Linear-time Approximations for Dominating Sets and Independent Dominating Sets in Unit Disk Graphs

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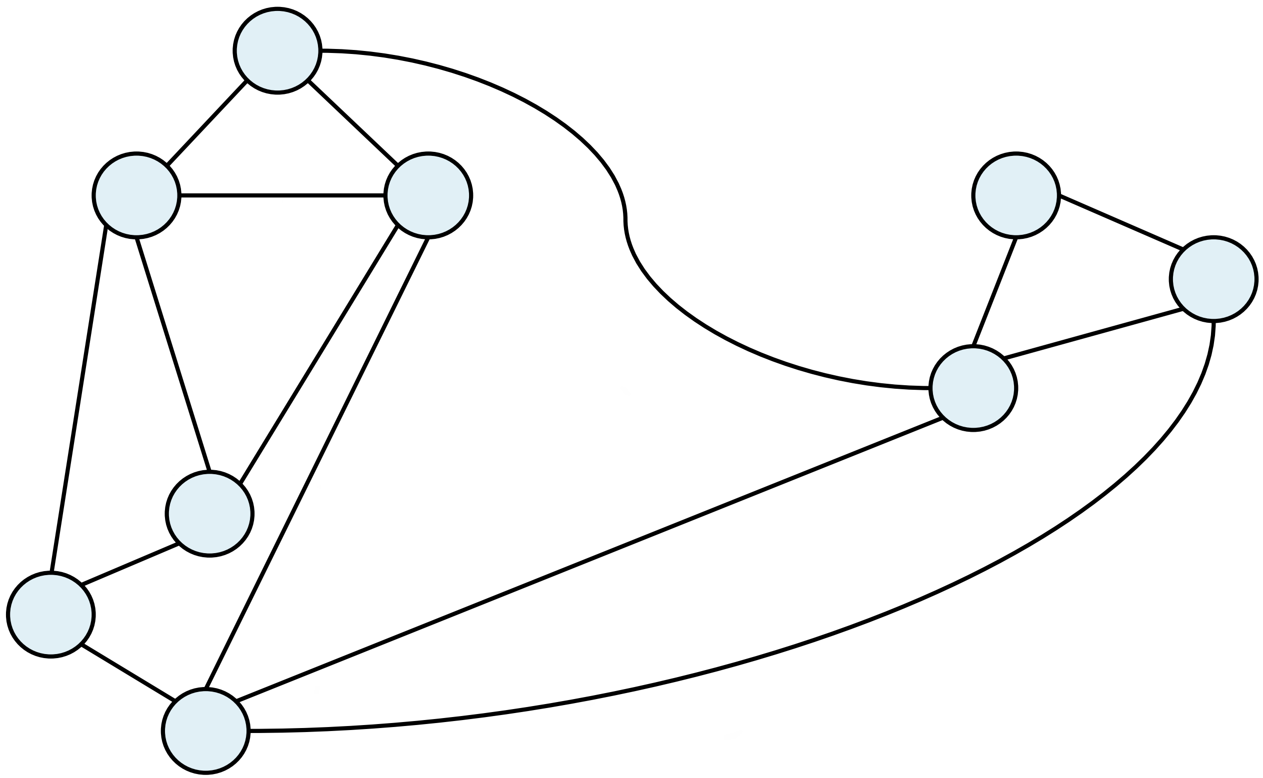
Dominating set

$G\ (V, E)$



$D \subseteq V$

D dominating set $\iff$ $\forall\, w \in V \setminus D,\ \exists\, v \in D \mid vw \in E$

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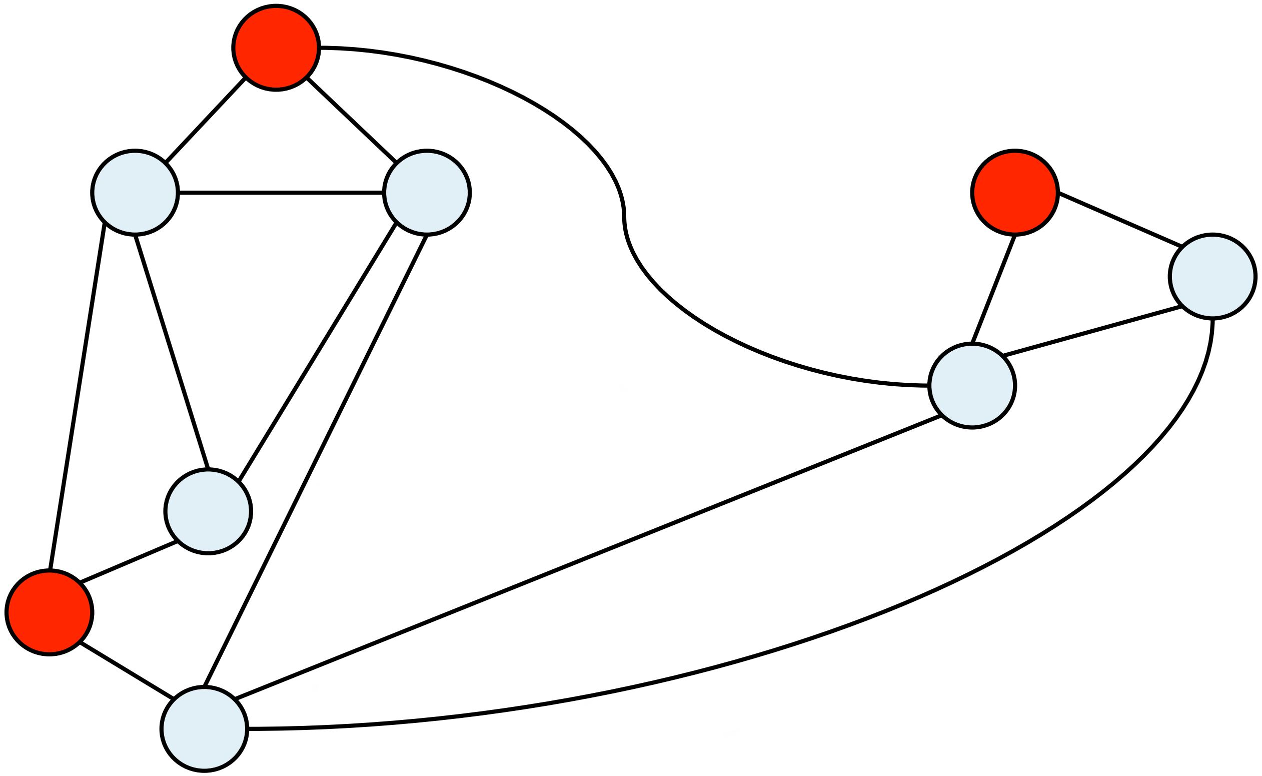

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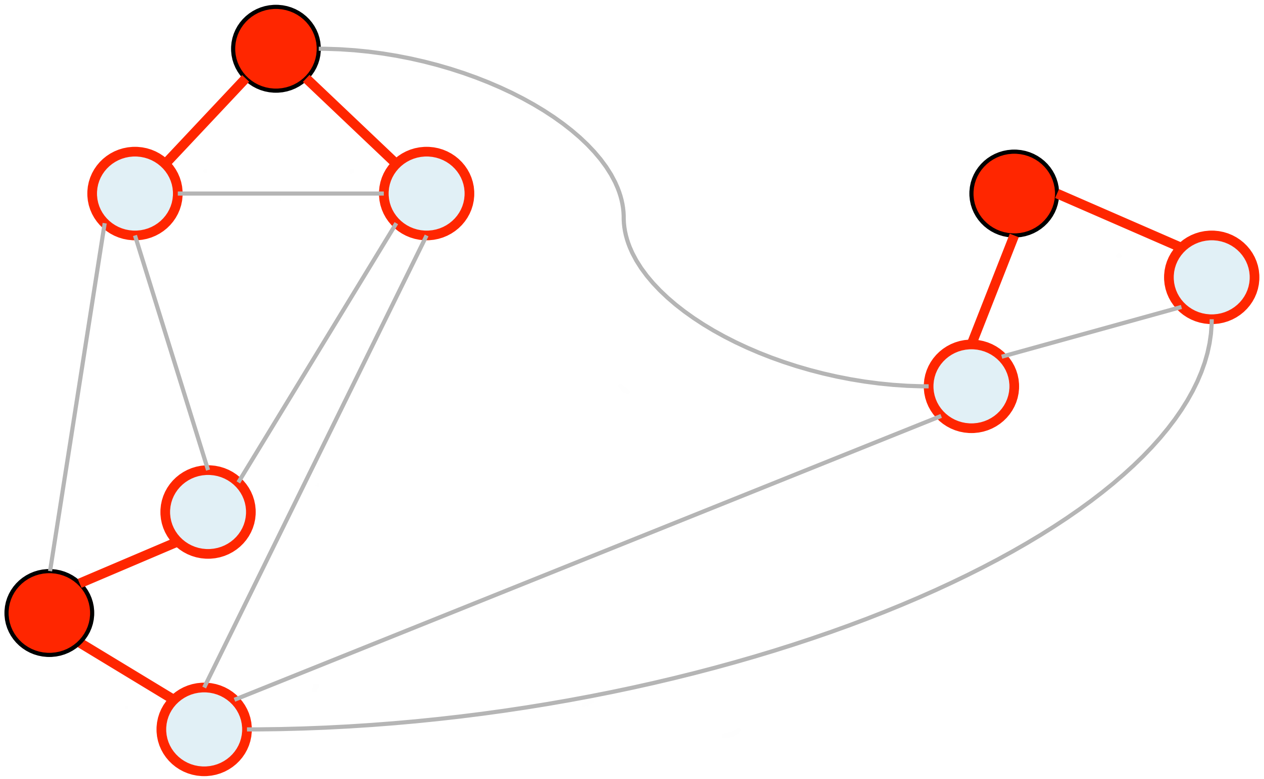

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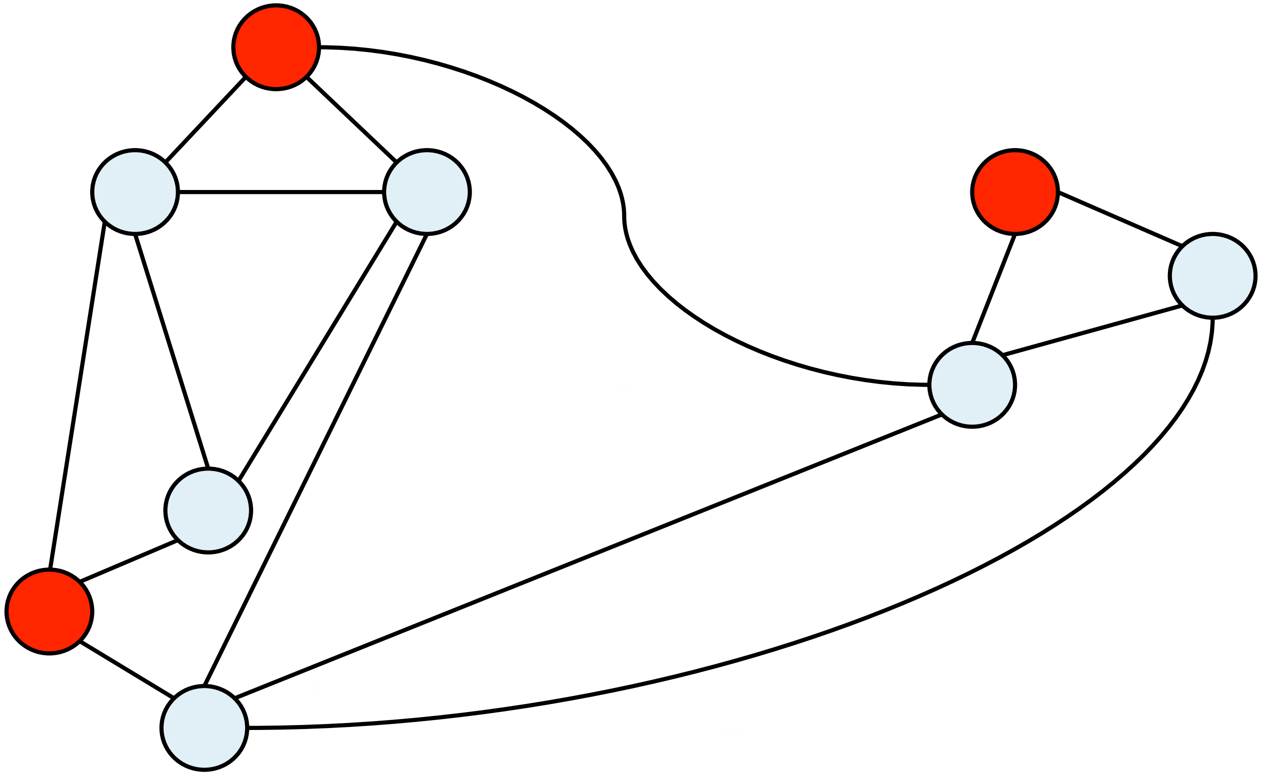

