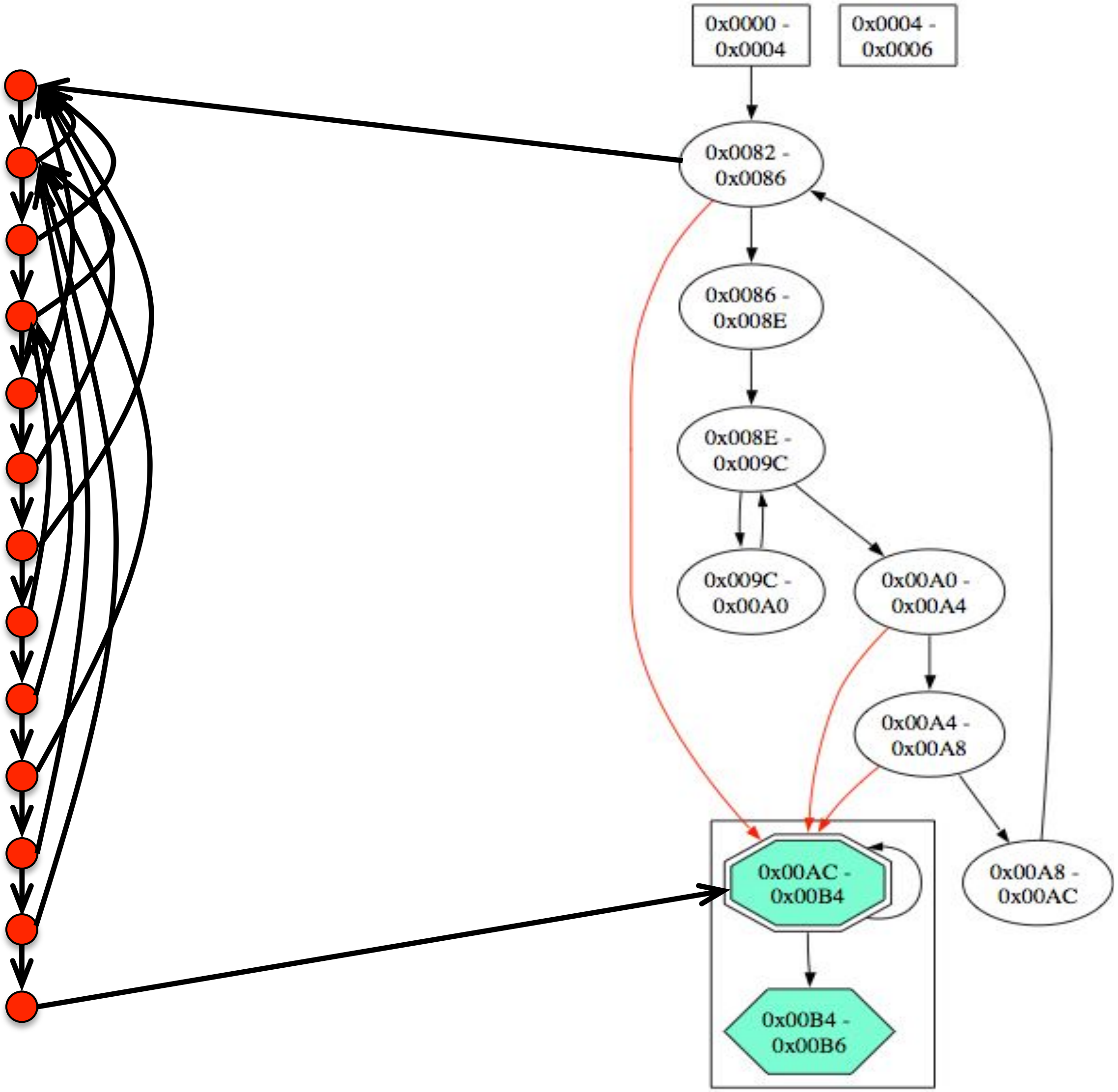


The codec from Chroni and Nikolopoulos



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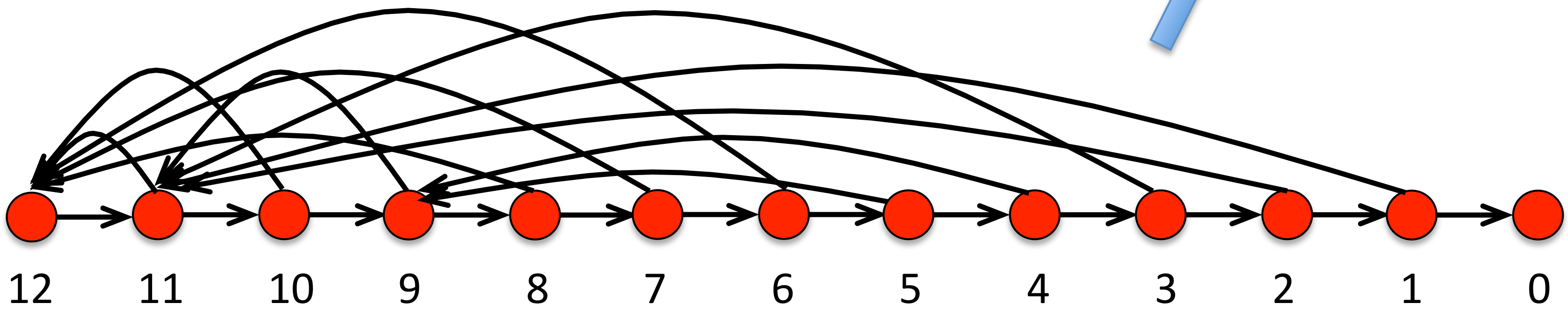
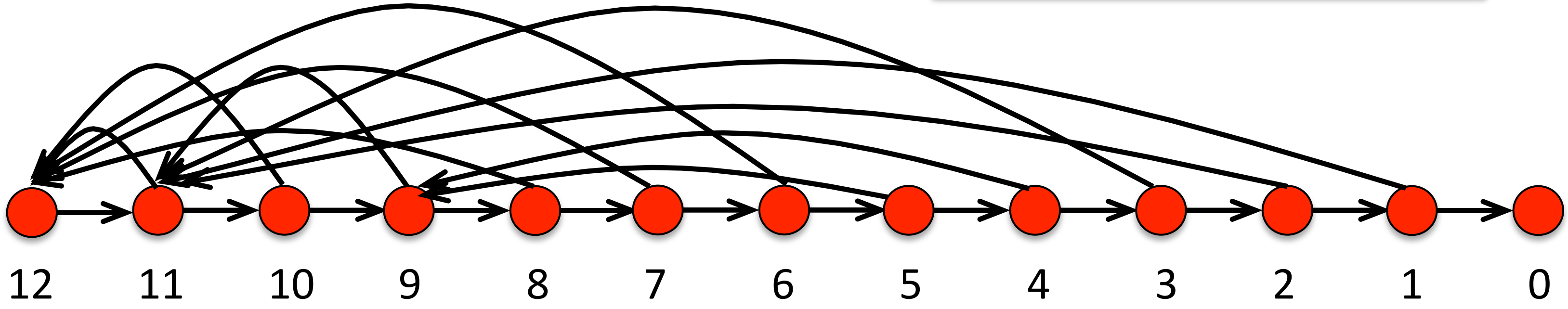


Our contribution

B., B., M., S., S. (WG 2013)



1. **full characterization of the class** of *canonical reducible permutation graphs* (the graphs produced by Chroni and Nikolopoulos's encoding algorithm)

2. a **linear-time recognition algorithm** for such graphs

3. a **new linear-time decoding algorithm** (graph → integer key) simpler, marginally faster and able to retrieve the correct key even after the malicious removal of $k \leq 2$ edges

4. a **tight bound for the resilience of the codec** against edge removals

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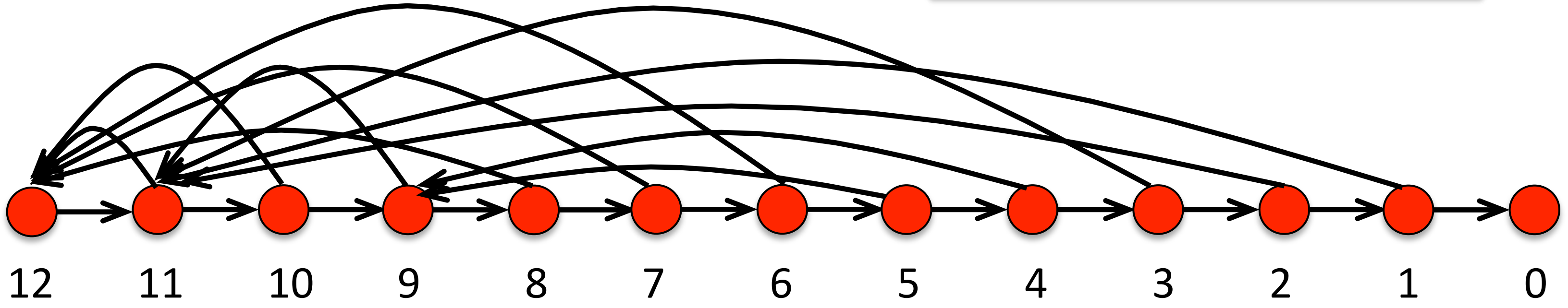
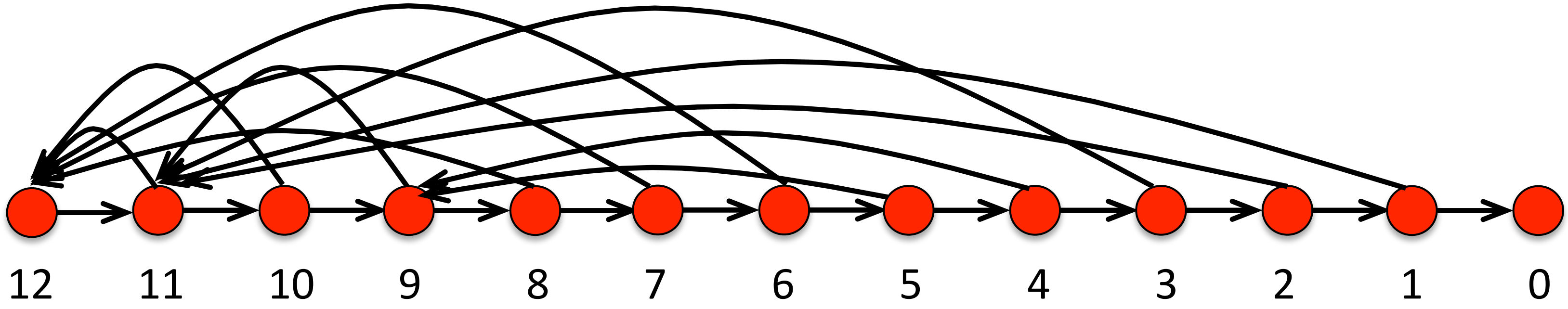
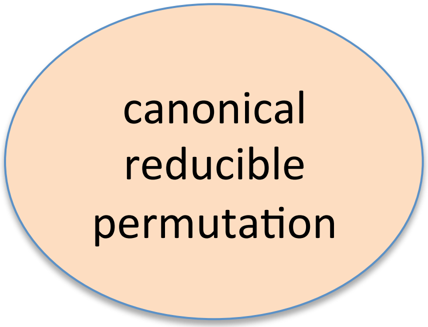


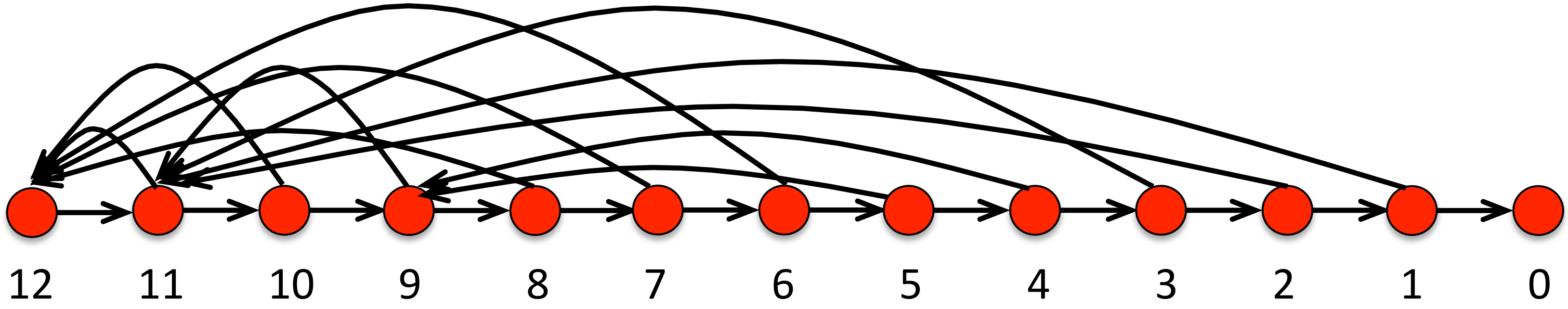
1. **formal definition of the class** of *canonical reducible permutation graphs* (precisely the graphs produced by Chroni and Nikolopoulos's encoding algorithm)
2. **characterization** and linear-time **recognition algorithm** for such graphs
3. a **new linear-time decoding algorithm** (graph → integer key) simpler, marginally faster and able to retrieve the correct key even after the malicious removal of $k \leq 2$ edges
4. a **tight bound for the resilience of the codec** against edge removals