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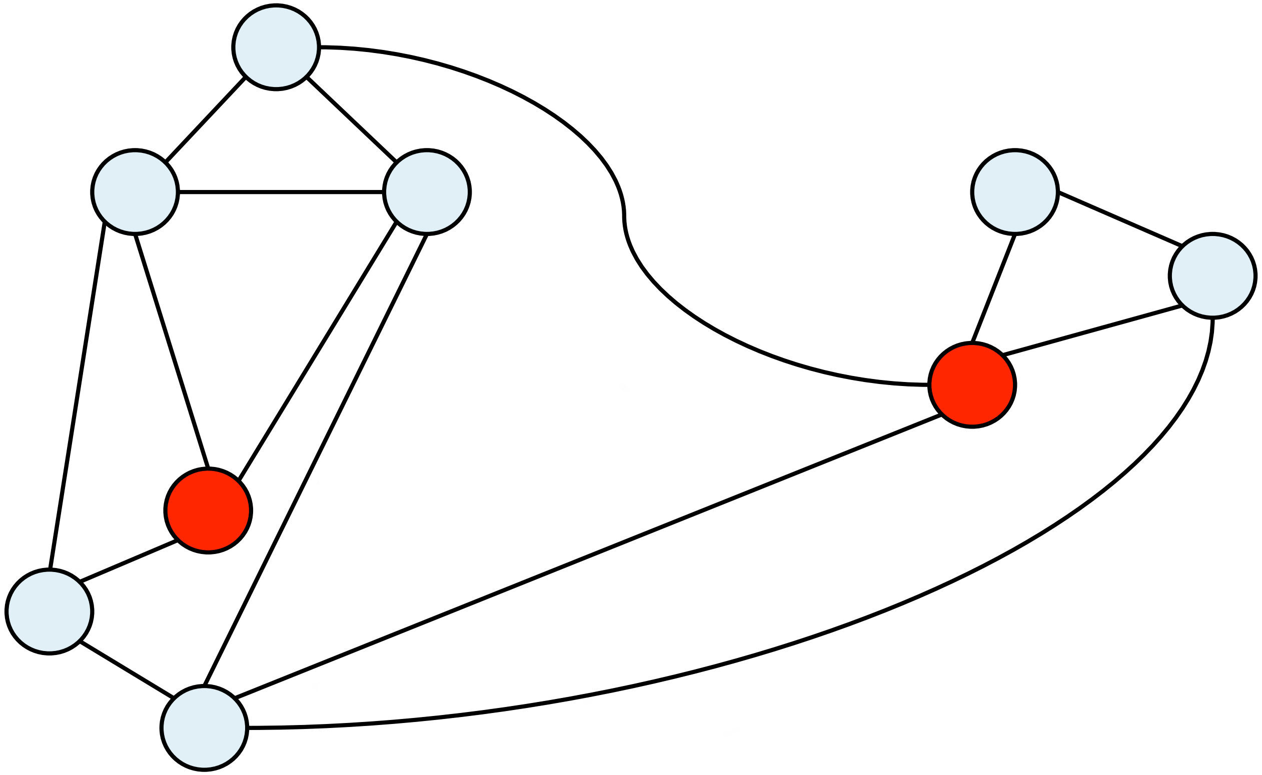

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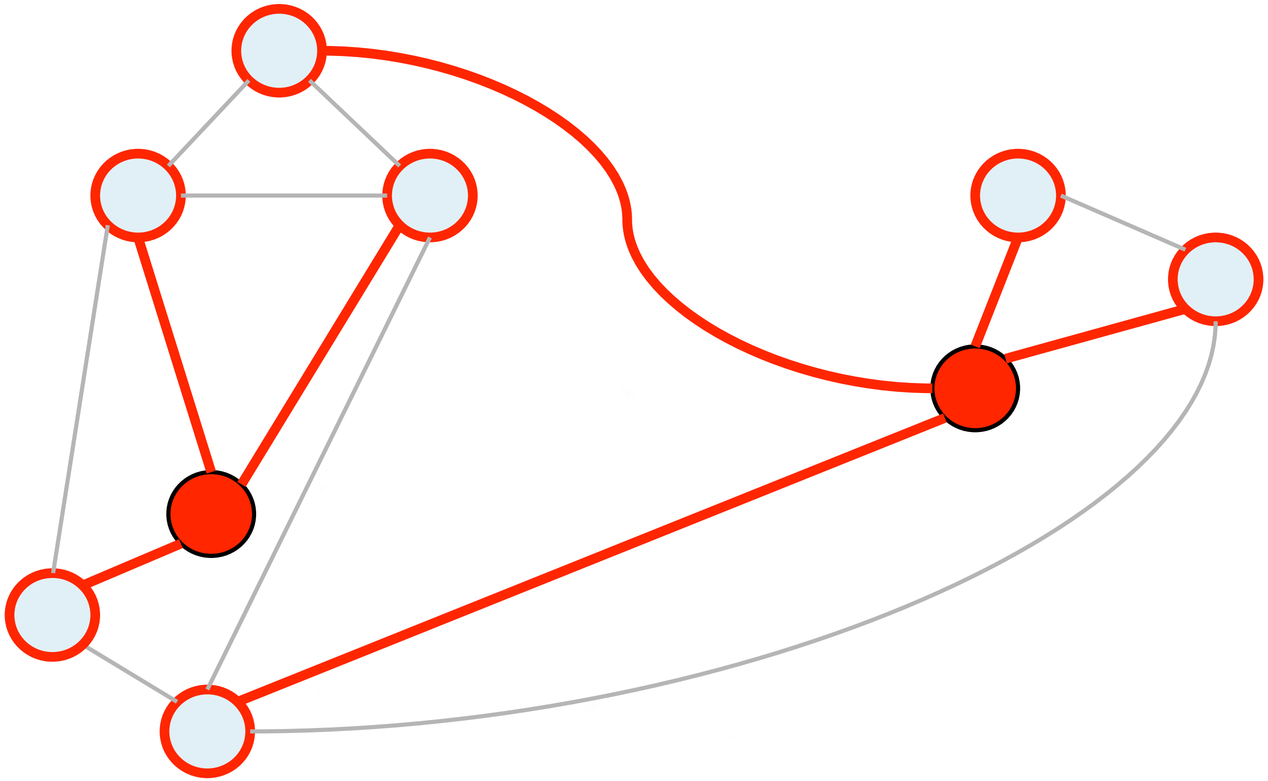

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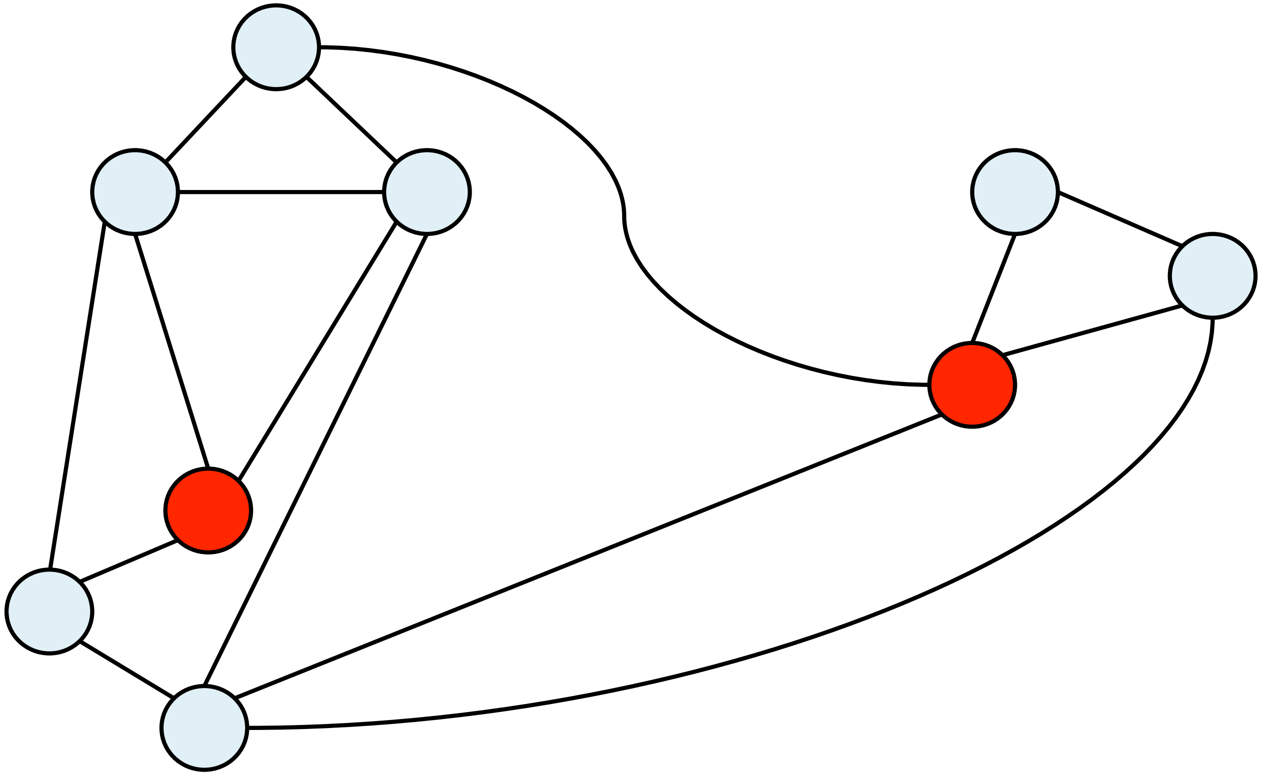

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Minimum dominating set problem

Input: graph G (V, E)
Output: dominating set D of G s.t. $|D|$ is minimum

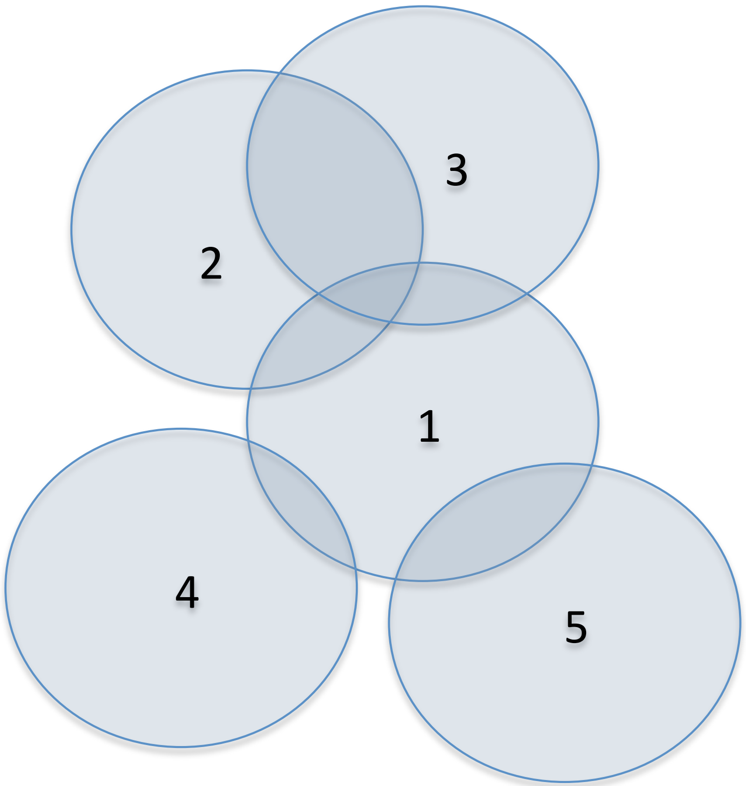
→ NP-hard

→ (1+log n)-approximation algorithm (Johnson 1974)

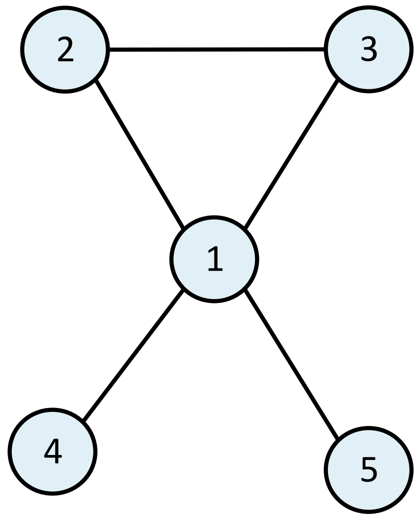
→ Not approximable within a (c log n) factor, for some $c > 0$
(Raz & Safra 1997)

Unit disk graph

Model of congruent disks



Graph
$G\ (V, E)$



Dominating sets in unit disk graphs

→ Several applications, e.g. ad-hoc wireless networks
(Marathe, Breu, Hunt III, Ravi & Rosenkrantz 1995)