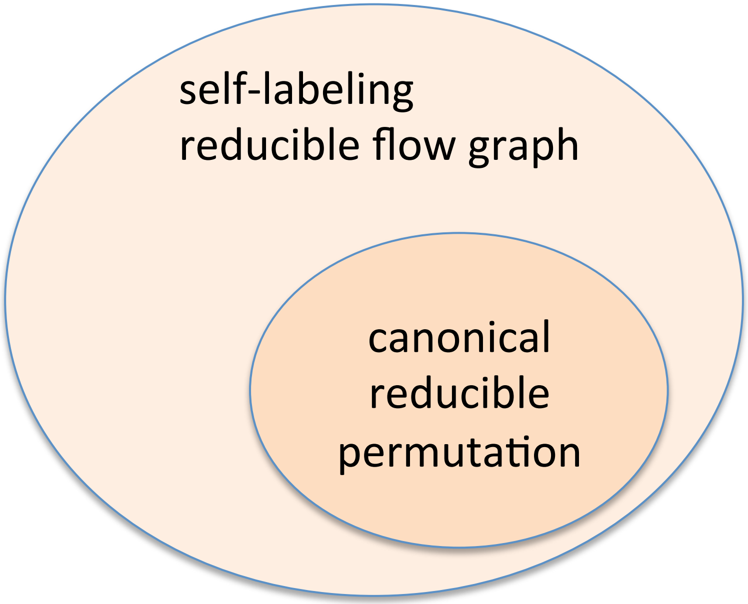
Canonical reducible permutation graphs





Canonical reducible permutation graphs

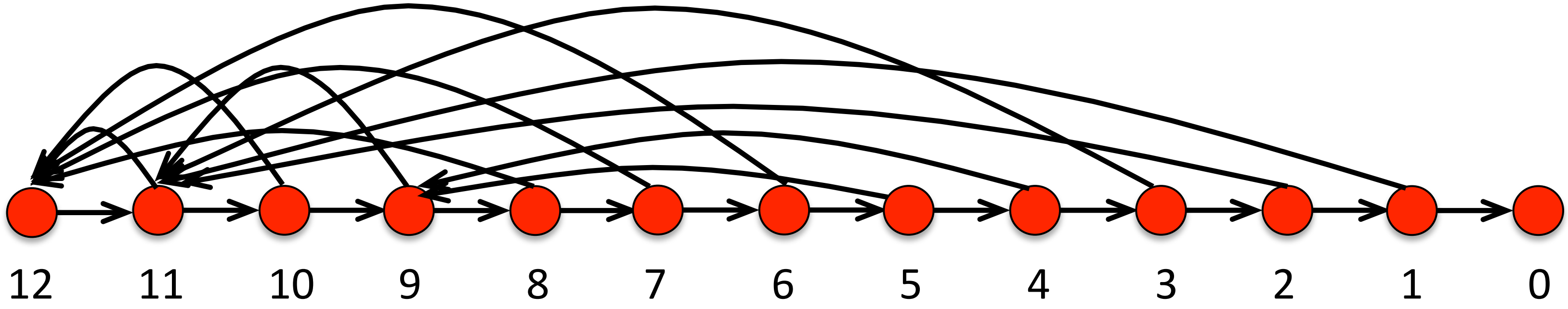


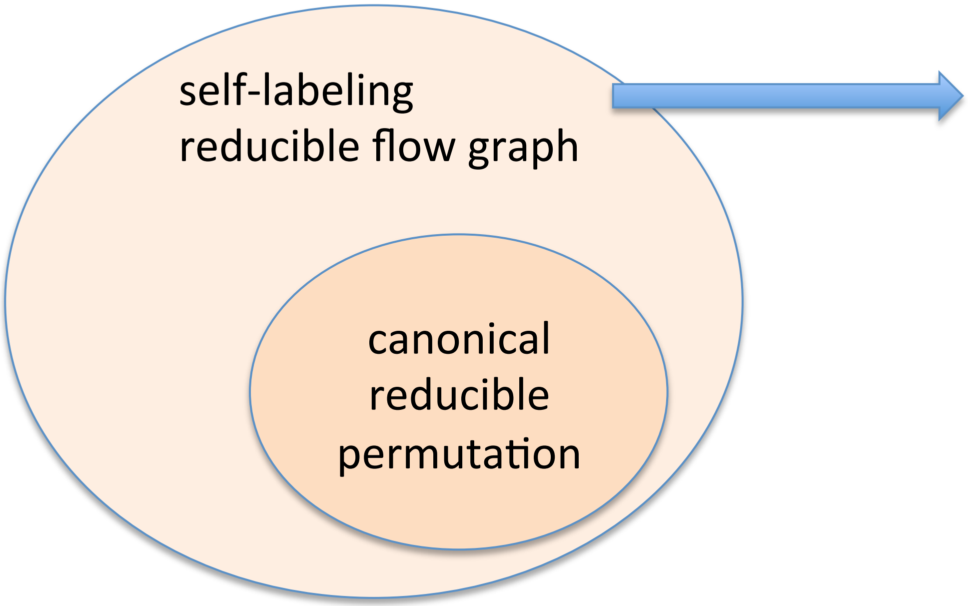


self-labeling
reducible flow graph

canonical
reducible
permutation

Canonical reducible permutation graphs





self-labeling reducible flow graph

canonical reducible permutation

Definition

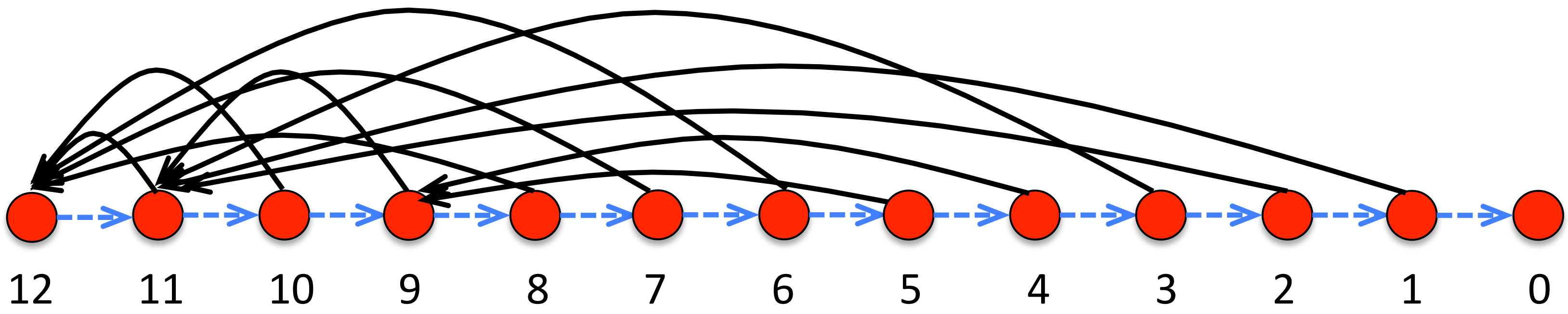
Self-labeling reducible flow graph $G(V,E)$:

- vertices 0, … , $|V|-1$
- exactly one Hamiltonian path
- v in $V \setminus \{0, |V|-1\} \implies N^+(v) = \{v-1, w\}$, for some $w > v$

$$v = 0 \implies N^+(v) = \{\ \}$$
$$N^-(v) = \{1\}$$

$$v = |V|-1 \implies N^+(v) = \{|V|-2\}$$
$$|N^-(v)| \geq 2$$

Canonical reducible permutation graphs



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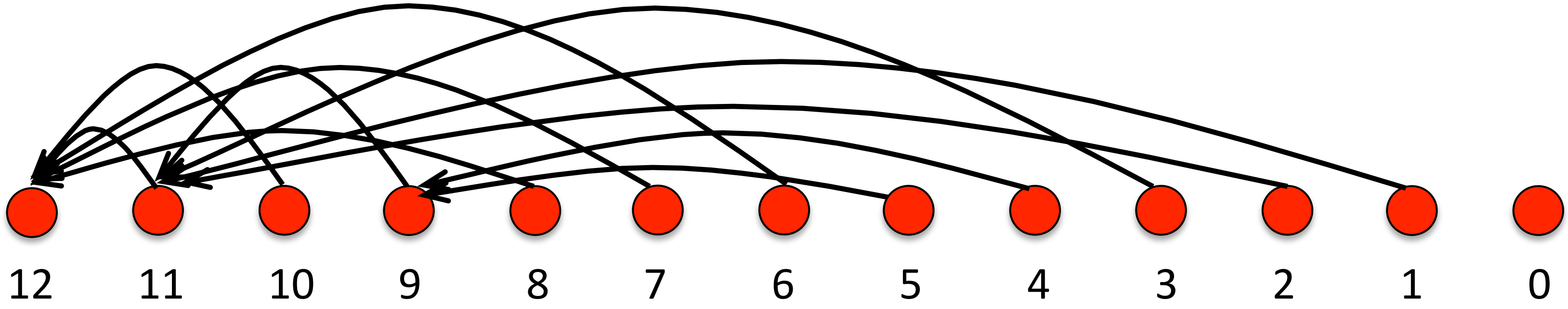

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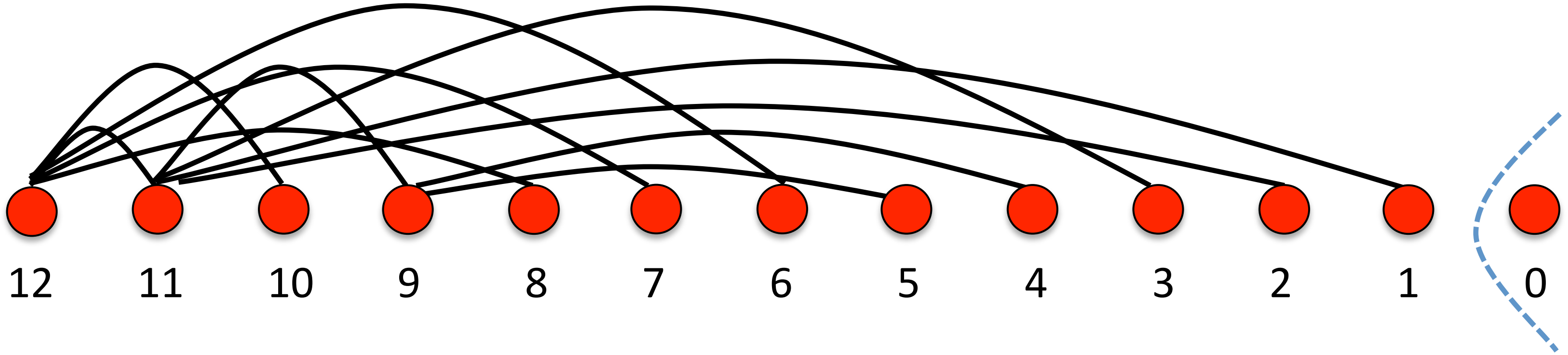
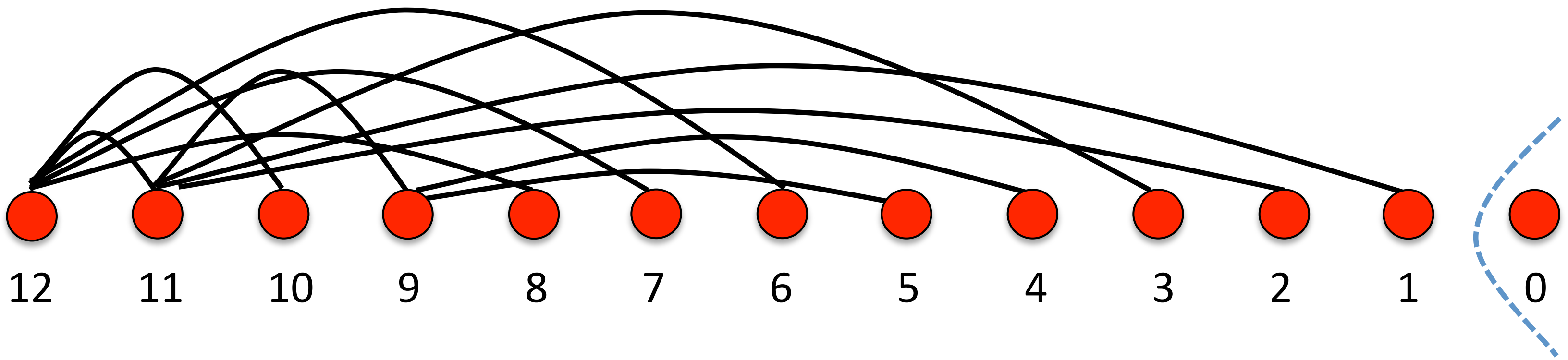
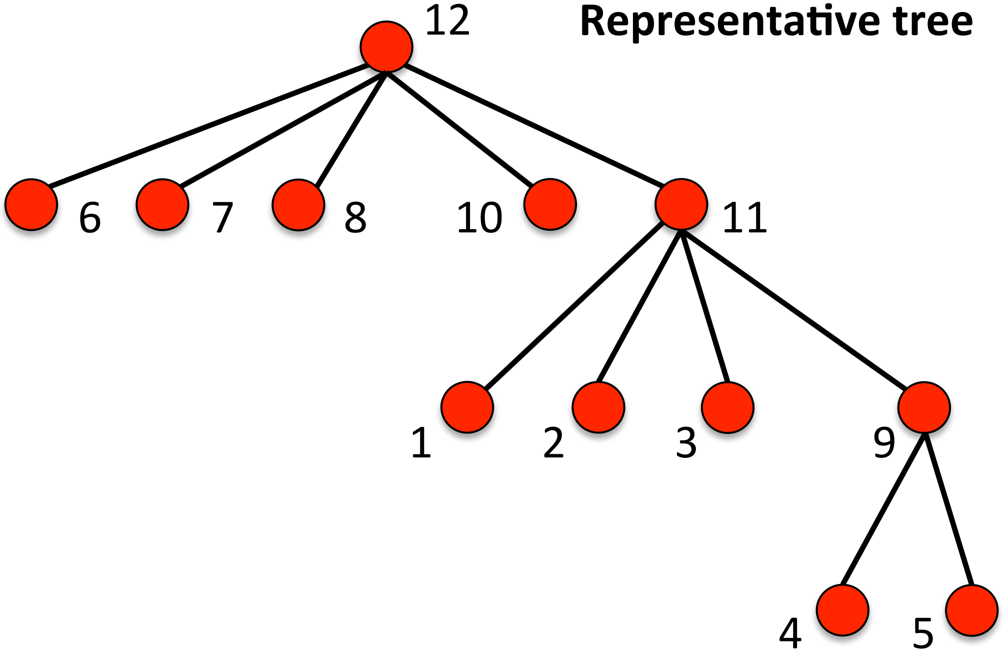


Canonical reducible permutation graphs

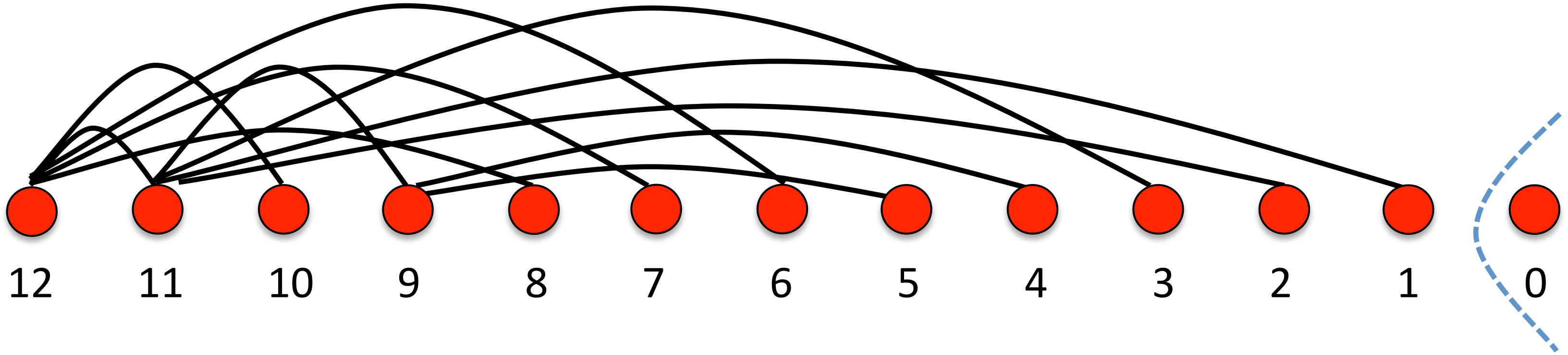


Representative tree



- children in ascending order
- max-heap property

Canonical reducible permutation graphs



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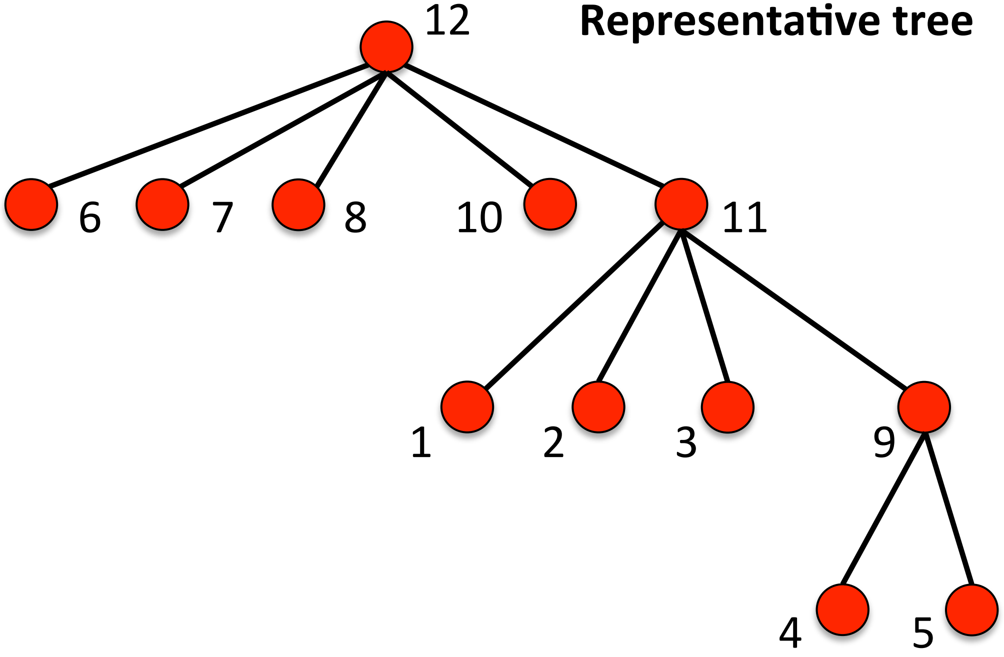
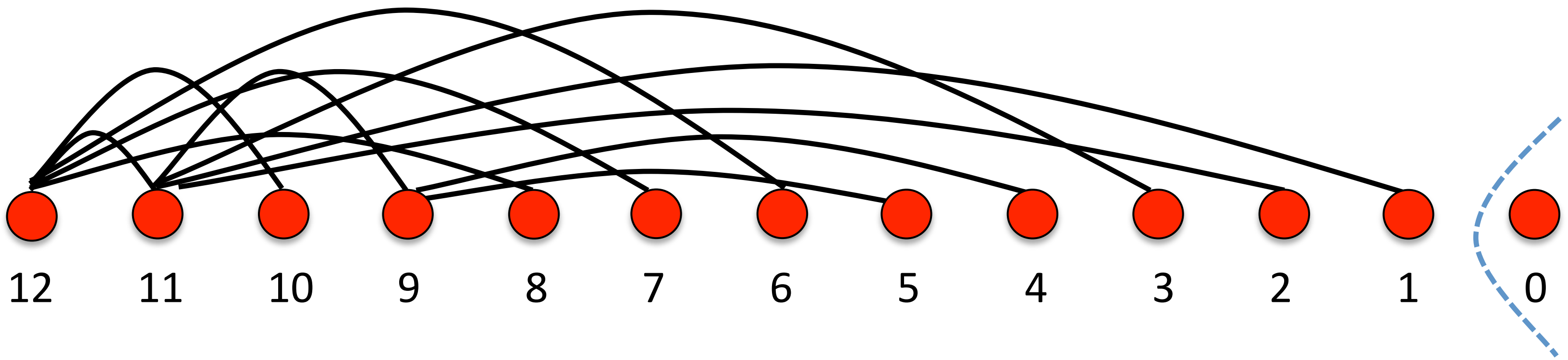


- children in ascending order
- max-heap property

root-free preorder traversal:
6, 7, 8, 10, 11, 1, 2, 3, 9, 4, 5

Canonical reducible permutation graphs



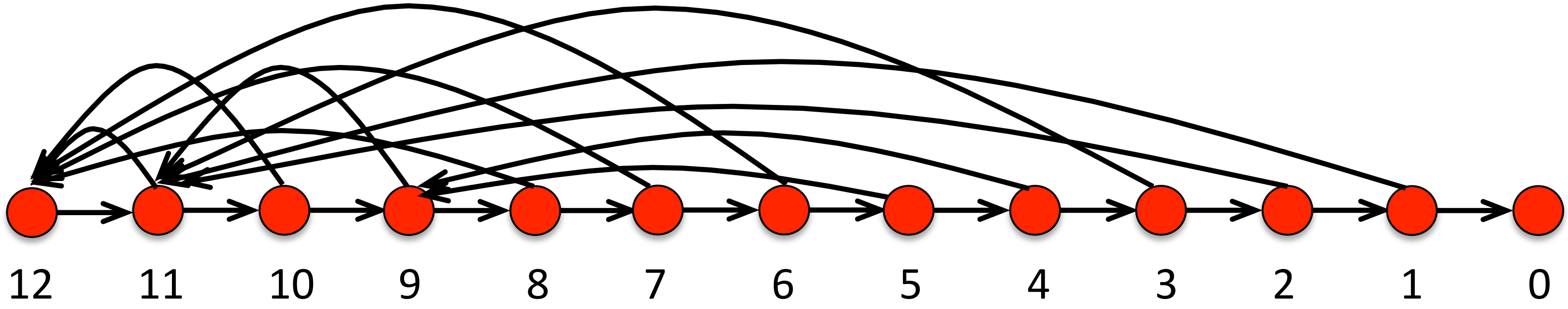
Definition

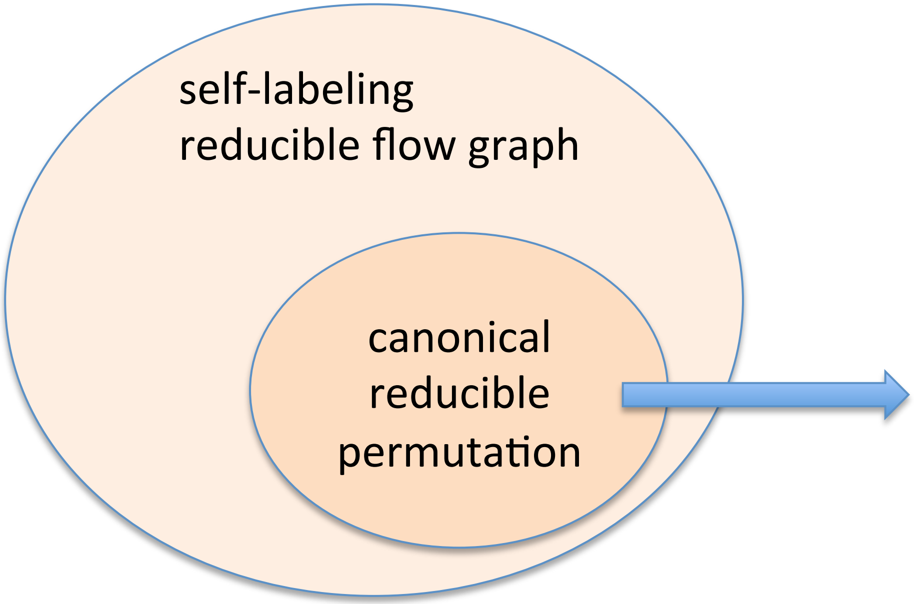
Canonical self-inverting permutation

- a self-inverting permutation
- elements $s_i = 1, 2, \ldots, 2n+1$
- exactly one fixed element
- each 2-cycle (s_i, s_j) satisfies

$$1 \leq i \leq n,\ s_i > s_j$$

Canonical reducible permutation graphs





self-labeling
reducible flow graph

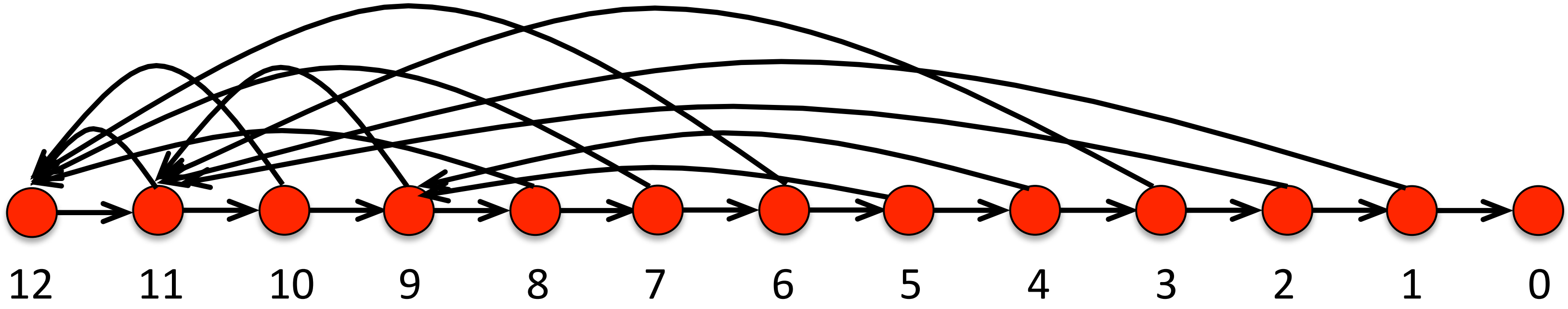
canonical
reducible
permutation

Definition

Canonical reducible permutation graph:

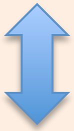
- a self-labeling reducible flow graph
- 2*n*+3 vertices
- its *representative tree* has a (root-free) preorder traversal which is a canonical self-inverting permutation

Canonical reducible permutation graphs



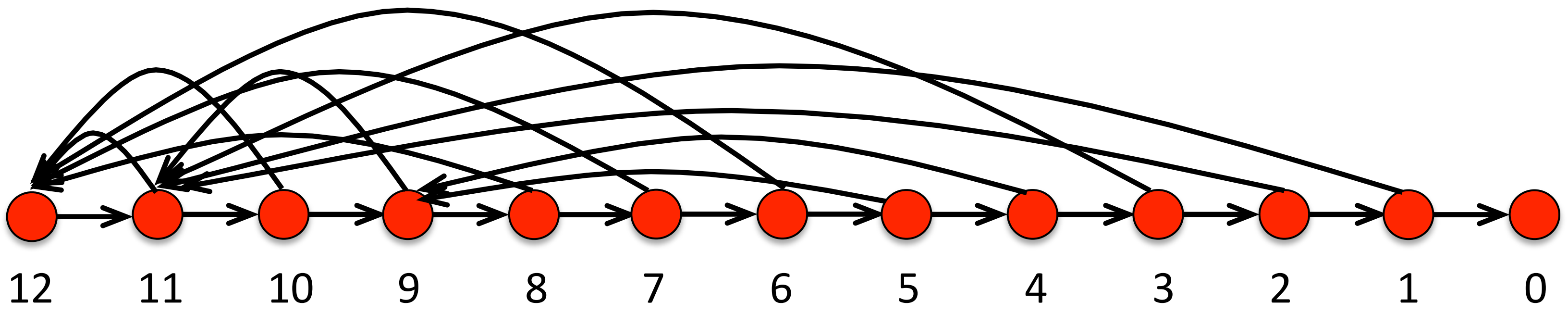
Theorem

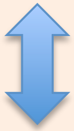
Watermark from Chroni and Nikolopoulos



Canonical reducible permutation graph

Canonical reducible permutation graphs





Theorem

Watermark from Chroni and Nikolopoulos

Canonical reducible permutation graph

(Proof by “we don’t want to know the details” argument)

Canonical reducible permutation graphs

Chroni and Nikolopoulos (2011)

key $\omega = 29$

$B = 11101$ $n = 5$

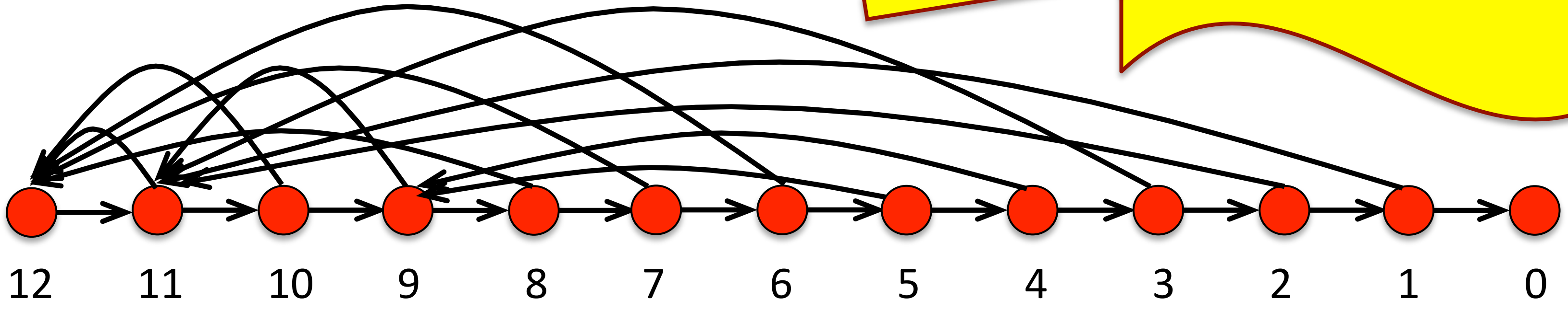
$\bar{B} = 00010$

$B^* = \underbrace{11111}_{n\ 1\text{'s}}\ 00010\ 0$

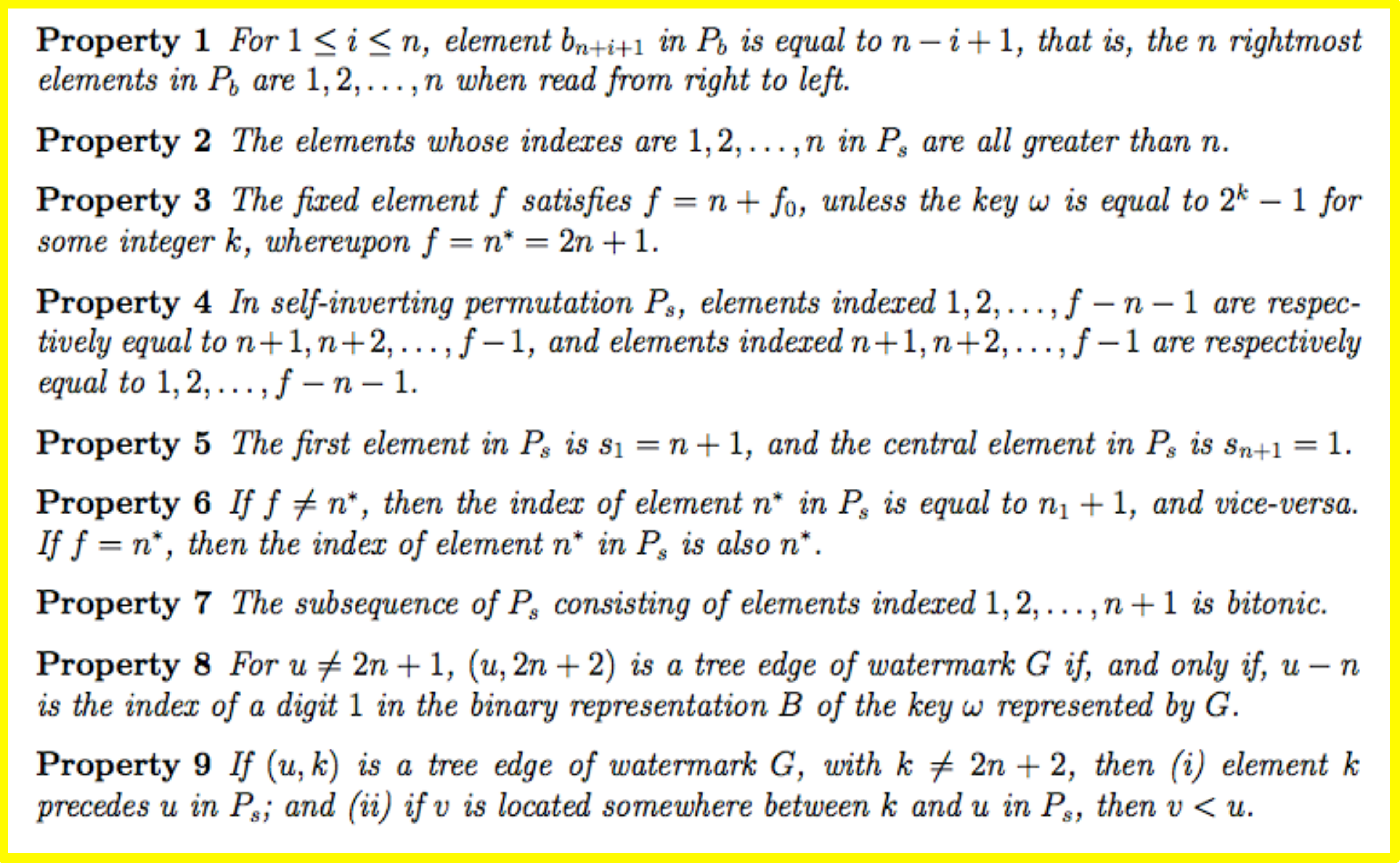
$P_b = \overbrace{6, 7, 8, 10, 11,}^{Z_0}\ \overbrace{9, 5, 4, 3, 2, 1}^{Z_1^R}$

$P_s = 6, 7, 8, 10, 11, 1, 2, 3, 9, 4, 5$

several structural properties



Codec properties



Property 1 *For* $1 \leq i \leq n$, *element* b_{n+i+1} *in* P_b *is equal to* $n-i+1$, *that is, the* n *rightmost elements in* P_b *are* $1, 2, \ldots, n$ *when read from right to left.*

Property 2 *The elements whose indexes are* $1, 2, \ldots, n$ *in* P_s *are all greater than* n.

Property 3 *The fixed element* f *satisfies* $f = n + f_0$, *unless the key* ω *is equal to* $2^k - 1$ *for some integer* k, *whereupon* $f = n^* = 2n + 1$.

Property 4 *In self-inverting permutation* P_s, *elements indexed* $1, 2, \ldots, f-n-1$ *are respectively equal to* $n+1, n+2, \ldots, f-1$, *and elements indexed* $n+1, n+2, \ldots, f-1$ *are respectively equal to* $1, 2, \ldots, f-n-1$.

Property 5 *The first element in* P_s *is* $s_1 = n+1$, *and the central element in* P_s *is* $s_{n+1} = 1$.

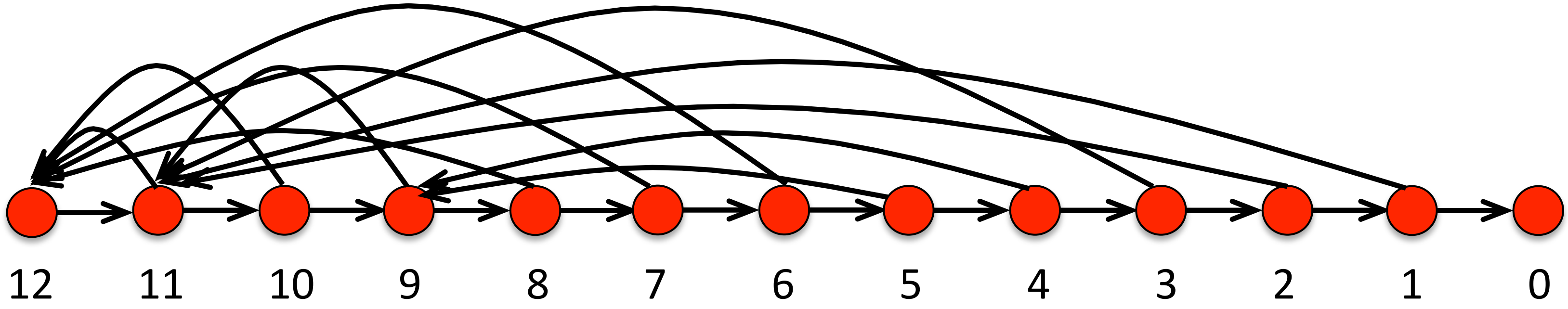
Property 6 *If* $f \neq n^*$, *then the index of element* n^* *in* P_s *is equal to* $n_1 + 1$, *and vice-versa. If* $f = n^*$, *then the index of element* n^* *in* P_s *is also* n^*.