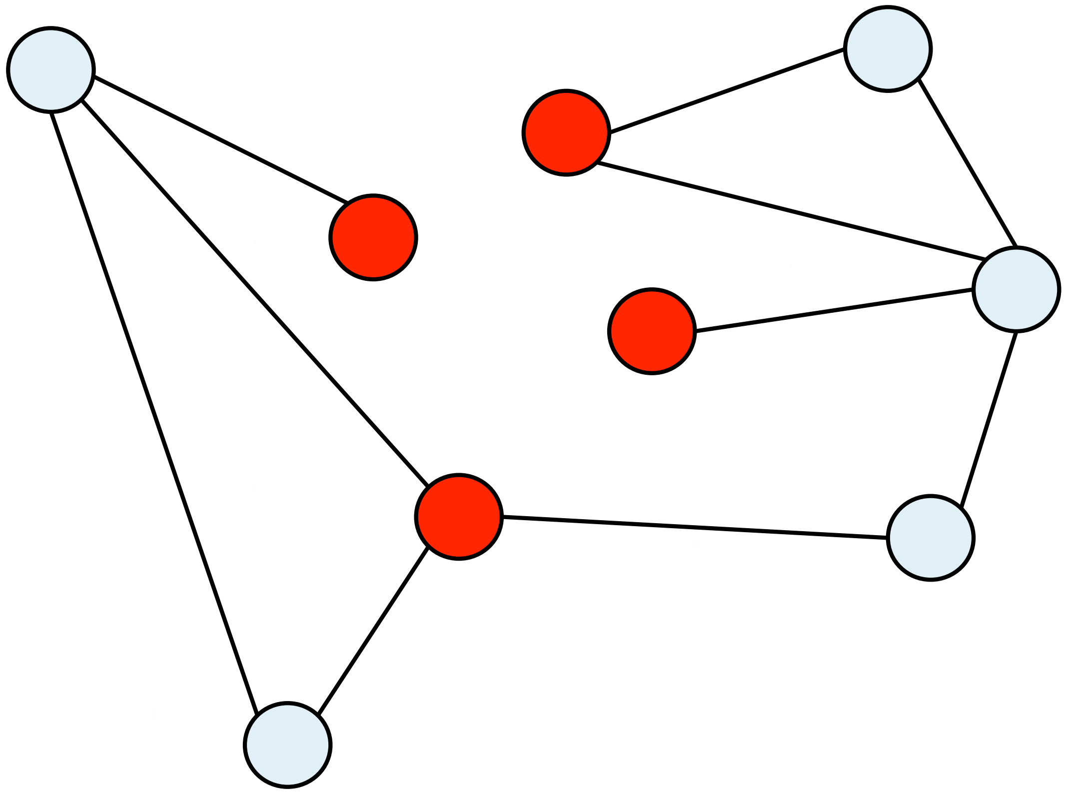
→ NP-hard nonetheless
(Clark, Colbourn & Johnson 1990)

→ Constant factor approximations (breaking the log *n* barrier),
and even PTAS

Two simple facts

1st fact:

Every maximal independent set is a dominating set.



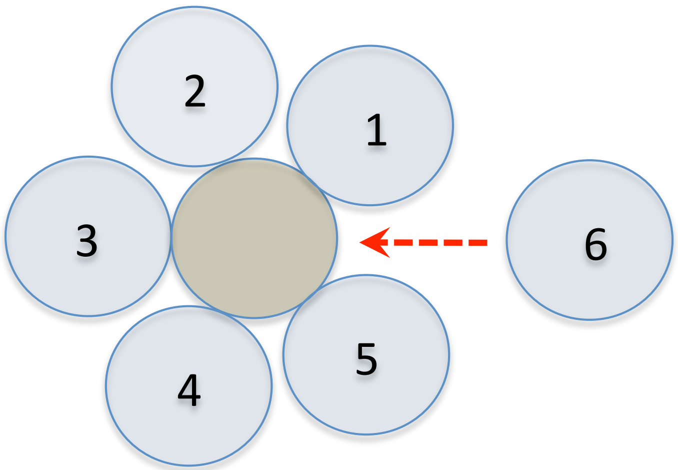
Two simple facts

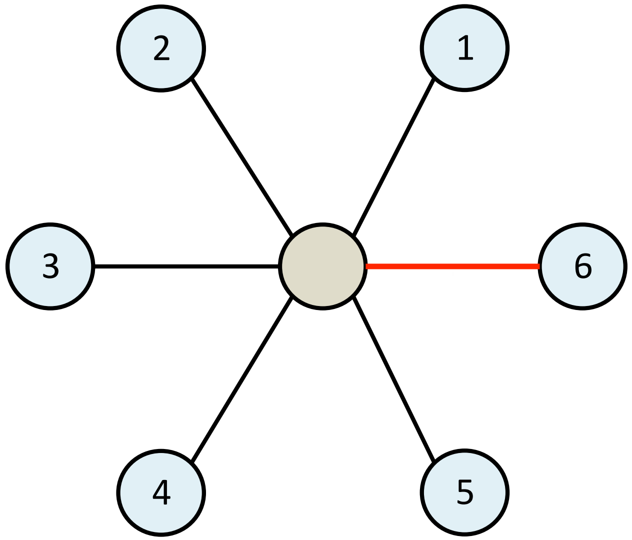
<u>1st fact</u>:

Every maximal independent set is a dominating set.

<u>2nd fact</u>:

A unit disk graph contains no $K_{1,6}$ as an induced subgraph.





$K_{1,6}$

5-approximation

1st fact:

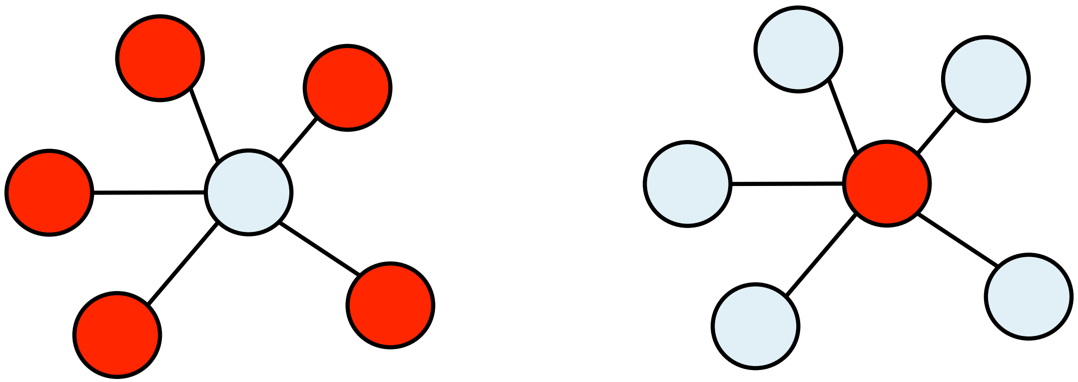
Every maximal independent set is a dominating set.

2nd fact:

A unit disk graph contains no $K_{1,6}$ as an induced subgraph.

Corollary:

If G is a unit disk graph, then
every maximal independent set S of G is a
5-approximation for the minimum independent set of G



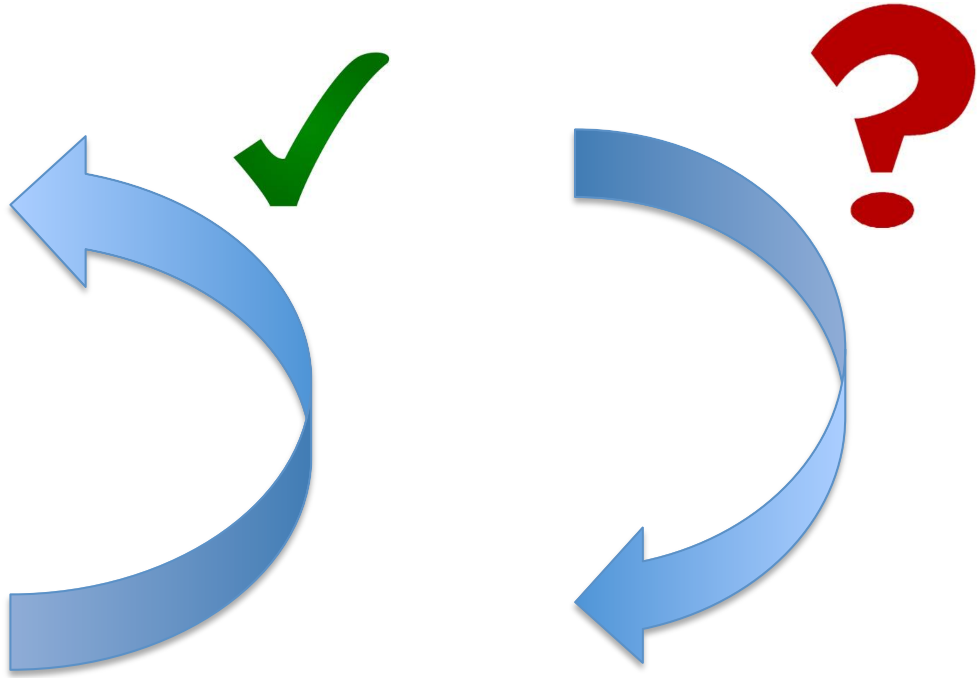
Algorithms for unit disk graphs

→ *Graph-based* algorithms

Input: a graph

→ *Geometric* algorithms

Input: a geometric model



Dominating sets in unit disk graphs

→ Vast literature on approximation algorithms:
(Marathe, Breu, Hunt III, Ravi & Rosenkrantz 1995)

O(n+m) graph-based 5-approximation (MBHRR'95)

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(Nieberg, Hurink & Kern 2008)*
(Gibson & Pirwani 2010)*– (general) disk graphs

(Hurink & Nieberg 2011)*– independent dominating set version

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*PTAS

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(De, Das & Nandy 2011)

$O(n+m)$ graph-based 5-approximation (MBHRR'95)
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$O(n^{9})$ geometric 4-approximation (DDN'11)
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Our contribution:
$O(n+m)$ graph-based 4.888... -approximation
$O(n \log n)$ geometric 4.888... -approximation
(FFMS'12)

100 meters world record

1995

Leroy Burrell (USA)
9.85 s

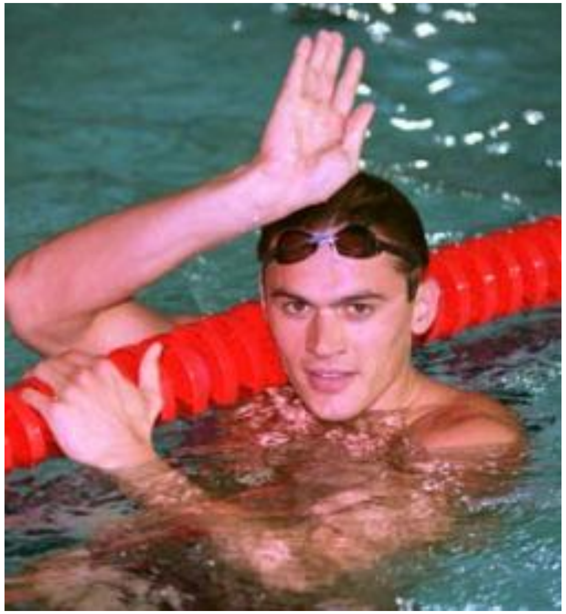
2012

Usain Bolt (JAMAICA)
9.58 s

- 2.7%

100 meters freestyle world record

1995



Alexander Popov (RUSSIA)
48.21 s

2012

César Cielo (BRAZIL)
46.91 s

- 2.6%

Dominating sets in unit disk graphs

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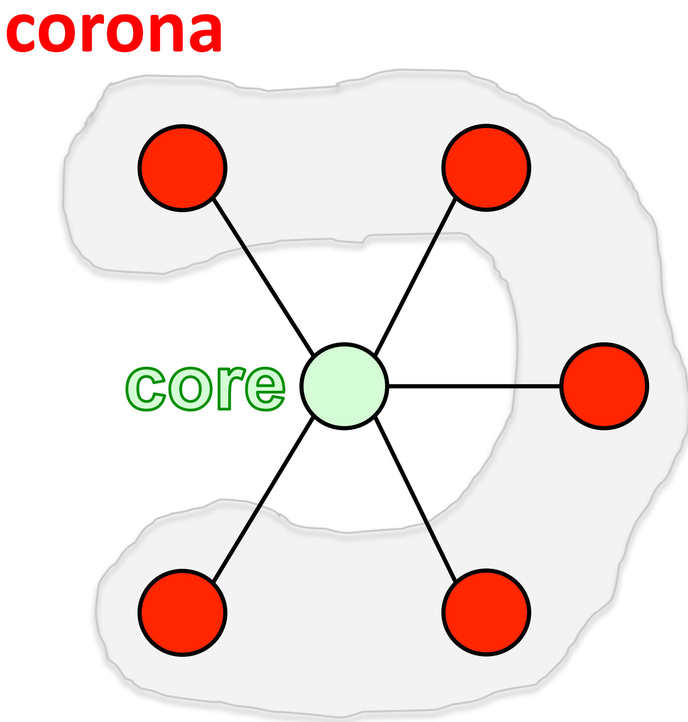
Our contribution:
$O(n+m)$ graph-based 4.888... -approxim
$O(n \log n)$ geometric 4.888... -approximation
(FFMS'12)

- 2.2%

Coronas and cores

Let D be a maximal independent set of graph $G(V,E)$.
A **corona** consists of exactly 5 (five) vertices of D presenting a common neighbor in $V \setminus D$, called a core.