Property 7 *The subsequence of* P_s *consisting of elements indexed* $1, 2, \ldots, n+1$ *is bitonic.*

Property 8 *For* $u \neq 2n+1$, $(u, 2n+2)$ *is a tree edge of watermark* G *if, and only if,* $u - n$ *is the index of a digit* 1 *in the binary representation* B *of the key* ω *represented by* G.

Property 9 *If* (u, k) *is a tree edge of watermark* G, *with* $k \neq 2n+2$, *then (i) element* k *precedes* u *in* P_s; *and (ii) if* v *is located somewhere between* k *and* u *in* P_s, *then* $v < u$.

Canonical reducible permutation graphs



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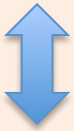
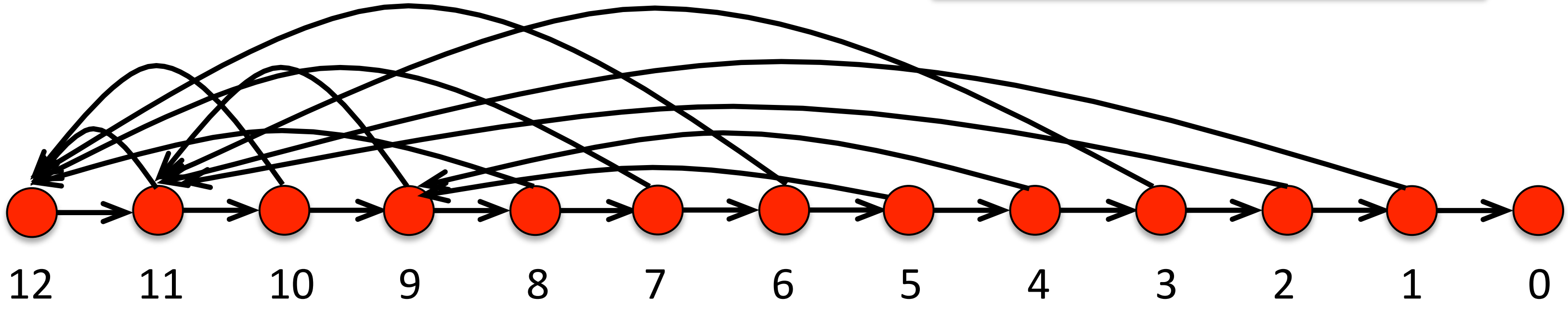


Canonical reducible permutation graph

(Proof by "we don't want to know the details" argument) ✓

Our contribution

B., B., M., S., S. (WG 2013)



1. **formal definition of the class** of *canonical reducible permutation graphs* (precisely the graphs produced by Chroni and Nikolopoulos's encoding algorithm)
2. **characterization** and **linear-time recognition algorithm** for such graphs
3. a **new linear-time decoding algorithm** (graph → integer key) simpler, marginally faster and able to retrieve the correct key even after the malicious removal of $k \leq 2$ edges
4. a **tight bound for the resilience of the codec** against edge removals

Characterizing the watermark graphs

(*canonical reducible permutation graphs*)

Theorem (characterization)

Canonical reducible permutation graph

⇕

Self-labeling reducible flow graph such that:

- *its fixed element is 2n+1, and*
  its representative tree is a “type-1” tree

or

- *its fixed element belongs to [n+2, 2n], and*
  its representative tree is a “type-2” tree

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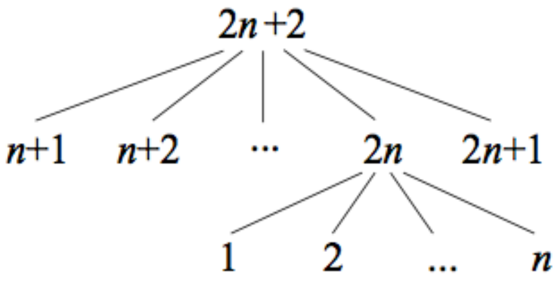
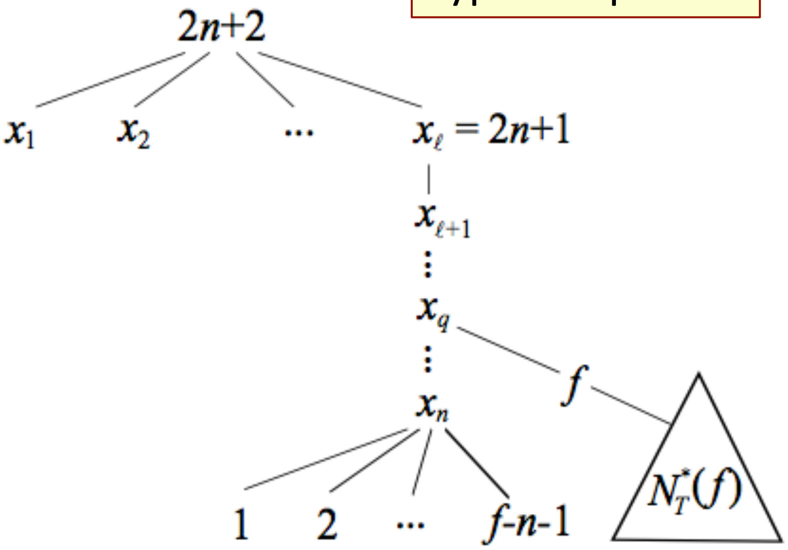
type-1 rep. tree



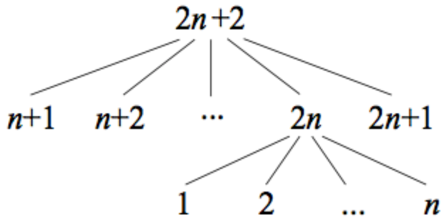
type-2 rep. tree



Types of representative trees

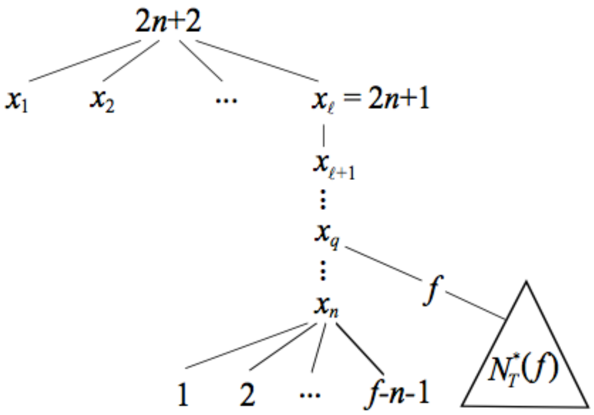
type-1

(i) $n+1, n+2, \ldots, 2n+1$ are children of the root $2n+2$ in T; and
(ii) $1, 2, \ldots, n$ are children of $2n$.



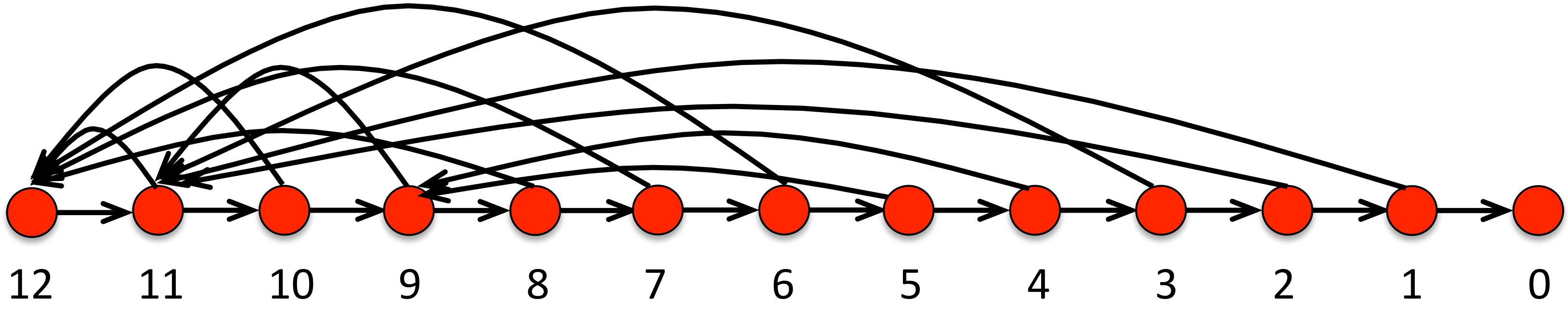
type-2

(i) $n+1 = x_1 < x_2 < \ldots < x_\ell = 2n+1$ are the children of $2n+2$, for some $\ell \in [2, n-1]$;
(ii) $x_i > x_{i+1}$ and x_i is the parent of x_{i+1}, for all $i \in [\ell, n-1]$;
(iii) $1, 2, \ldots, f-n-1$ are children of x_n;
(iv) $x_i = n+i$, for $1 \leq i \leq f-n-1$;
(v) f is a child of x_q, for some $q \in [\ell, n]$ satisfying $x_{q+1} < f$ whenever $q < n$; and
(vi) $N_T^*(f) = \{f-n, f-n+1, \ldots, n\}$ and $y_i \in N_T^*(f)$ has index $x_{y_i} - f + 1$ in the preorder traversal of $N_T^*[f]$.



(f denotes the unique fixed element)

Linear-time recognition



Due to the characterization theorem, it is an easy task to recognize a canonical reducible permutation graph.

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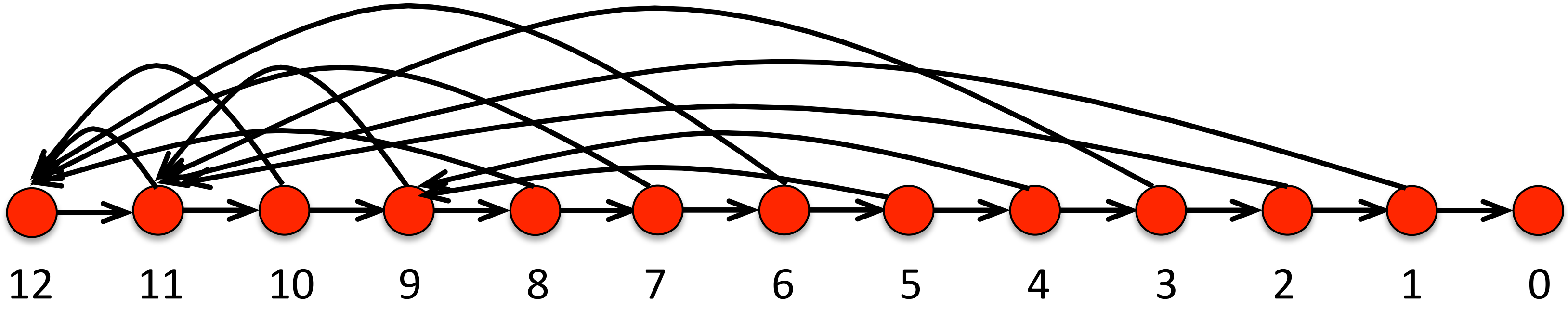

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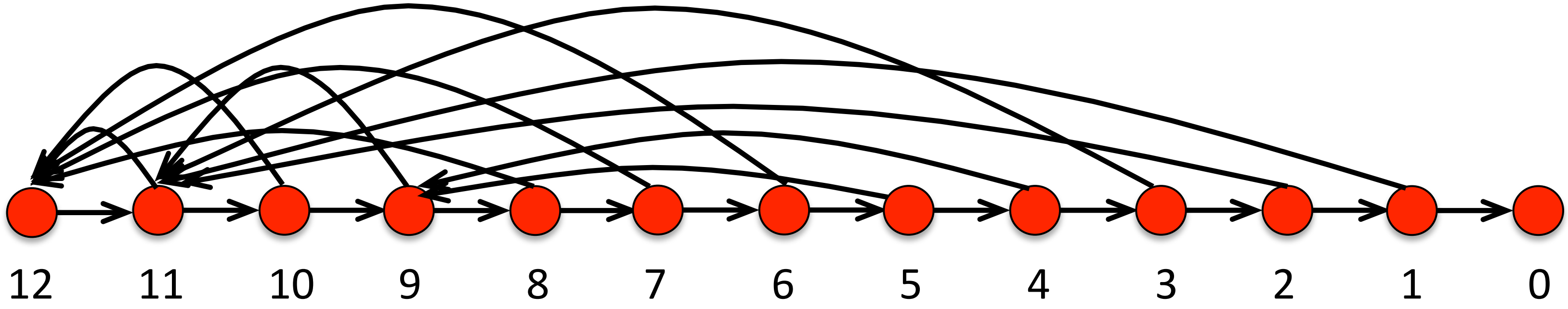

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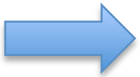
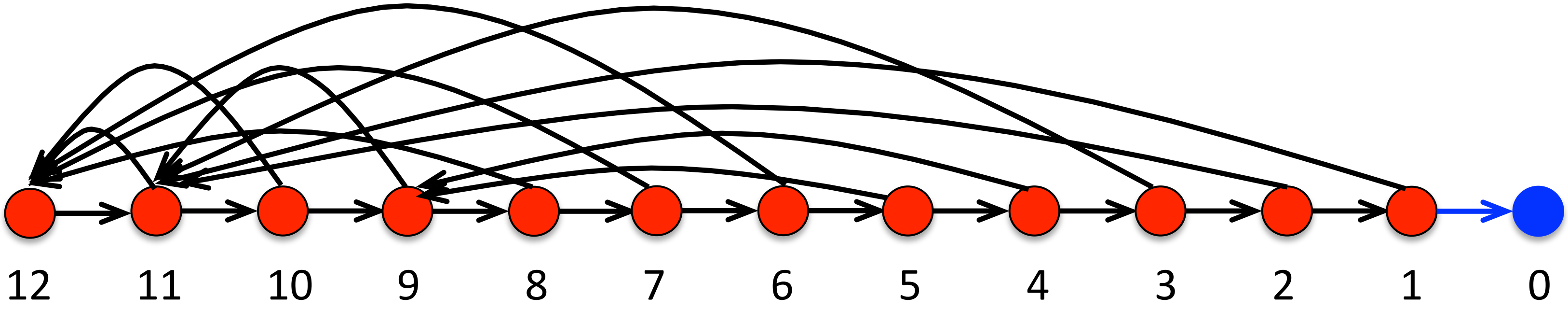


Linear-time algorithm to find the unique Hamiltonian path

Linear-time recognition



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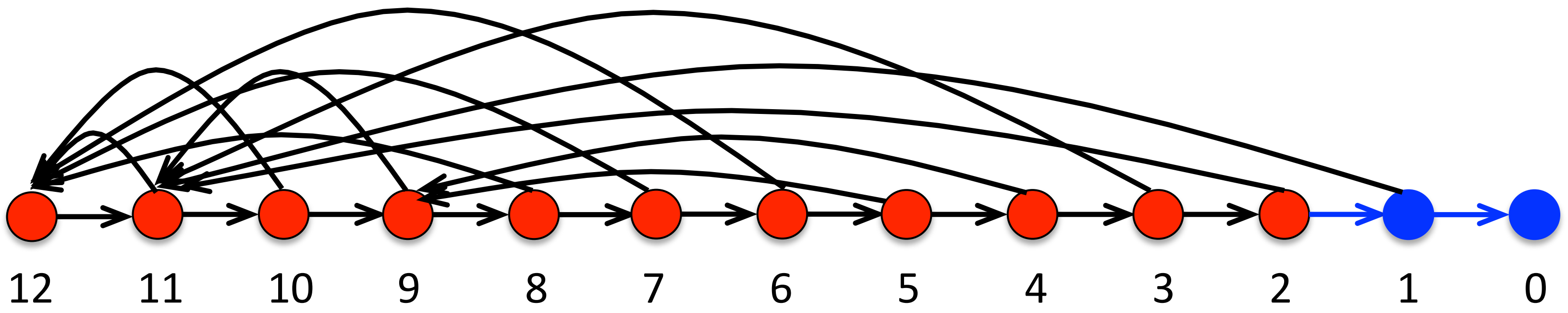

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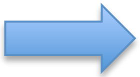

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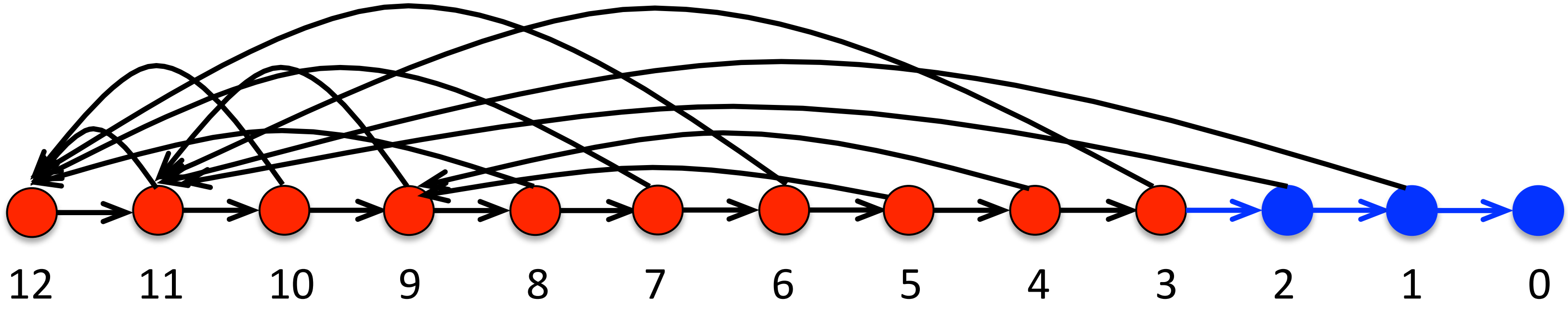

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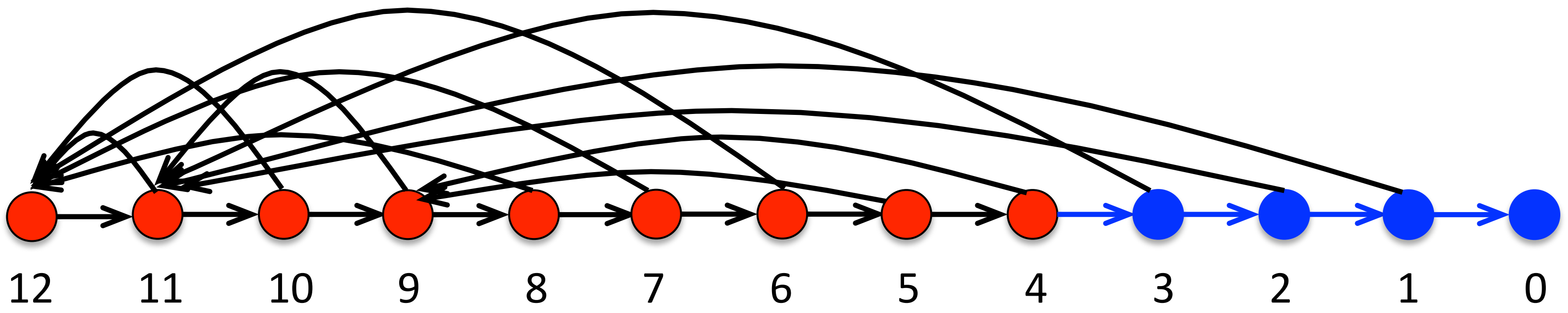

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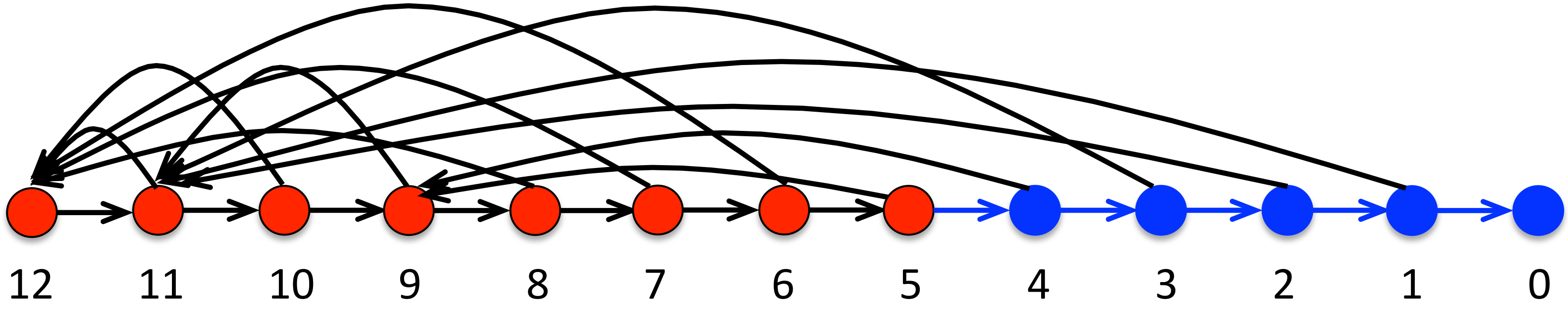

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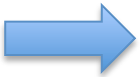

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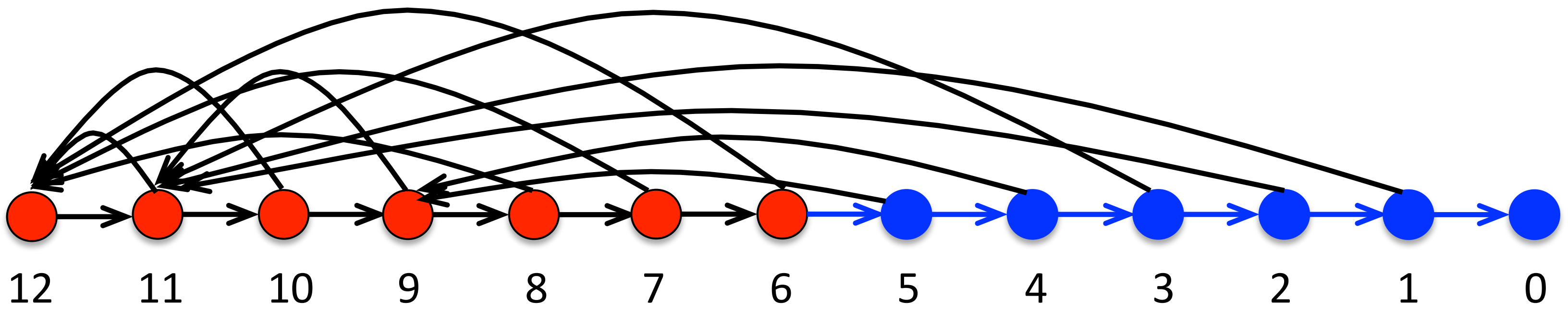

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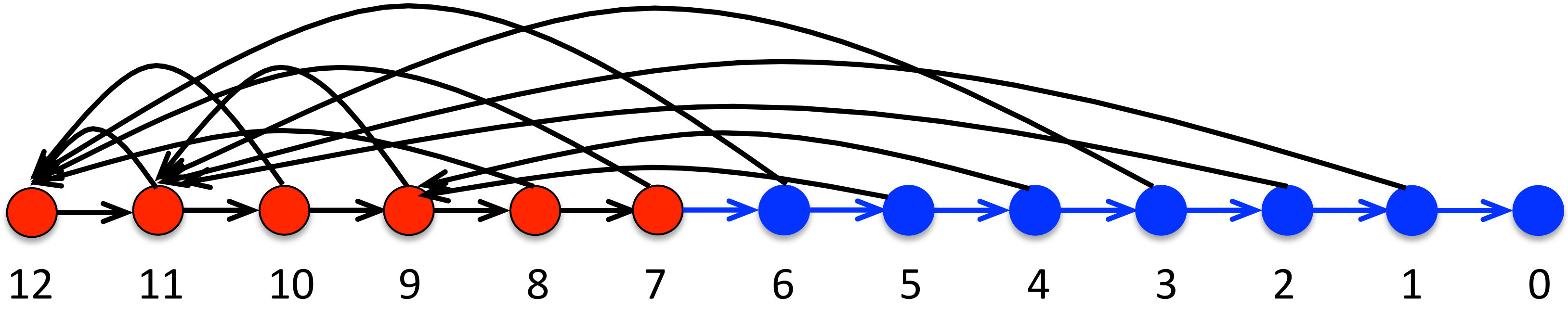

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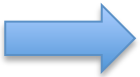

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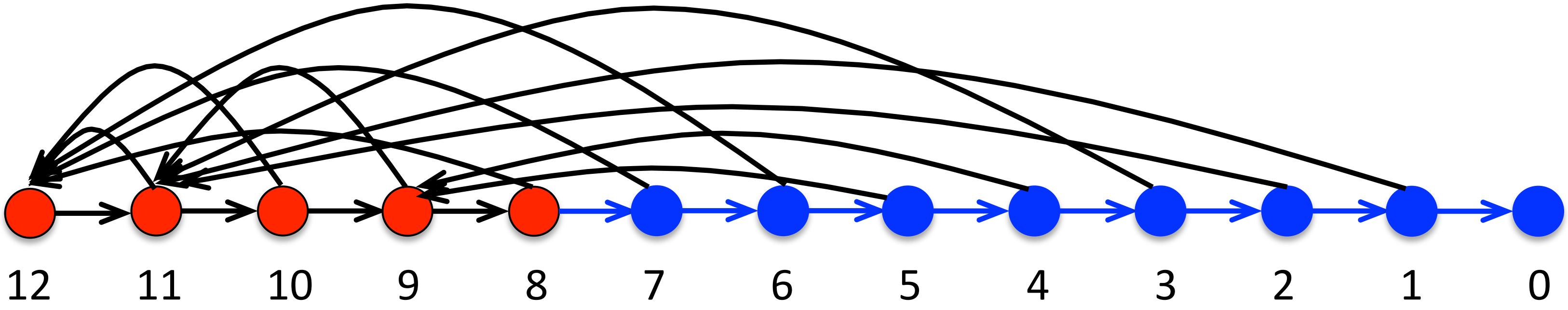

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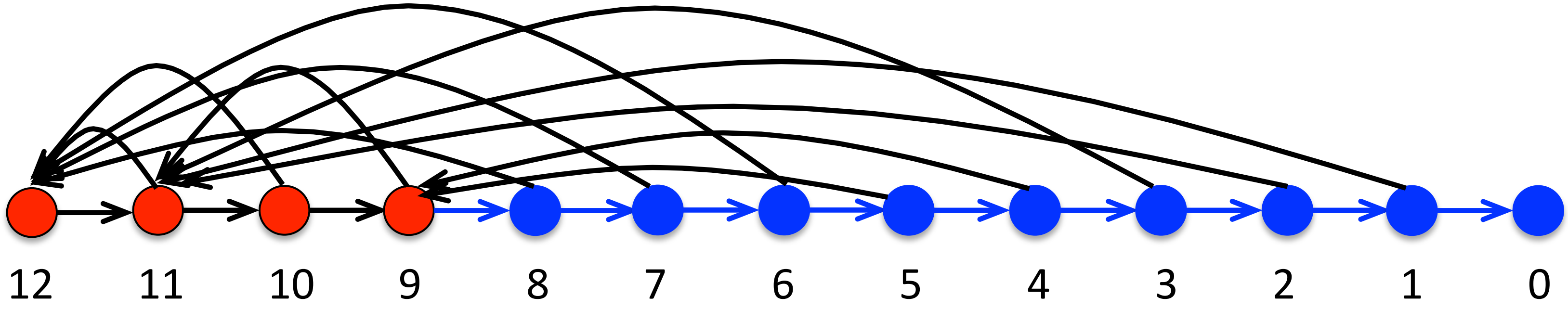

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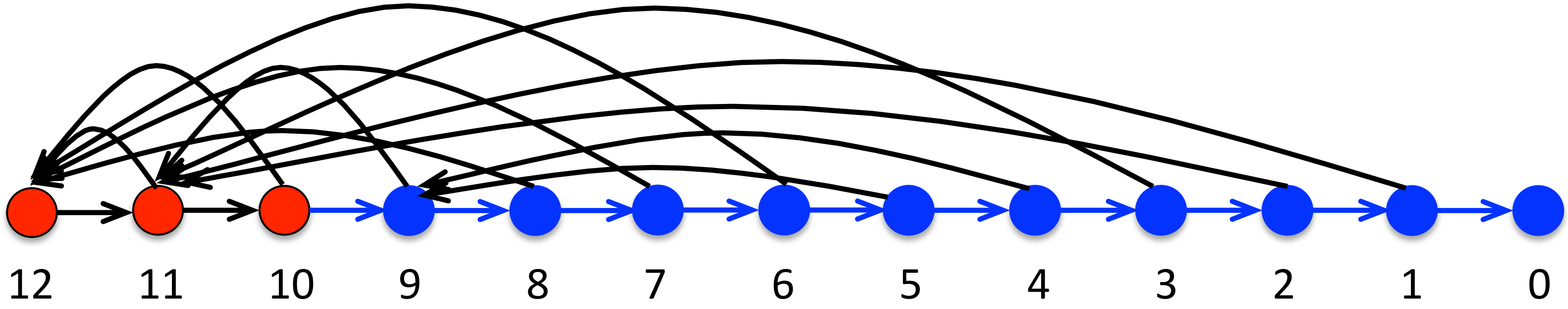

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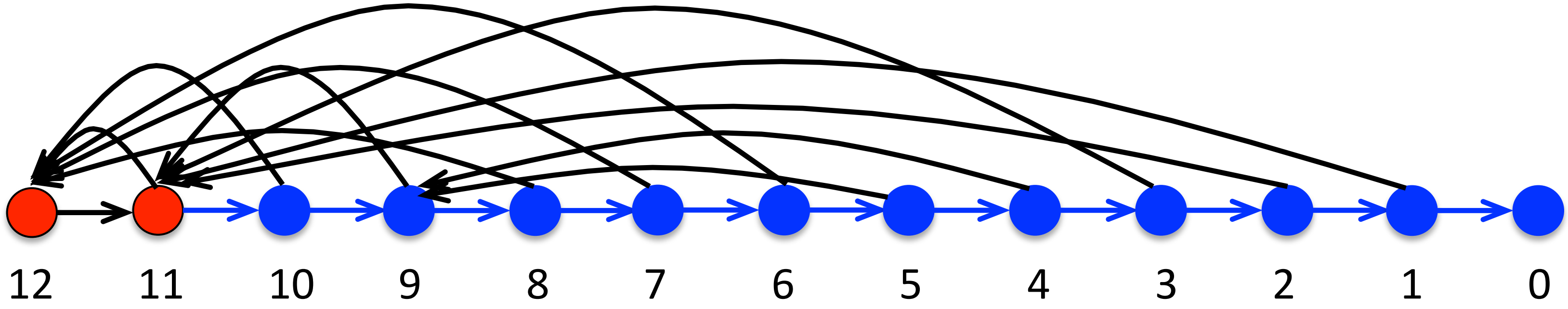

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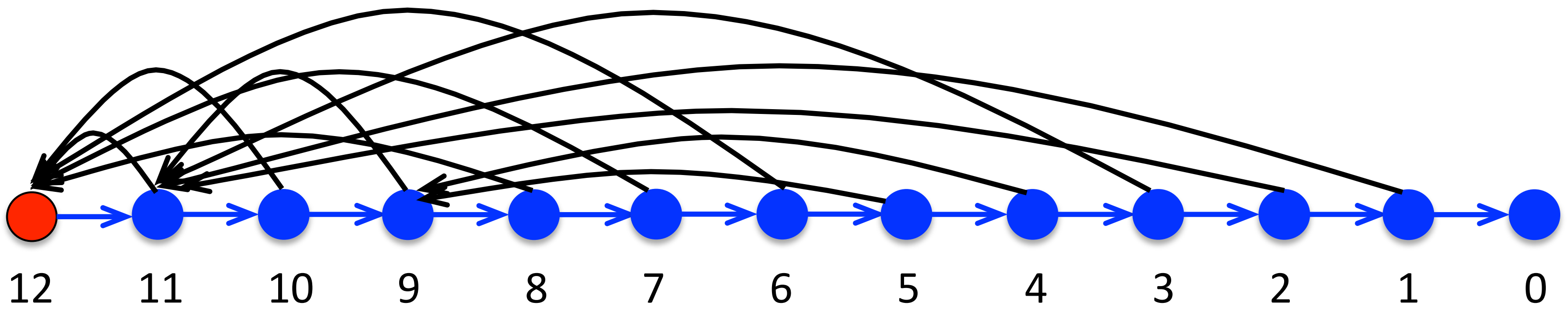