Coronas and cores

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A corona C can be

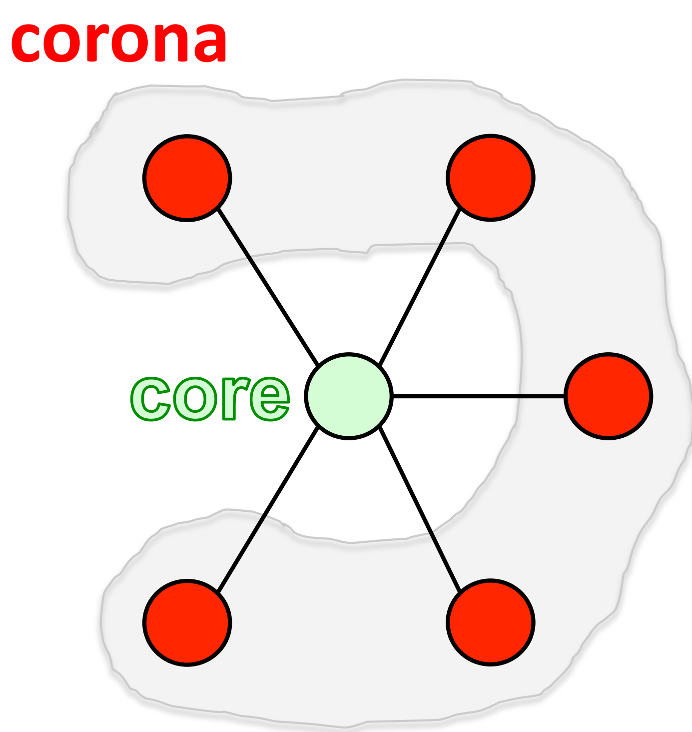
- reducible,

  if it has a core c s.t.
  $D \setminus C \cup \{c\}$ is still
  a dominating set of G;

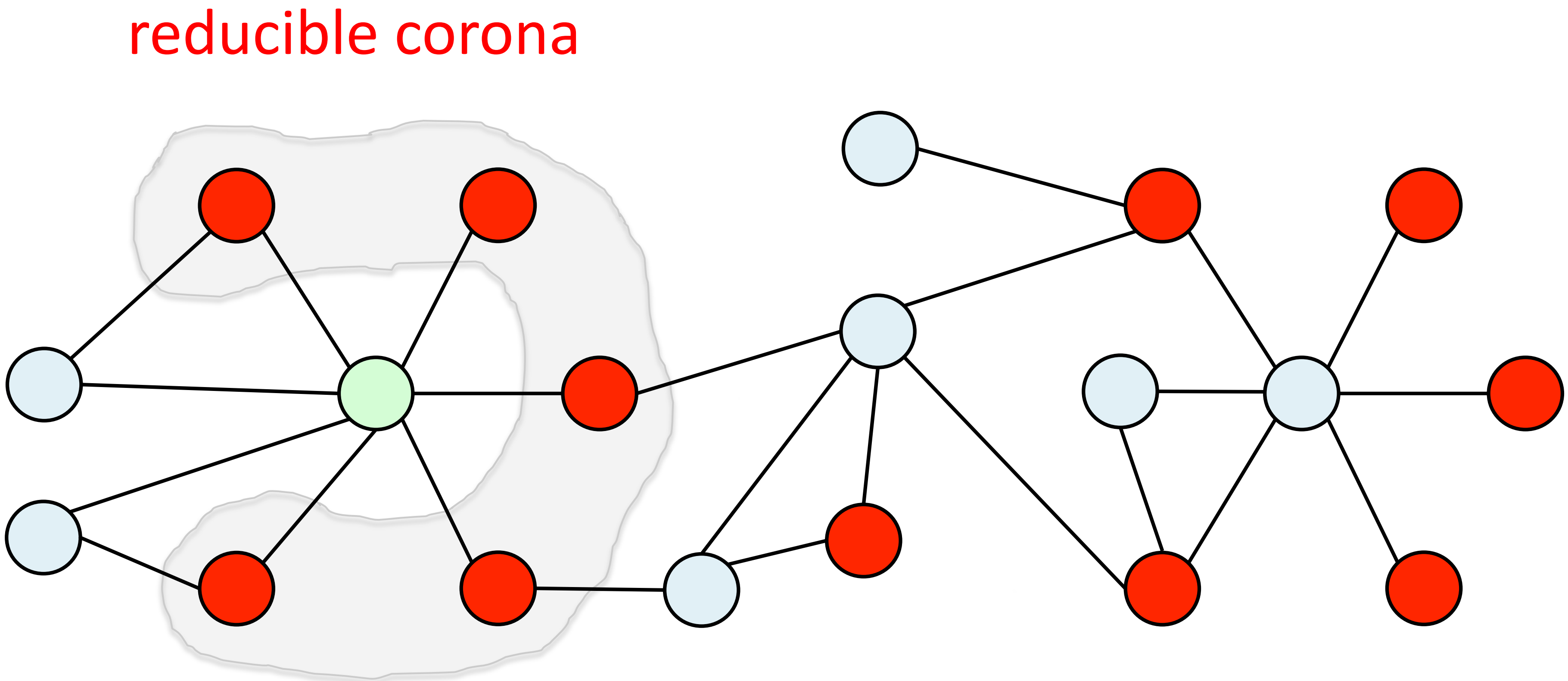
  or

- irreducible,

  otherwise.



Reducible and irreducible coronas



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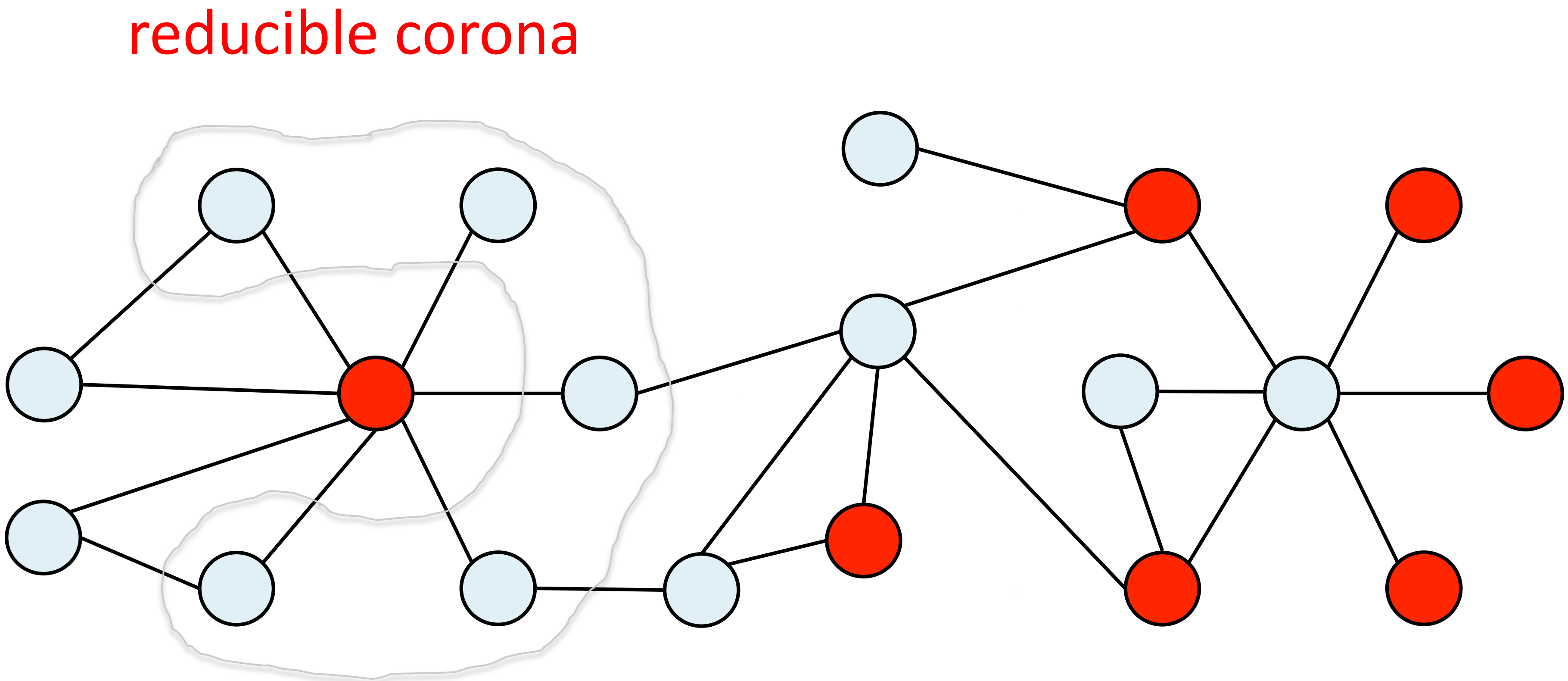

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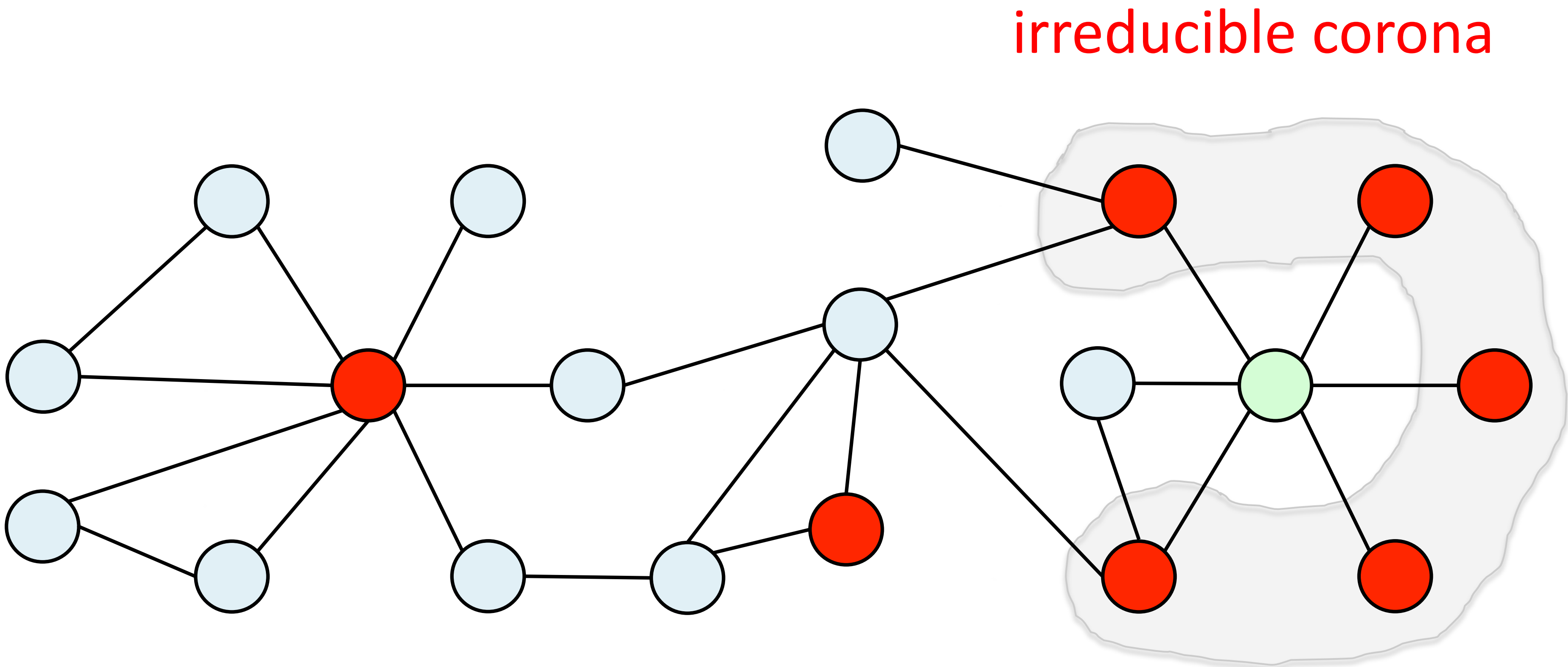
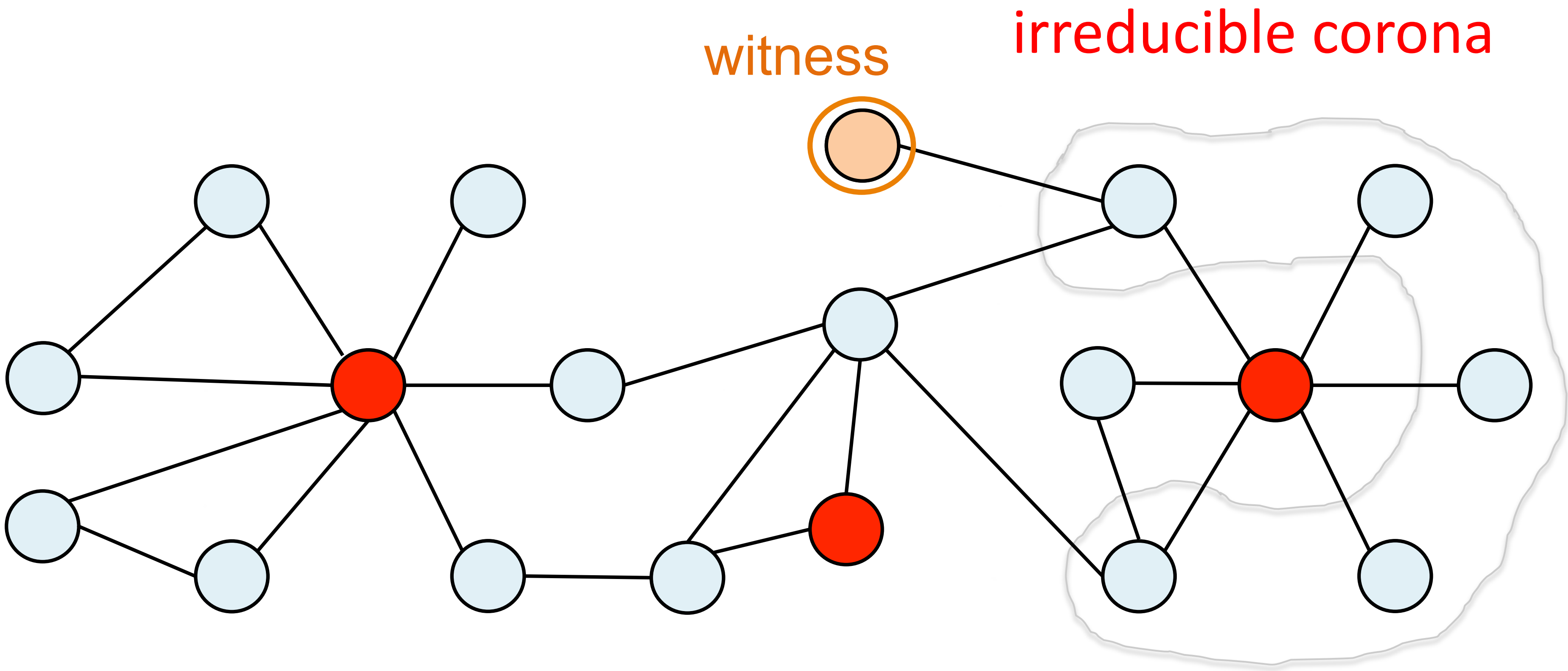