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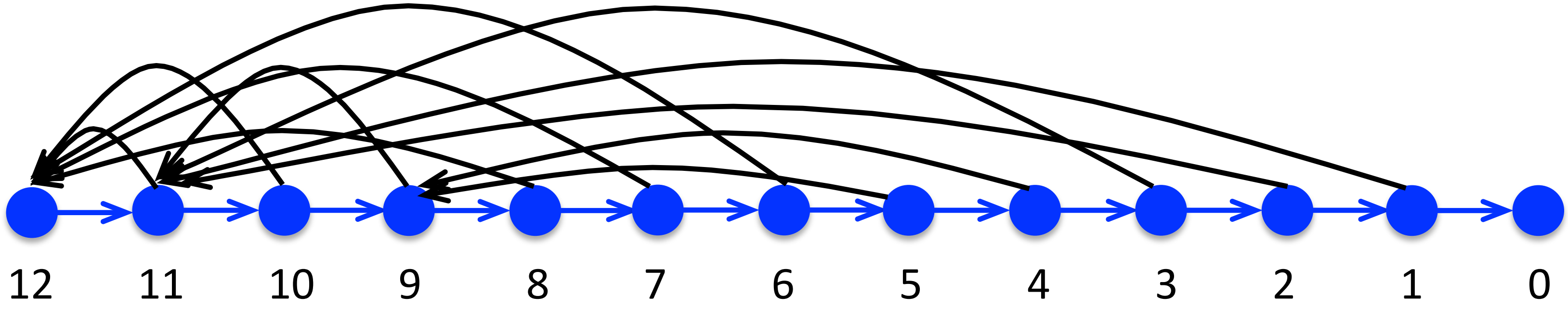

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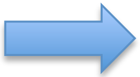

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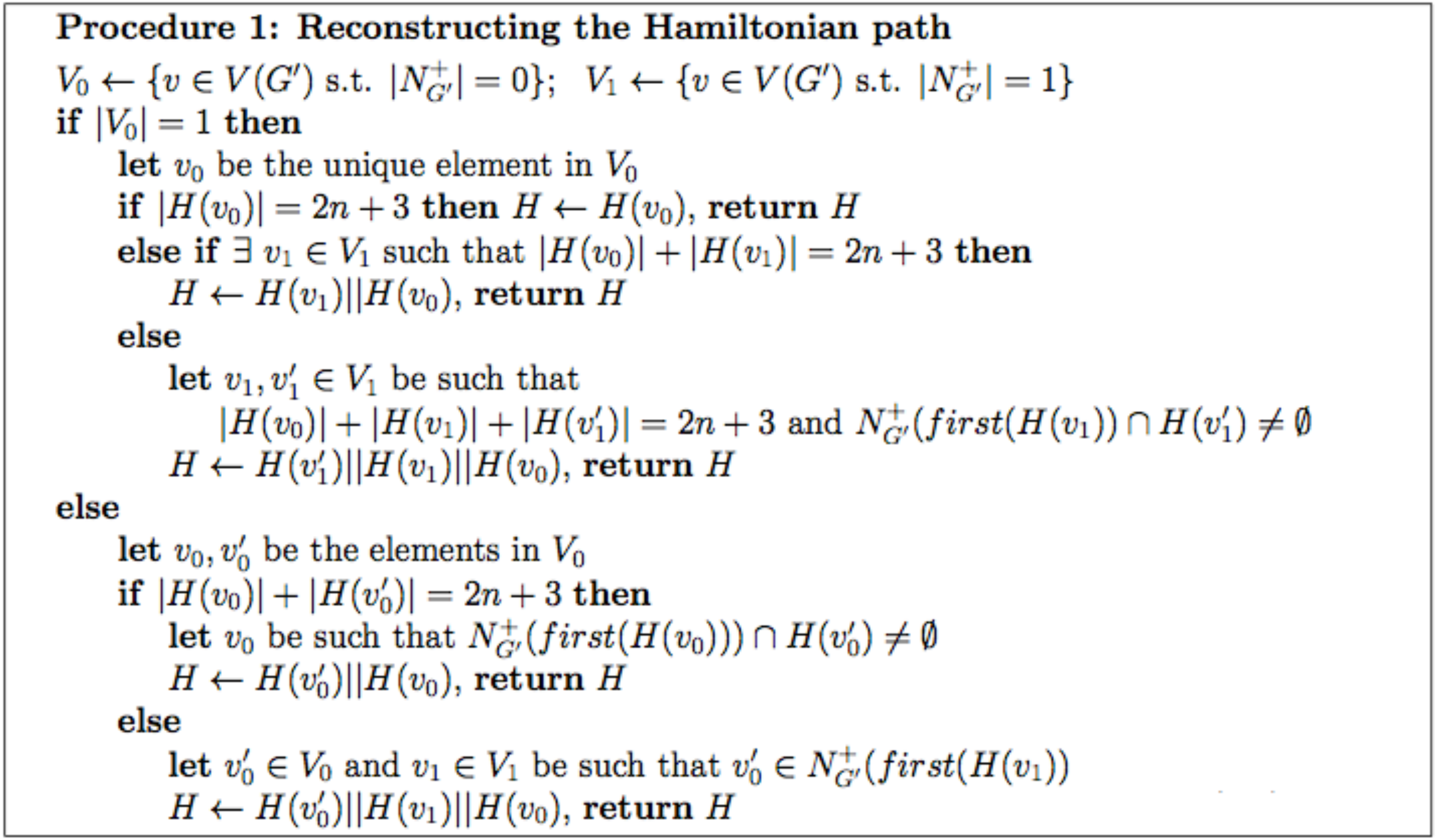

```
Procedure 1: Reconstructing the Hamiltonian path
$V_0 \leftarrow \{v \in V(G') \text{ s.t. } |N^+_{G'}| = 0\}$;  $V_1 \leftarrow \{v \in V(G') \text{ s.t. } |N^+_{G'}| = 1\}$
if $|V_0| = 1$ then
    let $v_0$ be the unique element in $V_0$
    if $|H(v_0)| = 2n + 3$ then $H \leftarrow H(v_0)$, return $H$
    else if $\exists\, v_1 \in V_1$ such that $|H(v_0)| + |H(v_1)| = 2n + 3$ then
        $H \leftarrow H(v_1)||H(v_0)$, return $H$
    else
        let $v_1, v_1' \in V_1$ be such that
            $|H(v_0)| + |H(v_1)| + |H(v_1')| = 2n + 3$ and $N^+_{G'}(first(H(v_1)) \cap H(v_1') \neq \emptyset$
        $H \leftarrow H(v_1')||H(v_1)||H(v_0)$, return $H$
else
    let $v_0, v_0'$ be the elements in $V_0$
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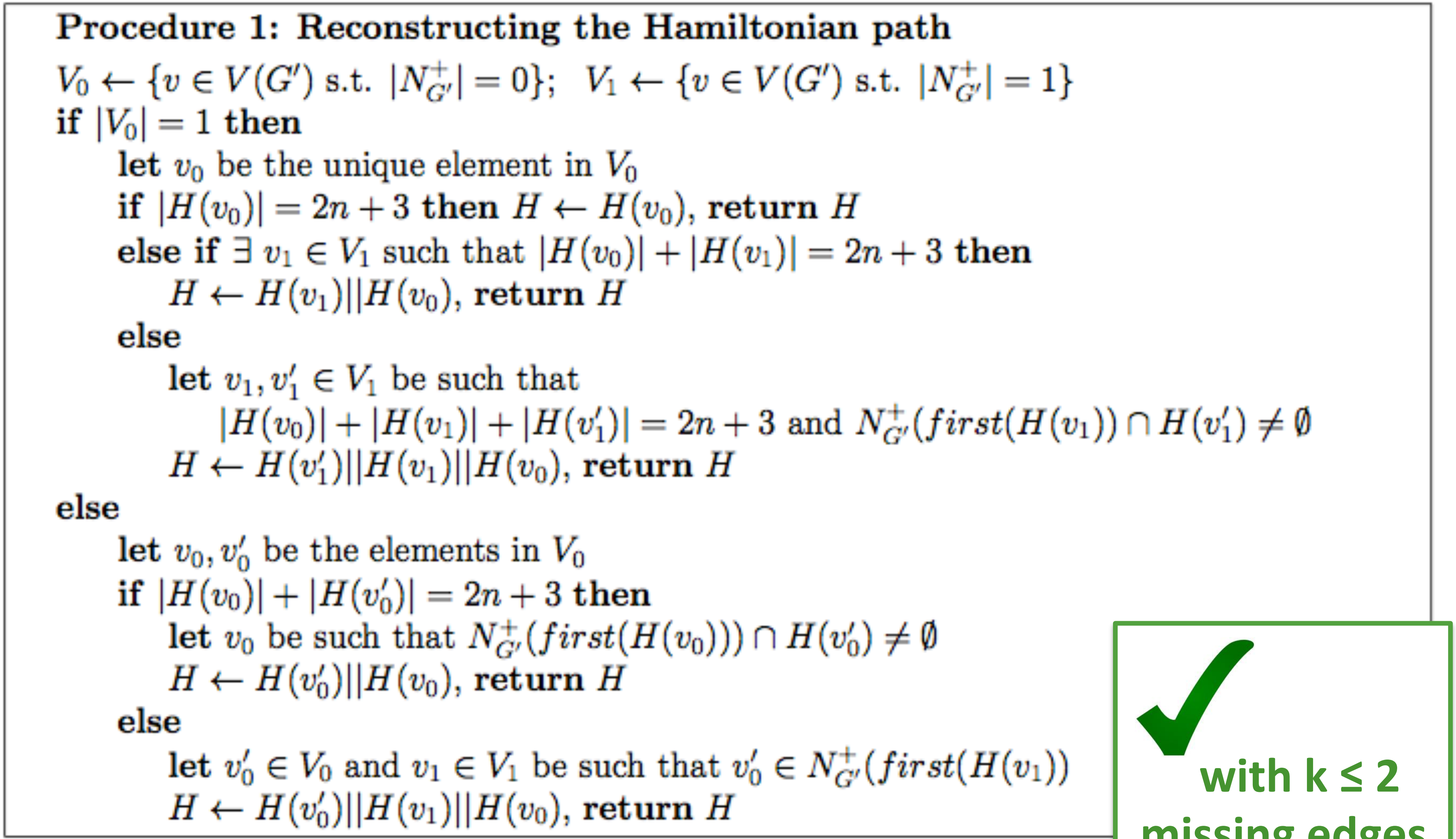
        $H \leftarrow H(v_0')||H(v_0)$, return $H$
    else
        let $v_0' \in V_0$ and $v_1 \in V_1$ be such that $v_0' \in N^+_{G'}(first(H(v_1))$
        $H \leftarrow H(v_0')||H(v_1)||H(v_0)$, return $H$
```

Linear-time recognition



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        $H \leftarrow H(v'_1)||H(v_1)||H(v_0)$, return $H$
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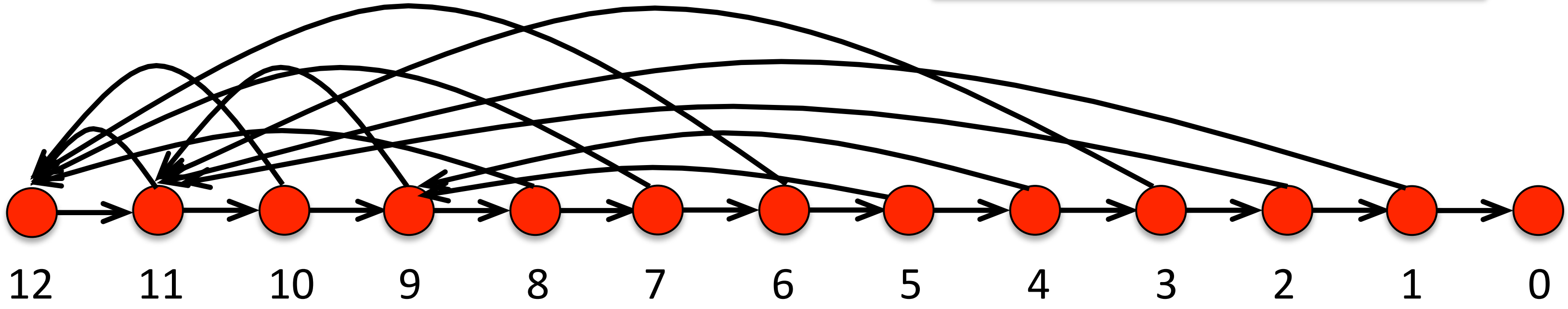
        $H \leftarrow H(v'_0)||H(v_1)||H(v_0)$, return $H$
```

✓ **with k ≤ 2 missing edges**

Our contribution

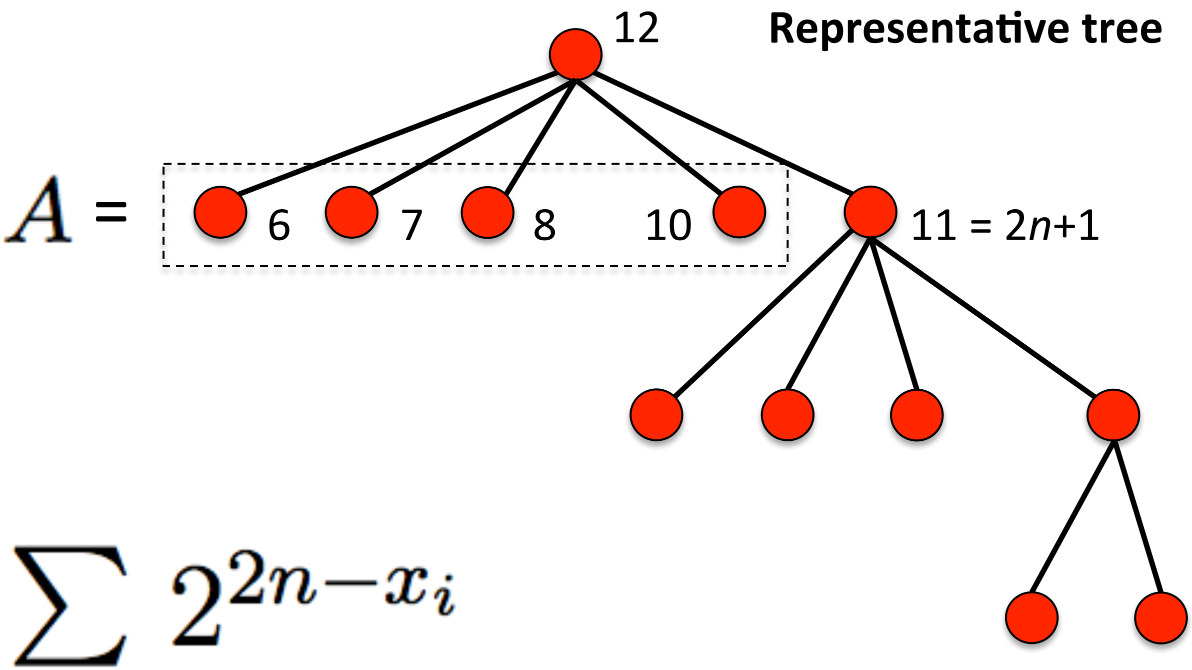
B., B., M., S., S. (WG 2013)



1. **formal definition of the class** of *canonical reducible permutation graphs* (precisely the graphs produced by Chroni and Nikolopoulos's encoding algorithm)
2. **characterization** and **linear-time recognition algorithm** for such graphs
3. a **new linear-time decoding algorithm** (graph → integer key) simpler, marginally faster and able to retrieve the correct key even after the malicious removal of $k \leq 2$ edges
4. a **tight bound for the resilience of the codec** against edge removals

A new decoding algorithm

1. find the unique Hamiltonian path and label the vertices accordingly
2. find the fixed element *f*
3. find the set *A* of the child nodes of the root of the representative tree that are different from 2*n*+1
4. calculate the key as follows



$$\omega = \sum_{x_i \in A} 2^{2n - x_i}$$

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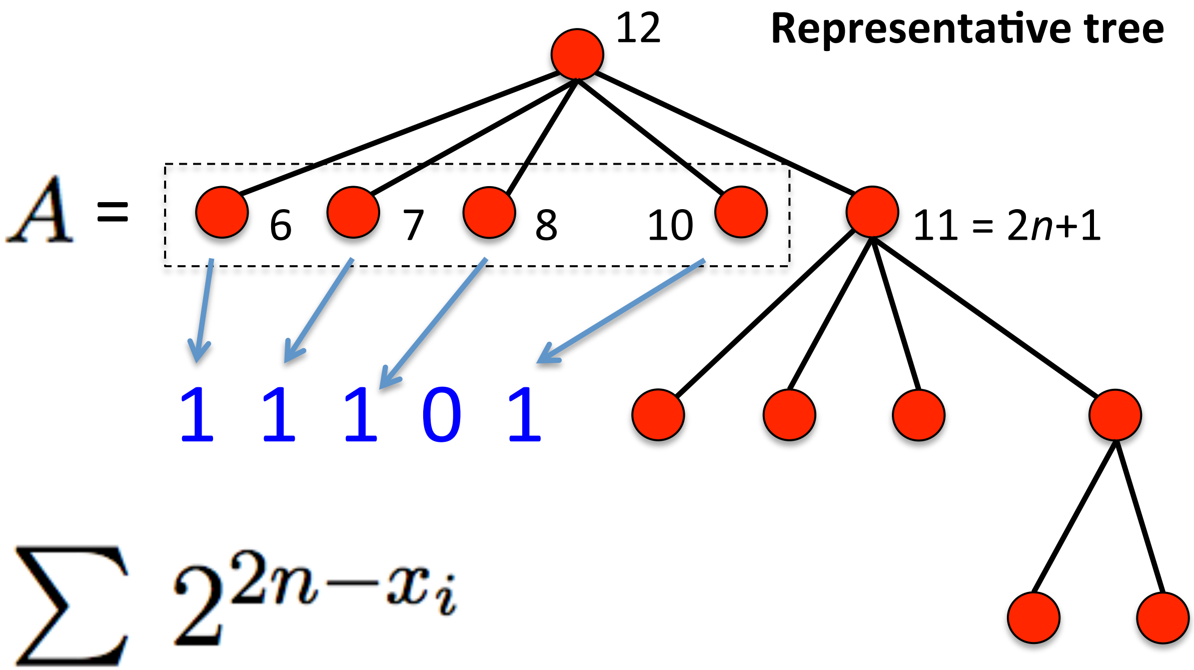

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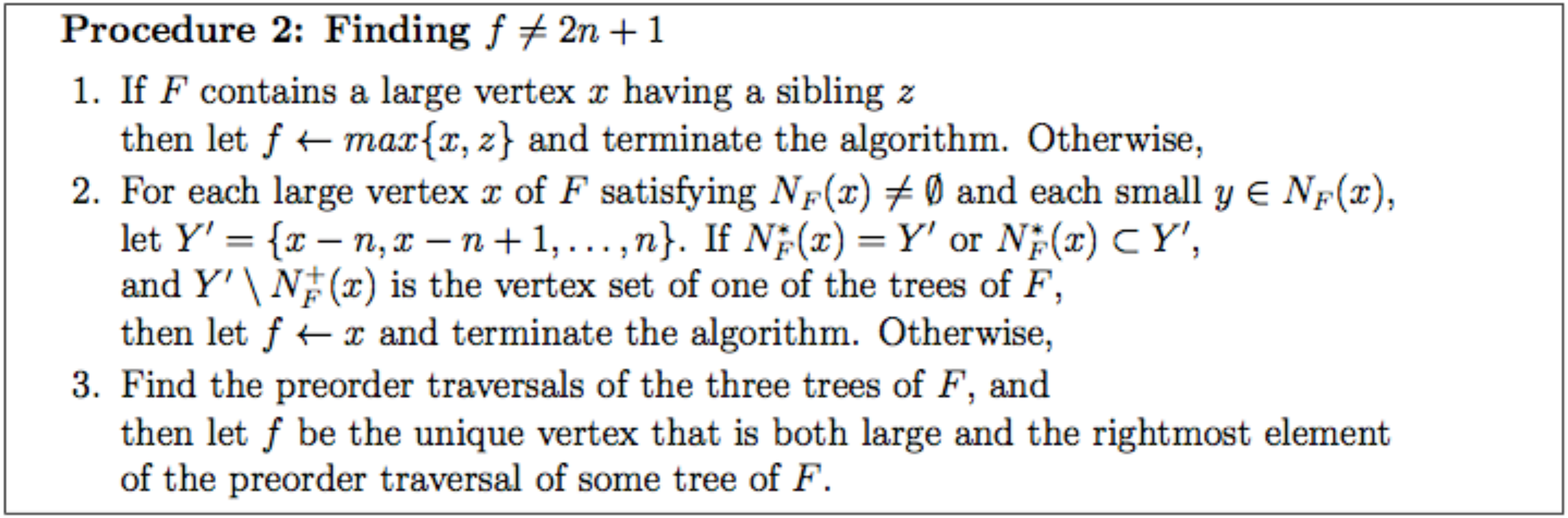

Procedure 2: Finding $f \neq 2n+1$

1. If F contains a large vertex x having a sibling z then let $f \leftarrow max\{x, z\}$ and terminate the algorithm. Otherwise,
2. For each large vertex x of F satisfying $N_F(x) \neq \emptyset$ and each small $y \in N_F(x)$, let $Y' = \{x-n, x-n+1, \ldots, n\}$. If $N_F^*(x) = Y'$ or $N_F^*(x) \subset Y'$, and $Y' \setminus N_F^+(x)$ is the vertex set of one of the trees of F, then let $f \leftarrow x$ and terminate the algorithm. Otherwise,
3. Find the preorder traversals of the three trees of F, and then let f be the unique vertex that is both large and the rightmost element of the preorder traversal of some tree of F.

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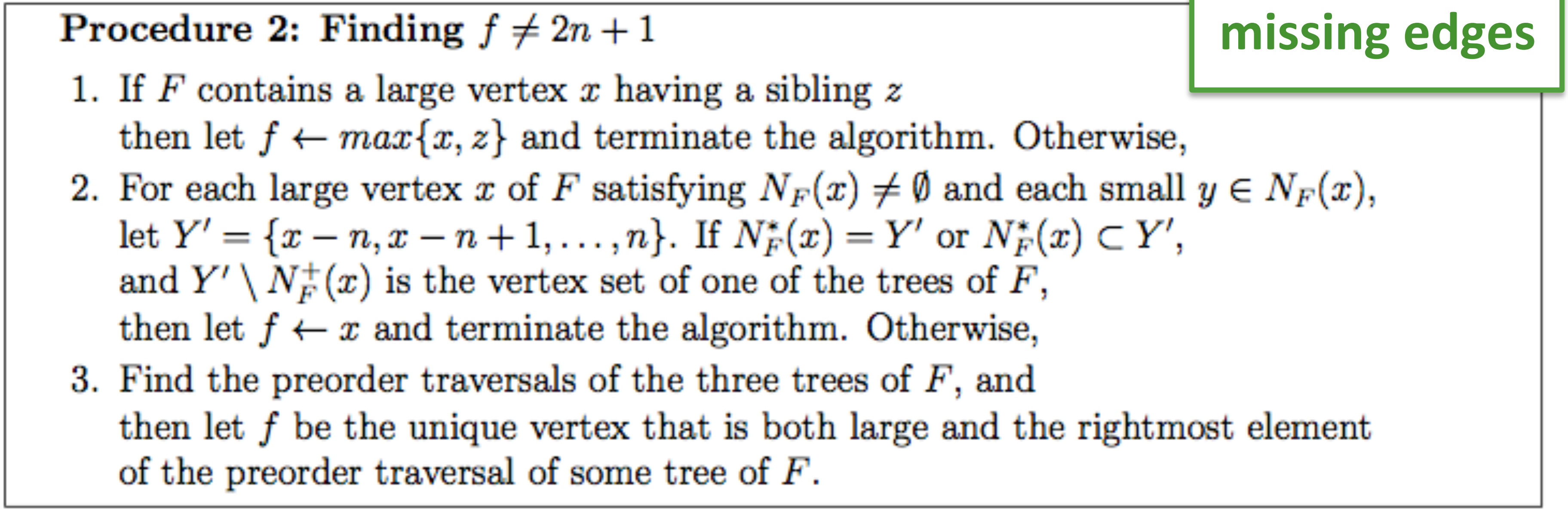

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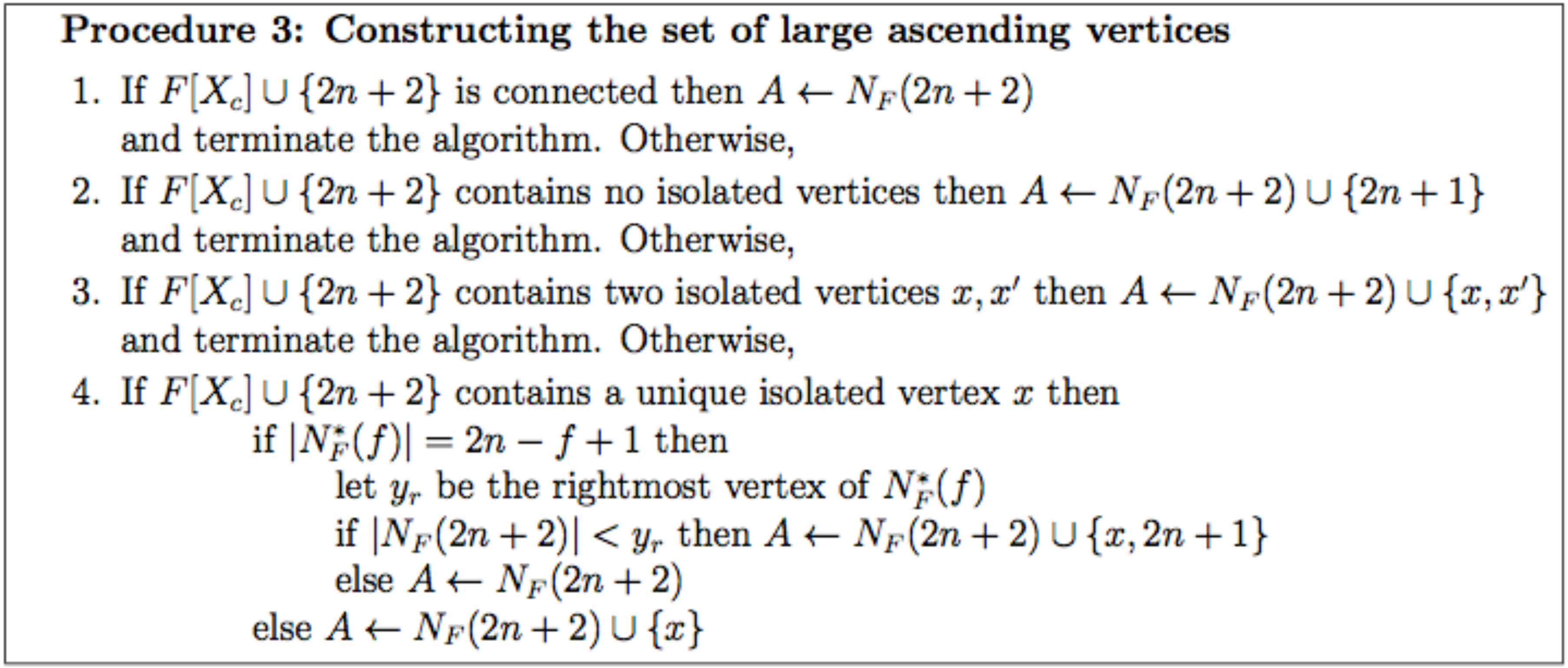

Procedure 3: Constructing the set of large ascending vertices

```
1. If $F[X_c] \cup \{2n+2\}$ is connected then $A \leftarrow N_F(2n+2)$
   and terminate the algorithm. Otherwise,
2. If $F[X_c] \cup \{2n+2\}$ contains no isolated vertices then $A \leftarrow N_F(2n+2) \cup \{2n+1\}$
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3. If $F[X_c] \cup \{2n+2\}$ contains two isolated vertices $x, x'$ then $A \leftarrow N_F(2n+2) \cup \{x, x'\}$
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4. If $F[X_c] \cup \{2n+2\}$ contains a unique isolated vertex $x$ then
           if $|N_F^*(f)| = 2n - f + 1$ then
               let $y_r$ be the rightmost vertex of $N_F^*(f)$
               if $|N_F(2n+2)| < y_r$ then $A \leftarrow N_F(2n+2) \cup \{x, 2n+1\}$
               else $A \leftarrow N_F(2n+2)$
           else $A \leftarrow N_F(2n+2) \cup \{x\}$
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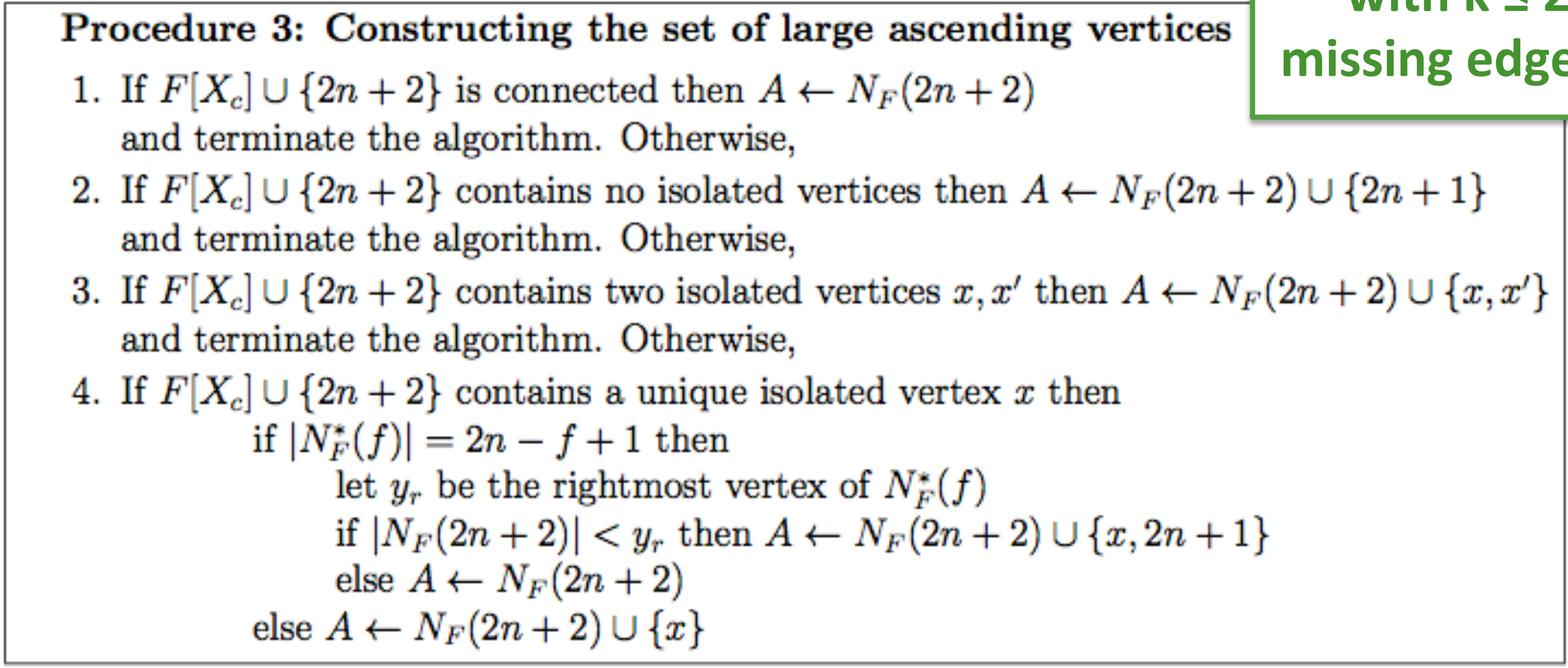