Witnesses



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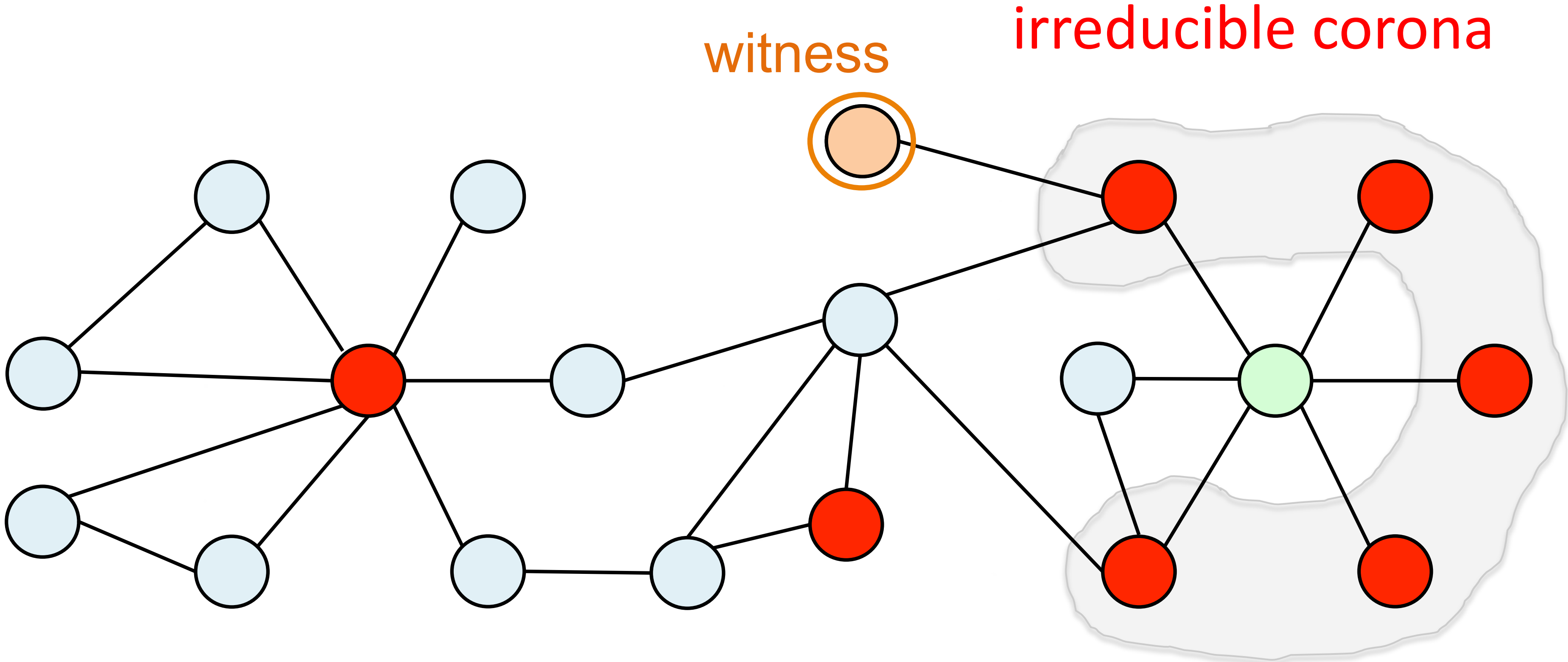
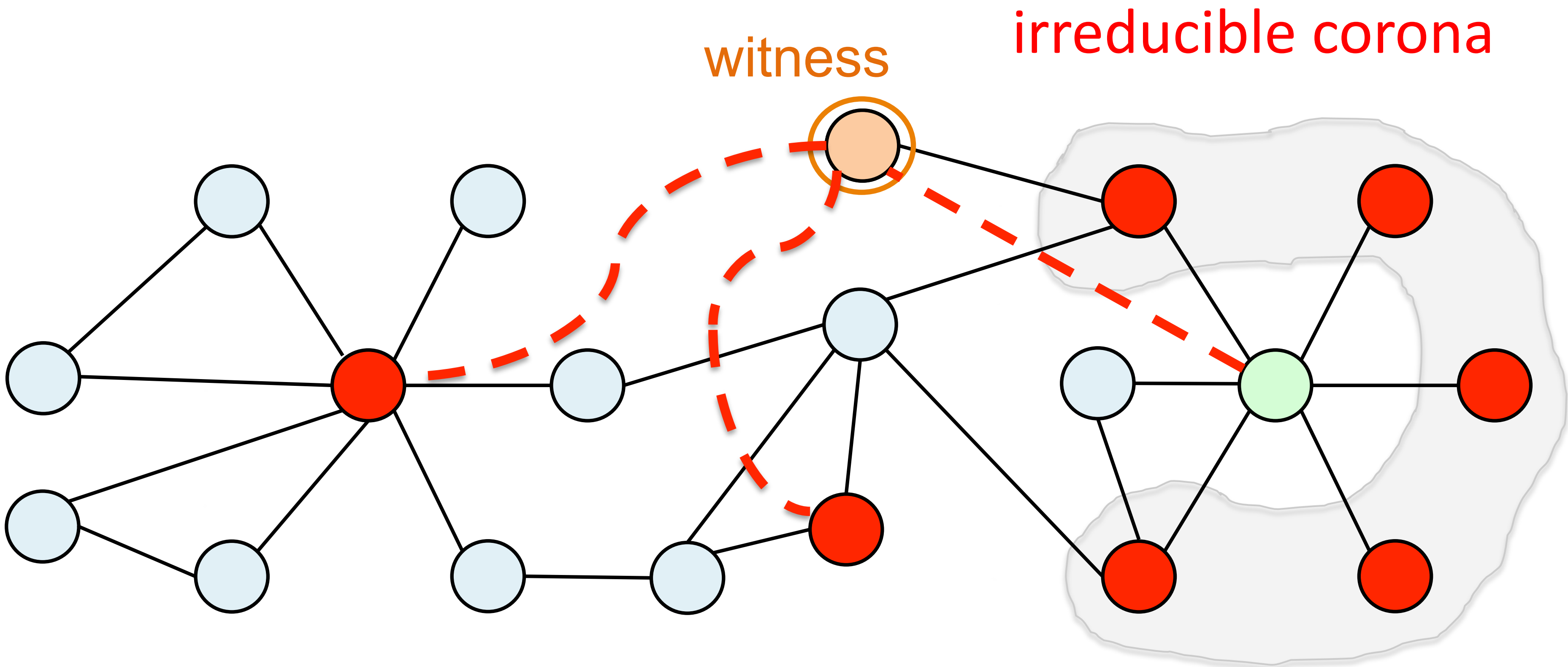


Witnesses

Let C be a corona of graph $G(V,E)$, and let c be a core of C.
A vertex w is a witness of c iff $cw \notin E$, and $N_D[w] \subseteq C$.



4.888...-approximation

1. Obtain a maximal independent set *D*
2. While there is a reducible corona *C* in *D*
3. Update *D* by reducing *C*
4. Return *D*

Input: adjacency lists (graph)
Time: O(n+m)

Input: center coordinates in Real RAM Model
Time: O(n log n)

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Input: adjacency lists (graph)
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Input: center coordinates in Real RAM Model
Time: O(n log n)

<ins>Lemma</ins>: a maximal independent set *D with no reducible coronas* is a 4.888…-approximation for the minimum (independent) dominating set.

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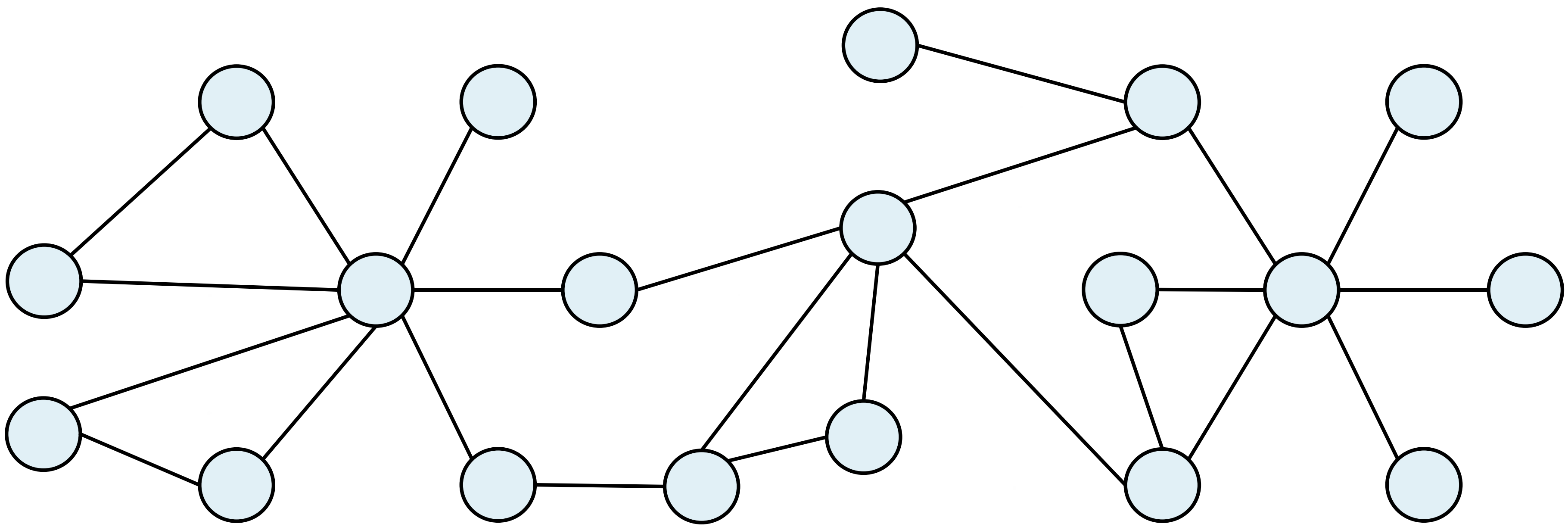

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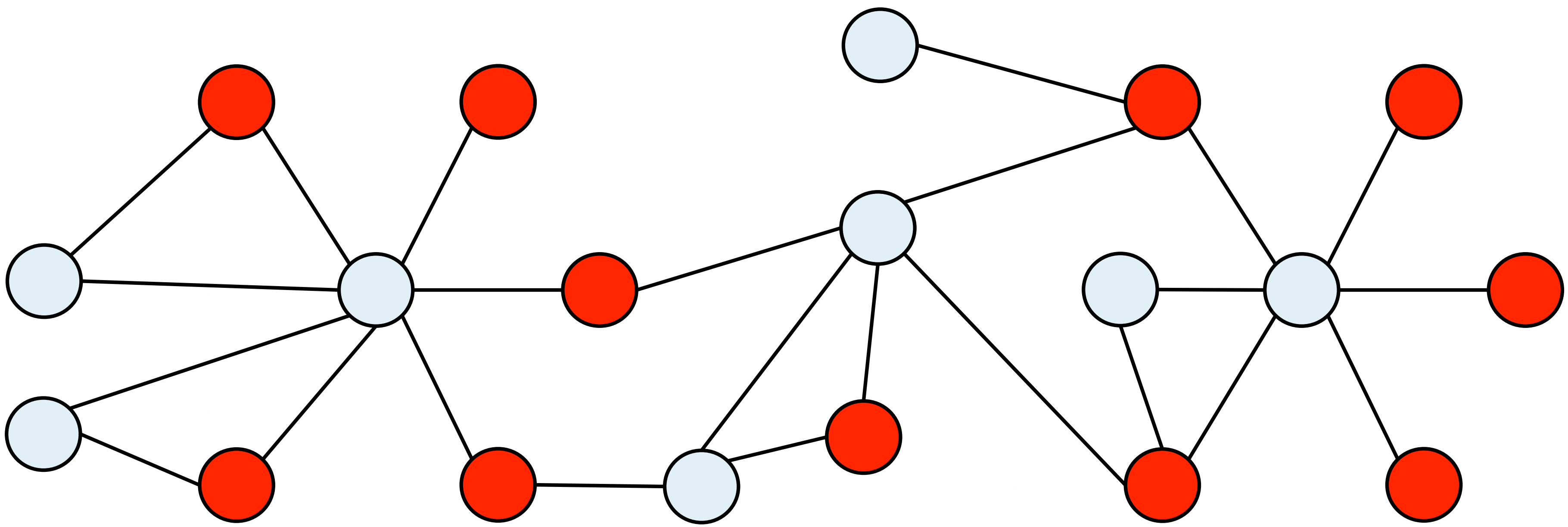

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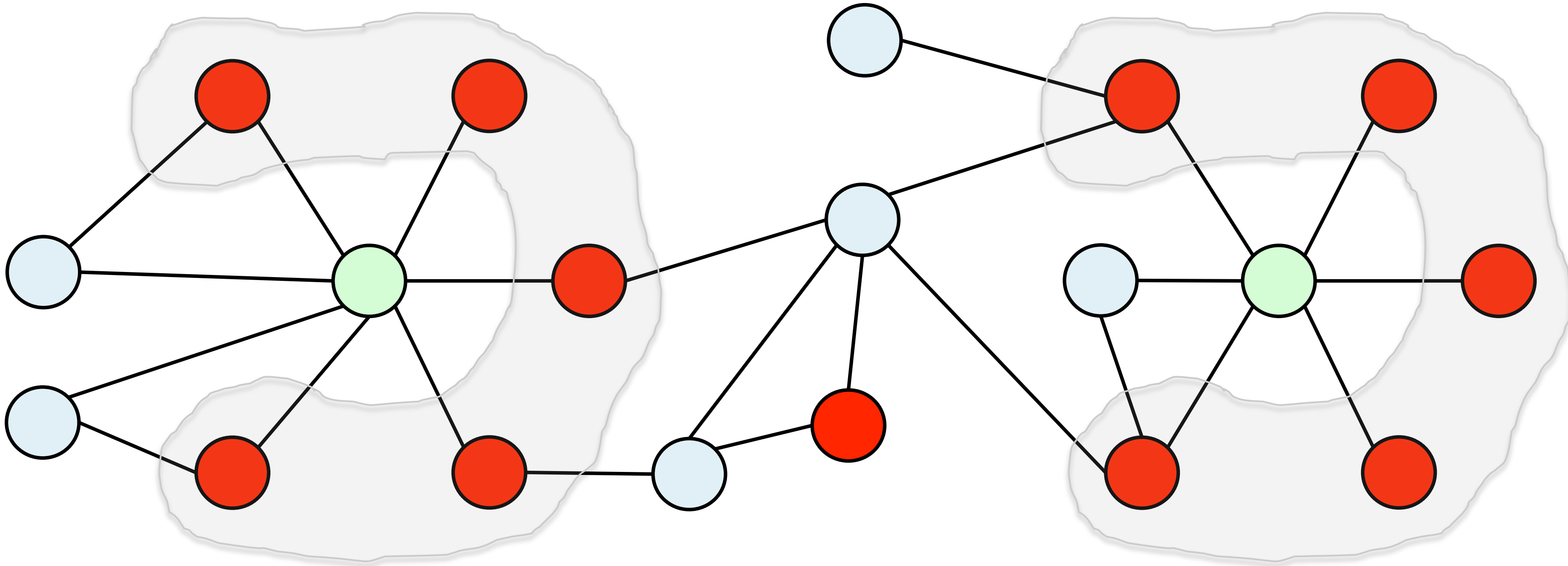

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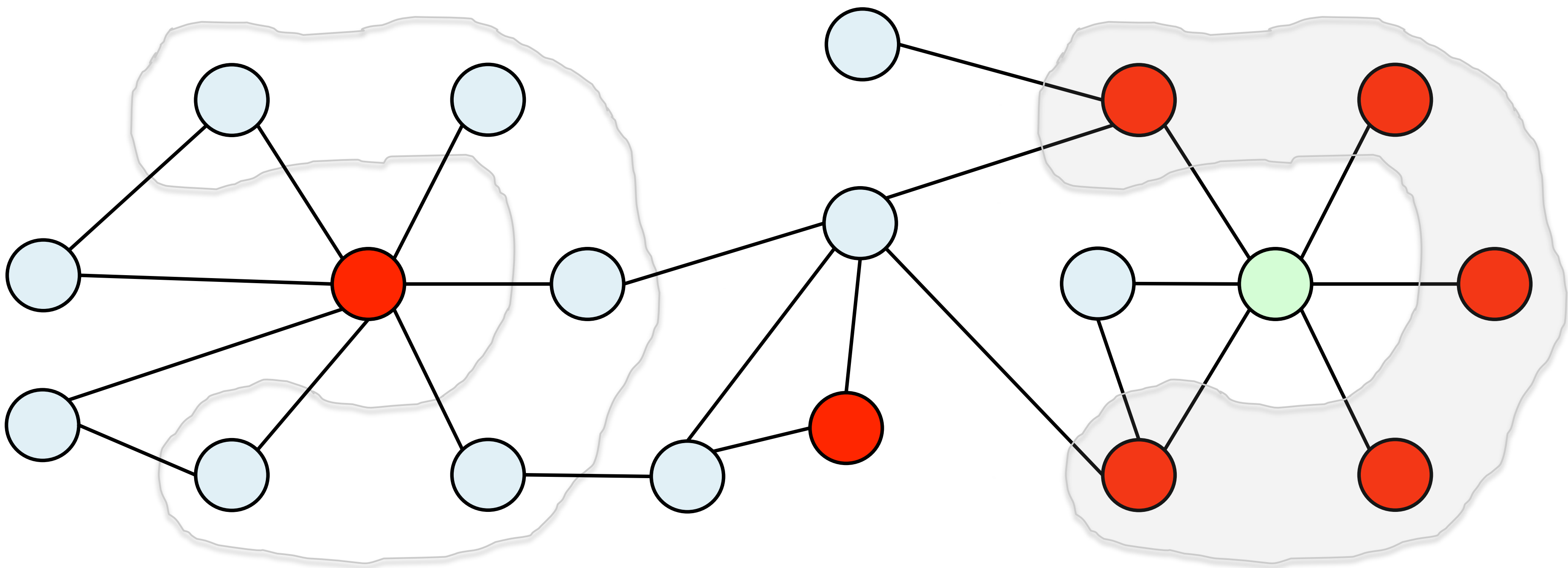
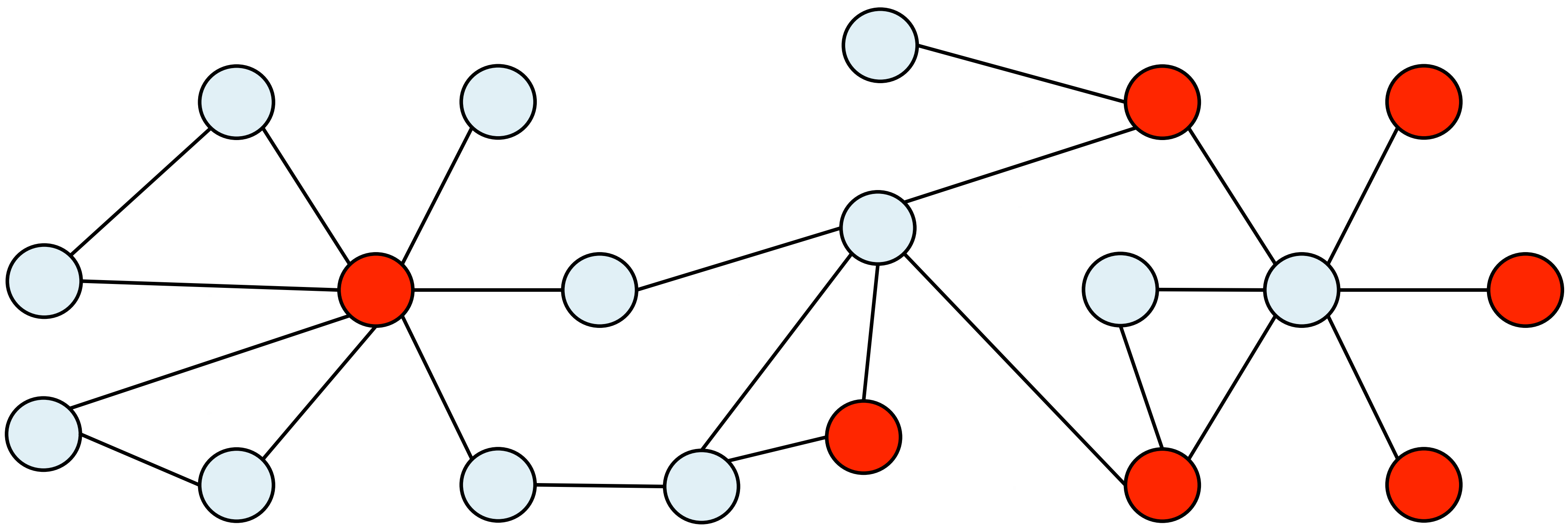


4.888...-approximation

D – a maximal independent set with no reducible coronas (algorithm output)



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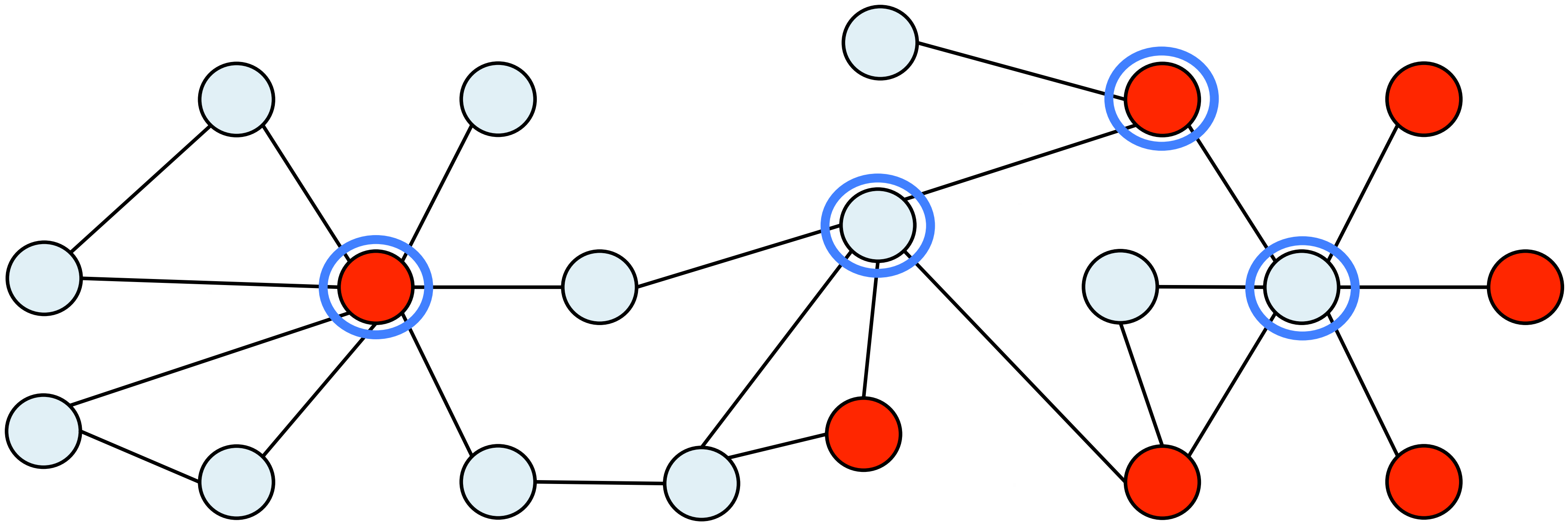
D – a maximal independent set with no reducible coronas (algorithm output)