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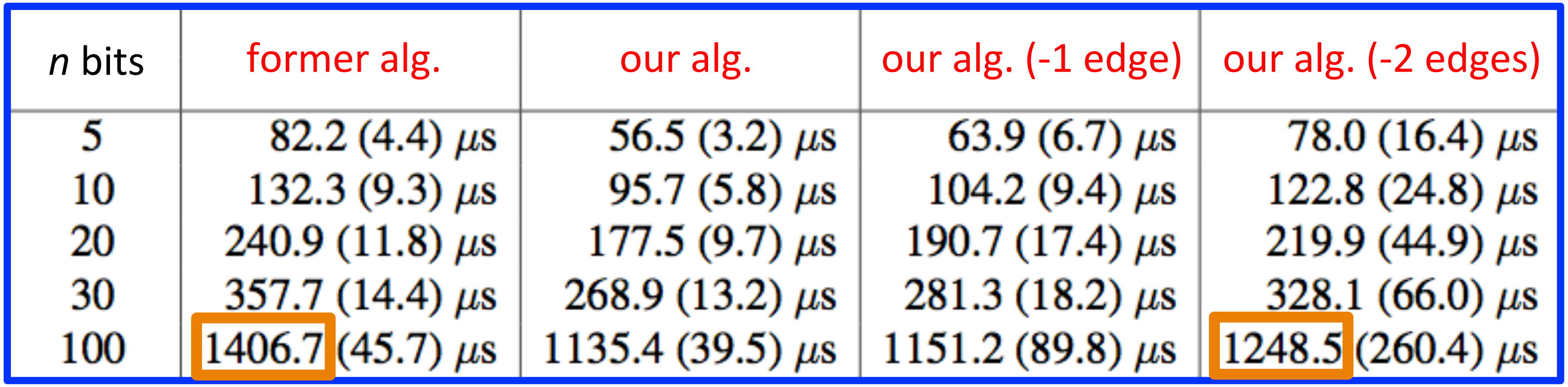
Experimental results

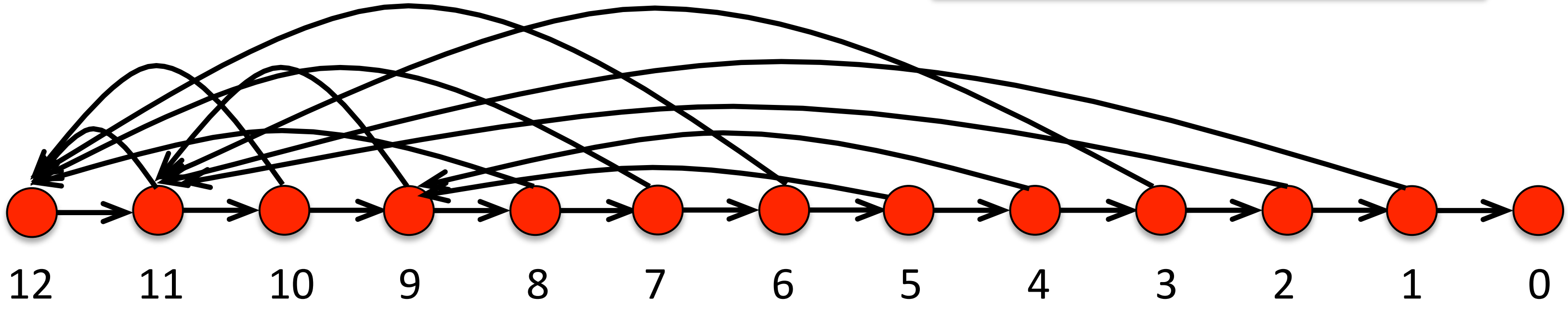


n bits	former alg.	our alg.	our alg. (-1 edge)	our alg. (-2 edges)
5	82.2 (4.4) μs	56.5 (3.2) μs	63.9 (6.7) μs	78.0 (16.4) μs
10	132.3 (9.3) μs	95.7 (5.8) μs	104.2 (9.4) μs	122.8 (24.8) μs
20	240.9 (11.8) μs	177.5 (9.7) μs	190.7 (17.4) μs	219.9 (44.9) μs
30	357.7 (14.4) μs	268.9 (13.2) μs	281.3 (18.2) μs	328.1 (66.0) μs
100	1406.7 (45.7) μs	1135.4 (39.5) μs	1151.2 (89.8) μs	1248.5 (260.4) μs

average time (standard deviation)

Our contribution

B., B., M., S., S. (WG 2013)



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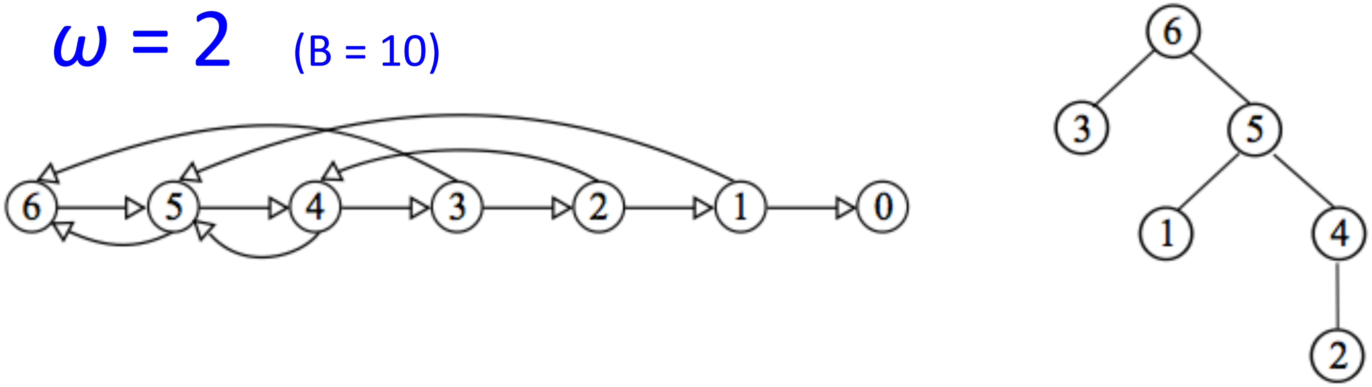
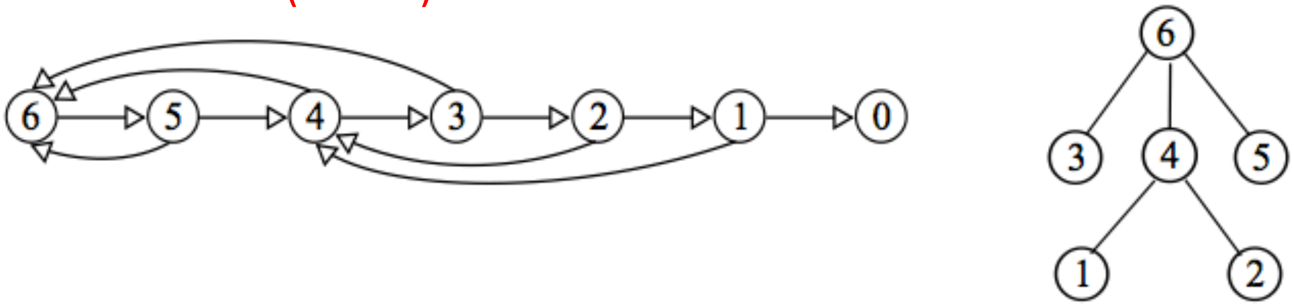
Resilience against edge modifications

$\omega = 2$ (B = 10)



$\omega = 3$ (B = 11)



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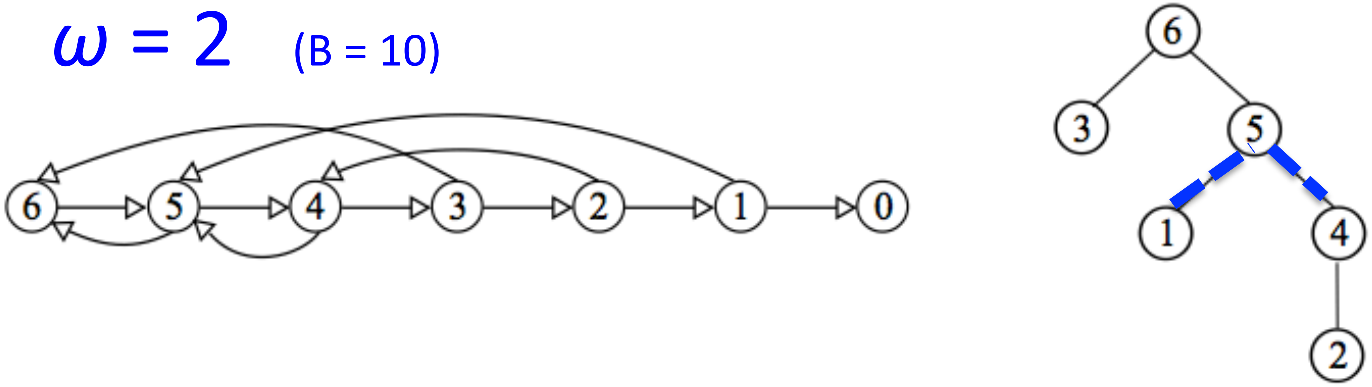

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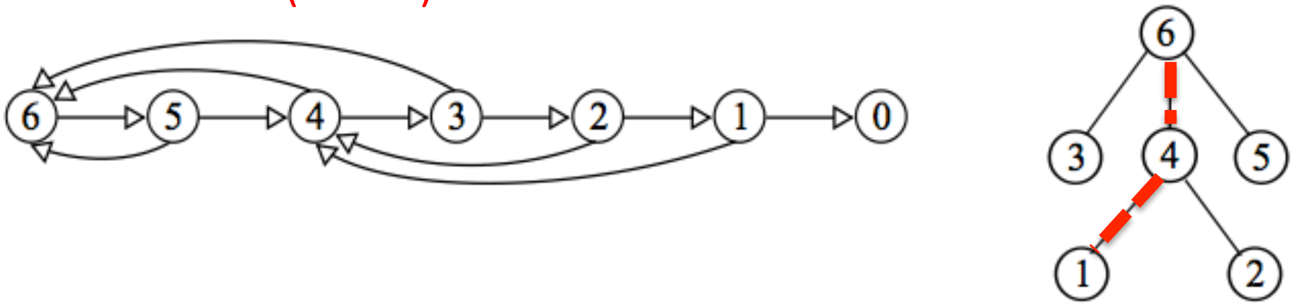

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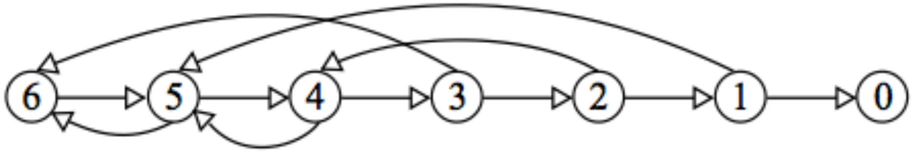

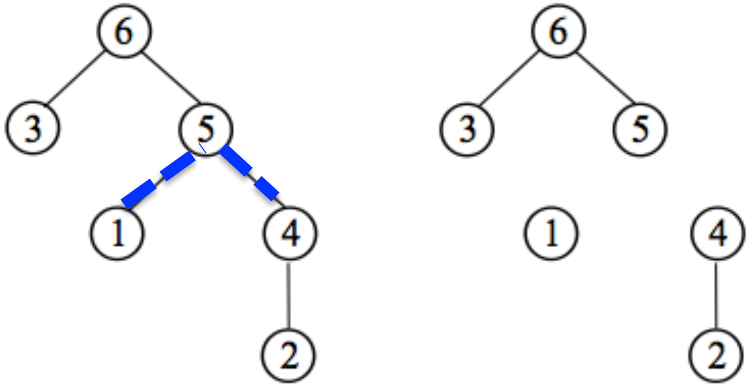

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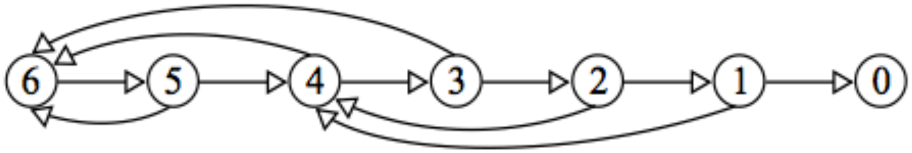

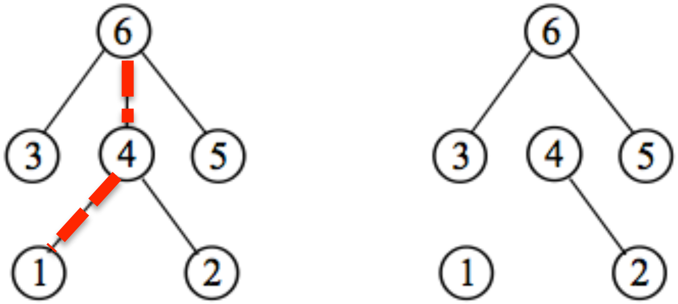

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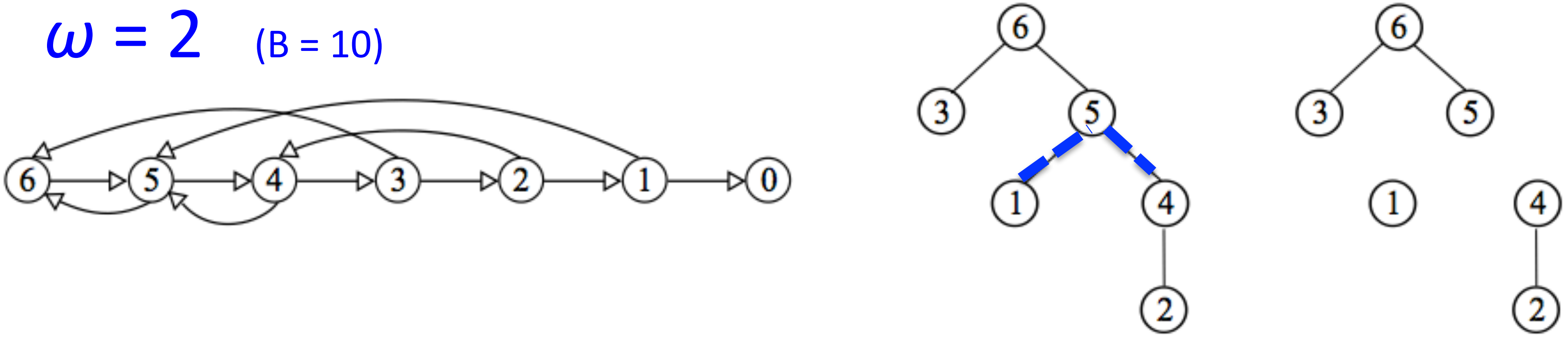

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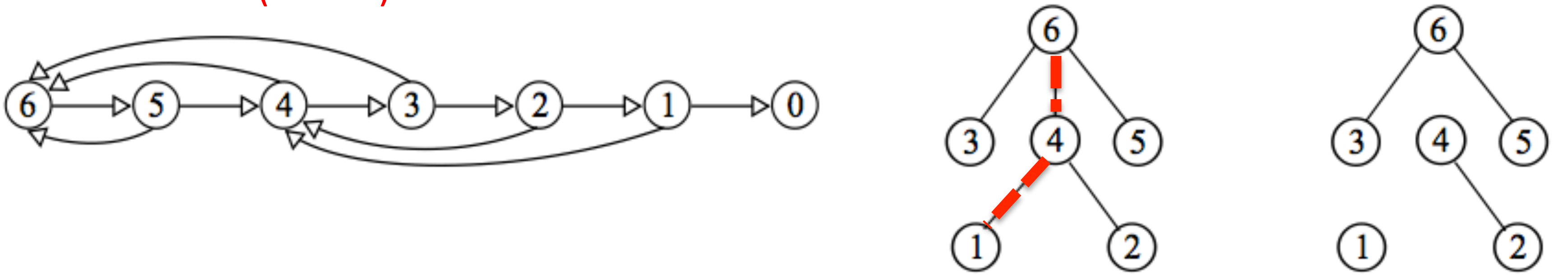
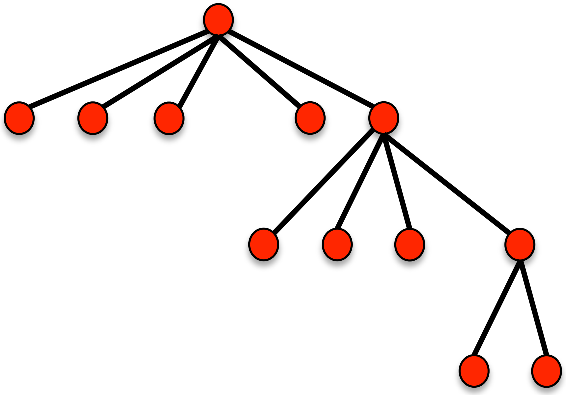


For *n* > 2 bits, it is possible to detect up to 5 edge insertions/deletions in polynomial time.
This bound is tight.



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Towards a provably resilient scheme for graph-based watermarking

Lucila Maria Souza Bento
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Vinícius Gusmão Pereira de Sá
Jayme Luiz Szwarcfiter

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