*D** – a minimum dominating set (optimum solution)

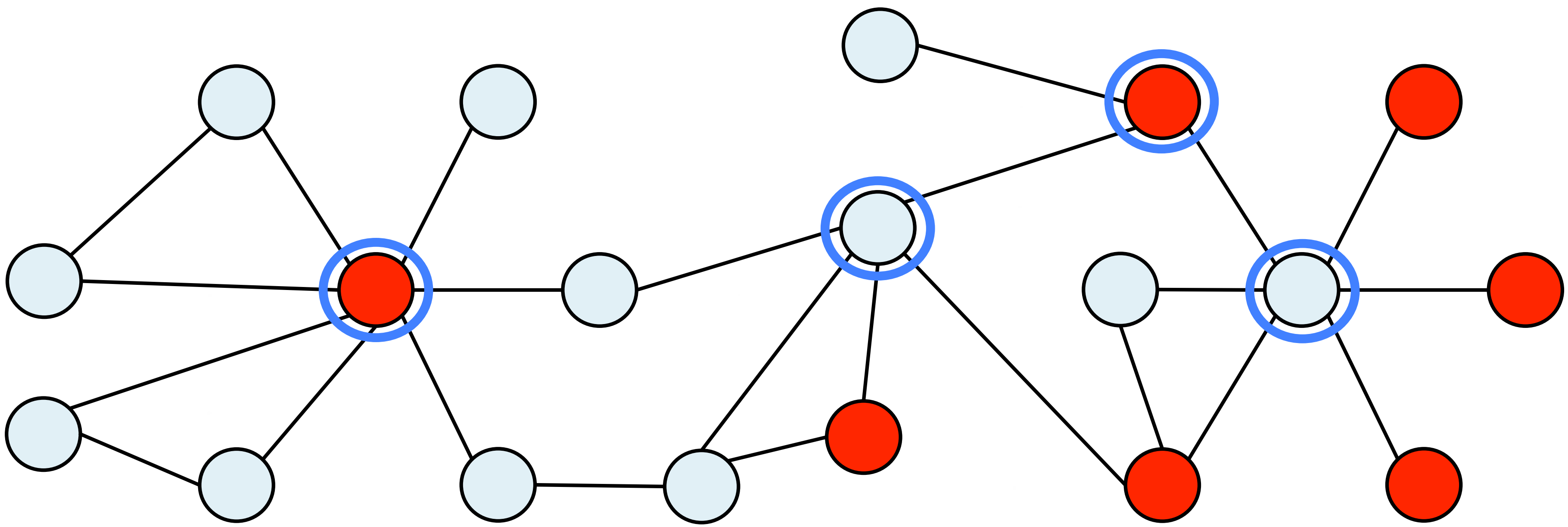


4.888...-approximation

D – a maximal independent set with no reducible coronas (algorithm output)

D^* – a minimum dominating set (optimum solution)

$$\frac{|D|}{|D^*|} \leq \text{??}$$



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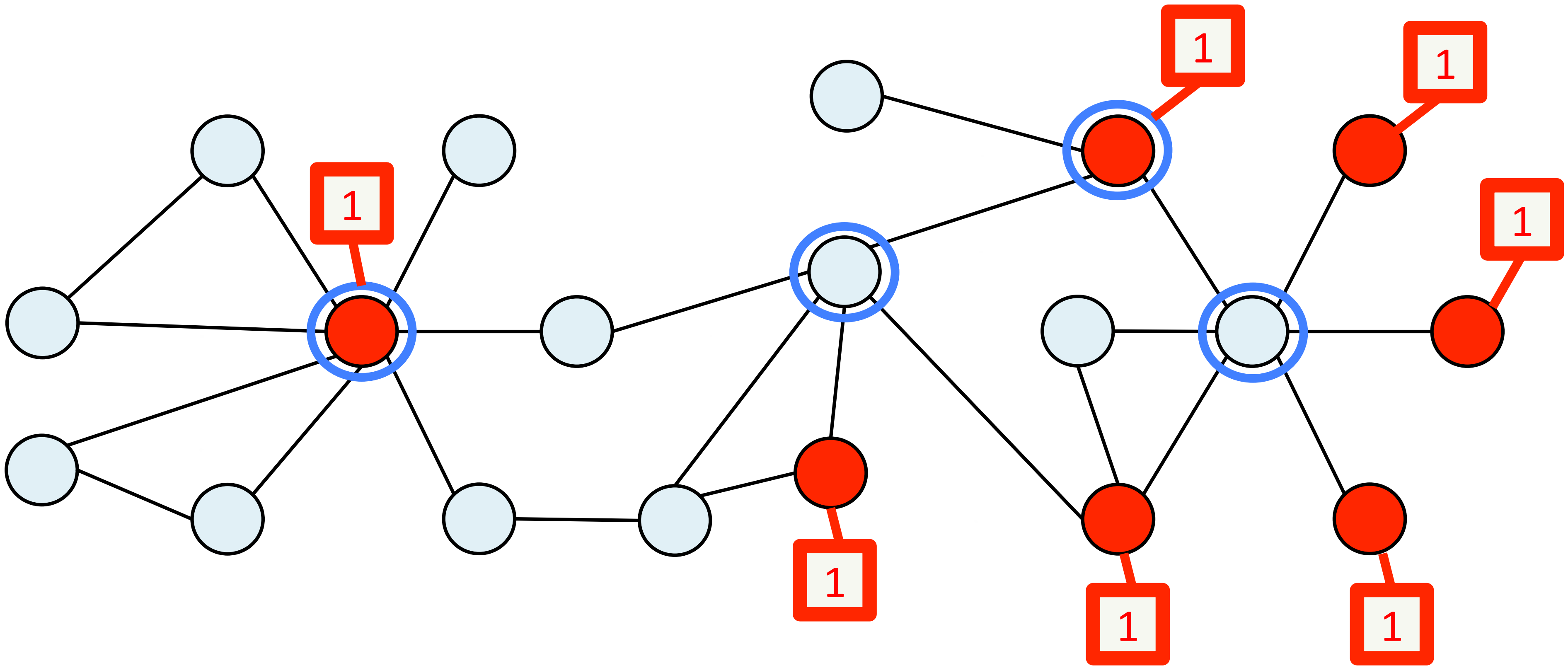

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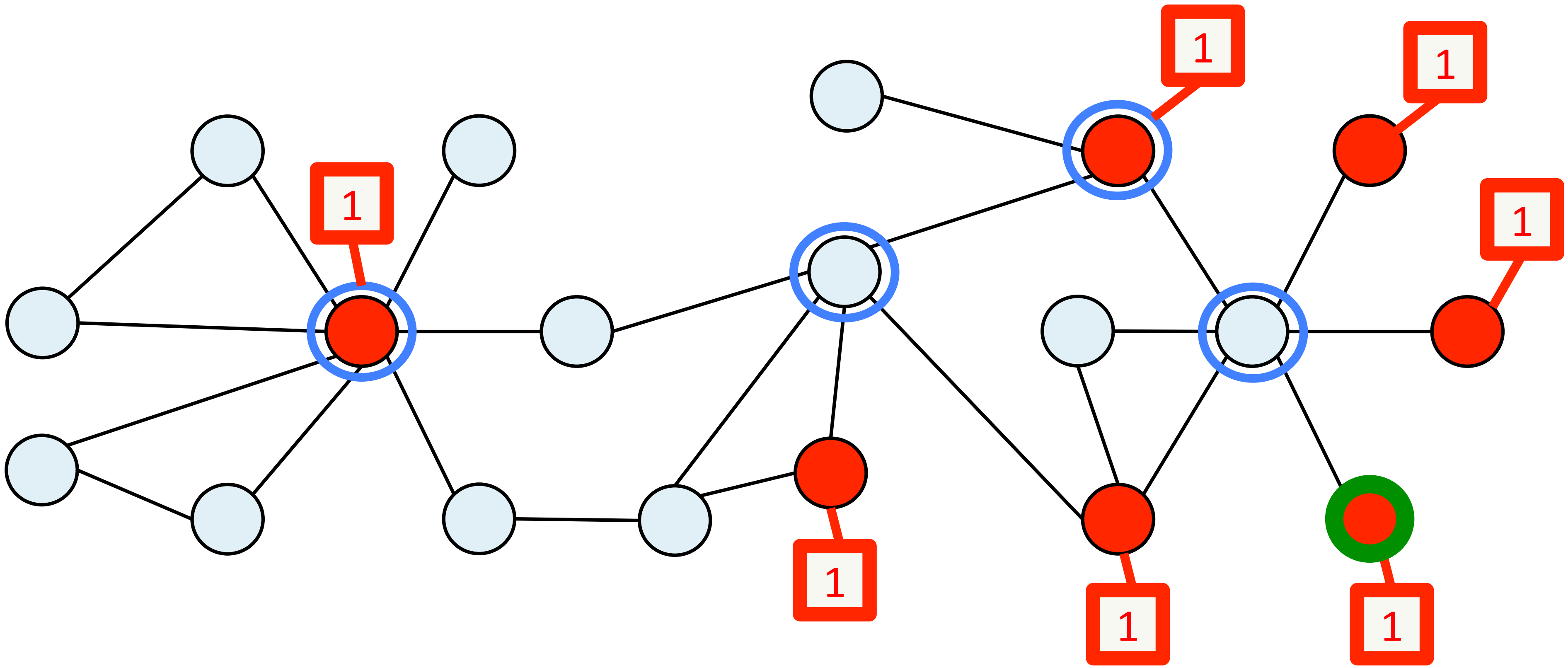

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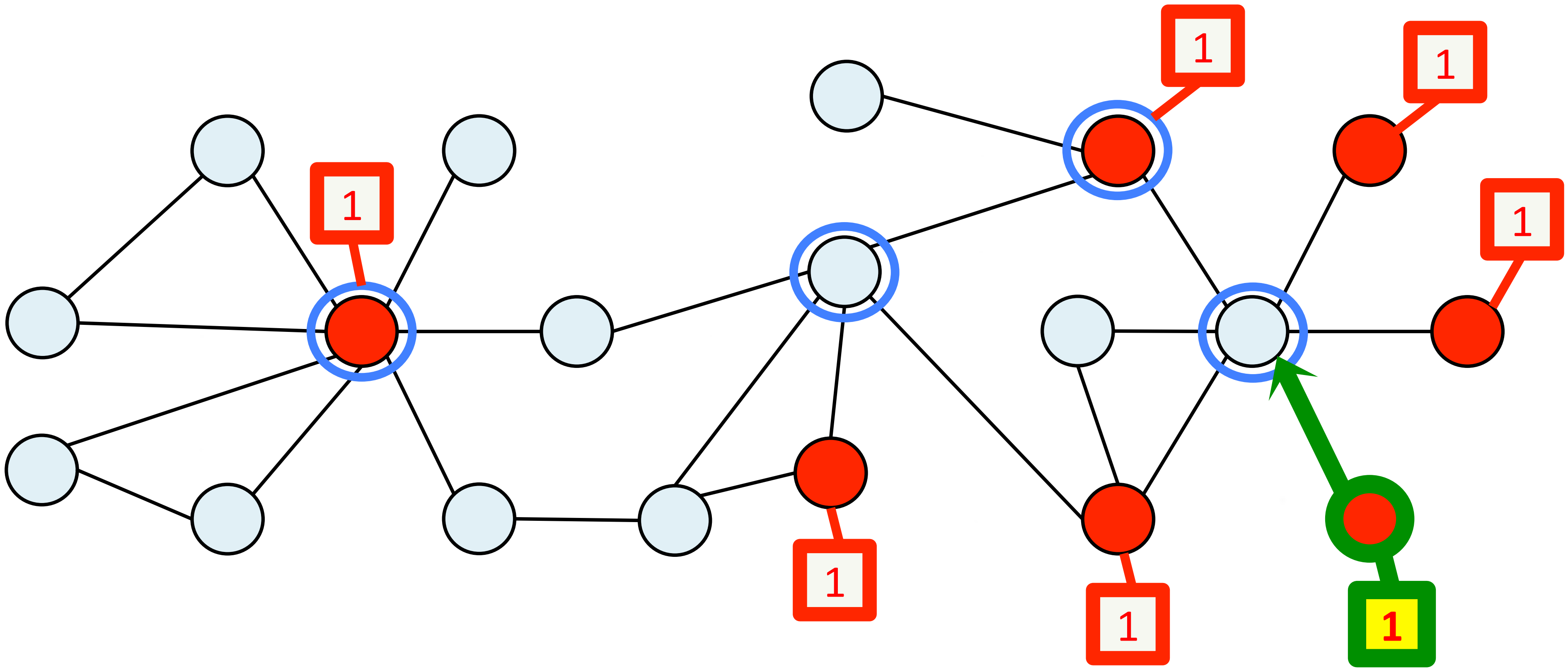

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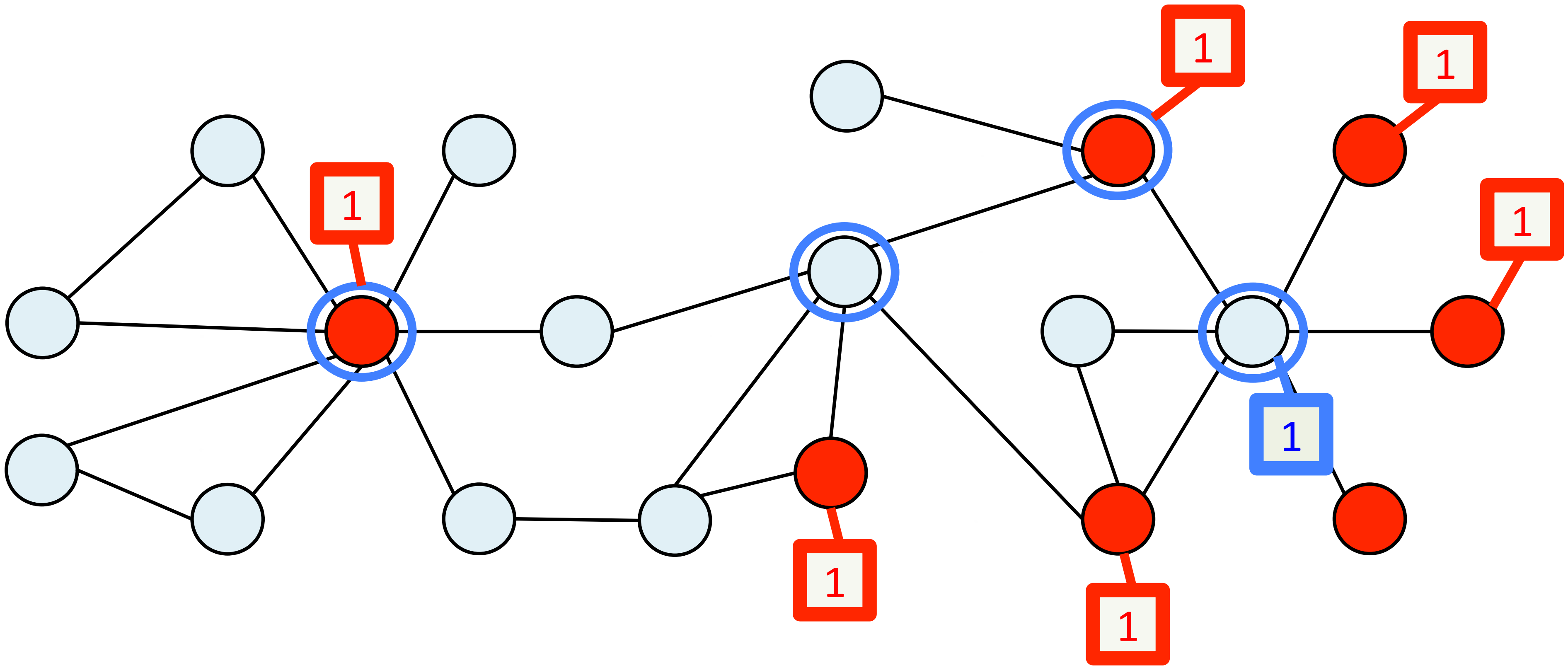

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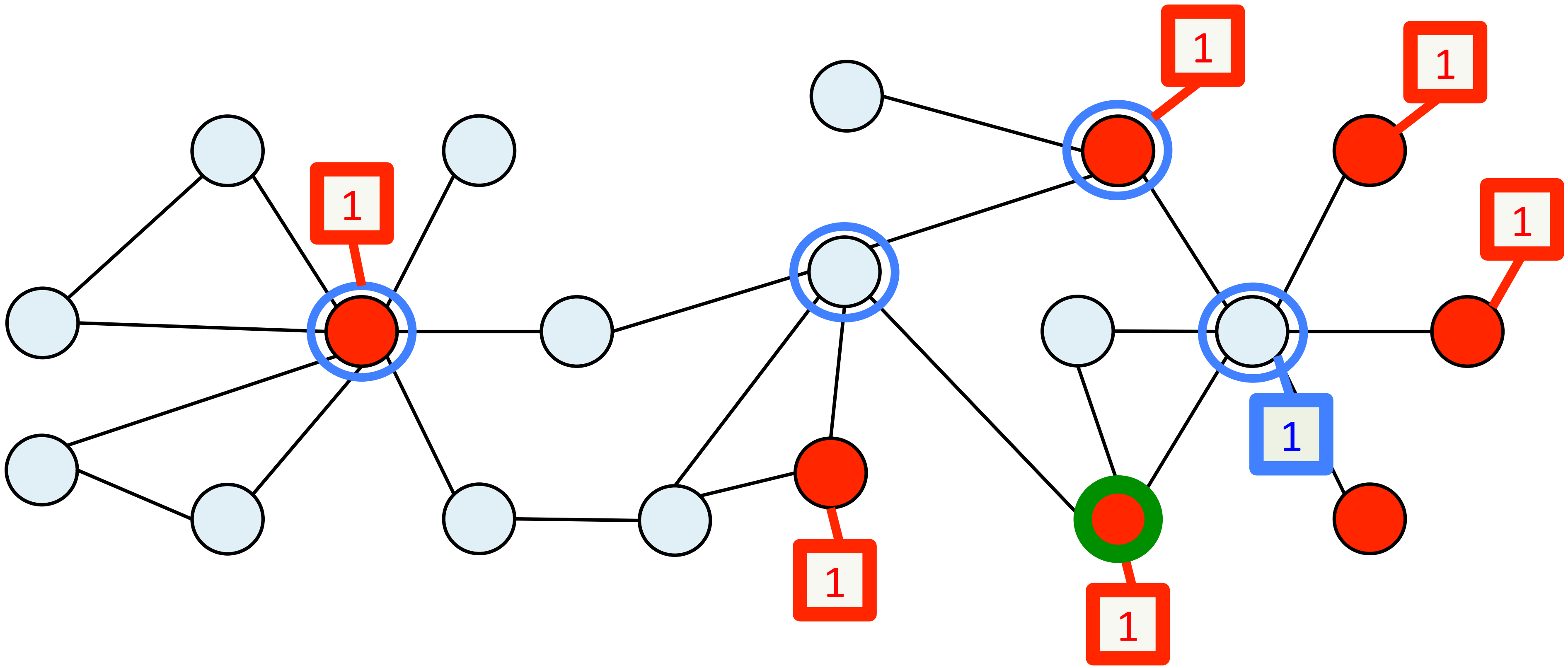

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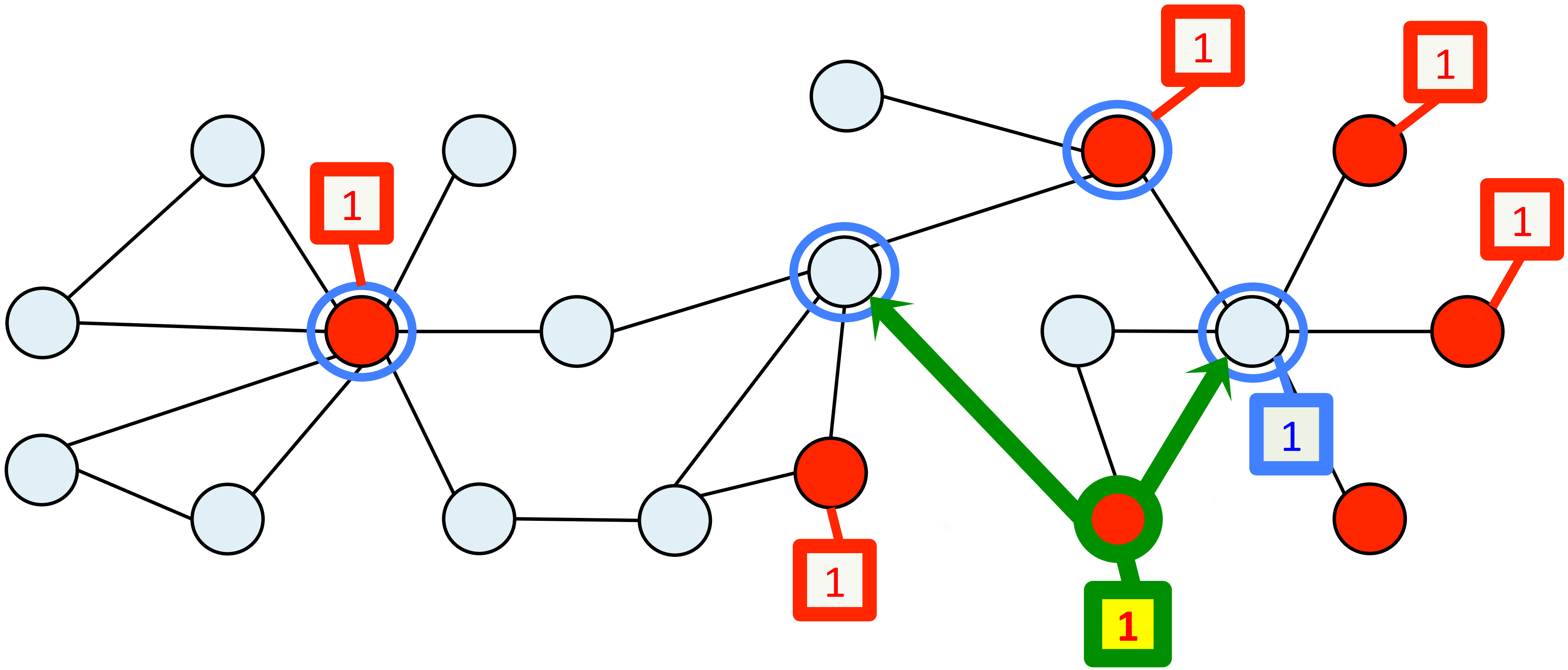

4.888...-approximation

- *D* – a maximal independent set with no reducible coronas (algorithm output)
- *D** – a minimum dominating set (optimum solution)

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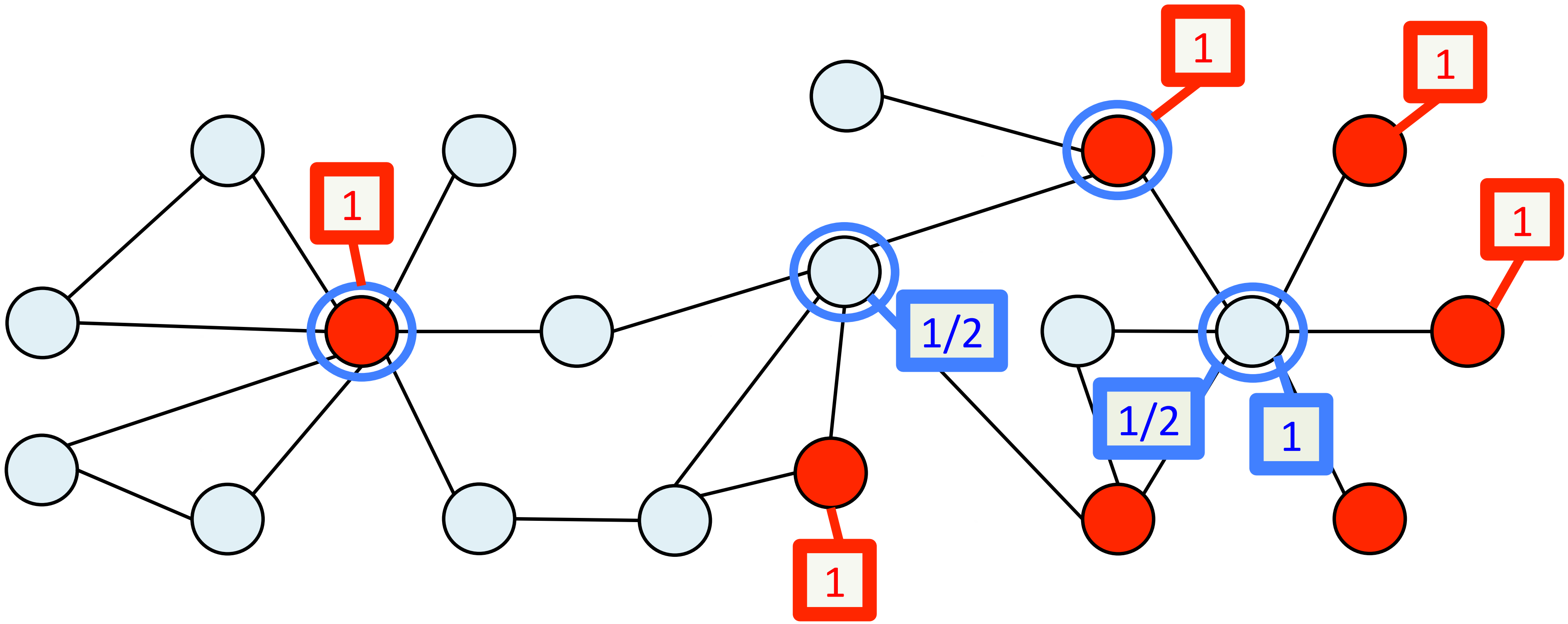

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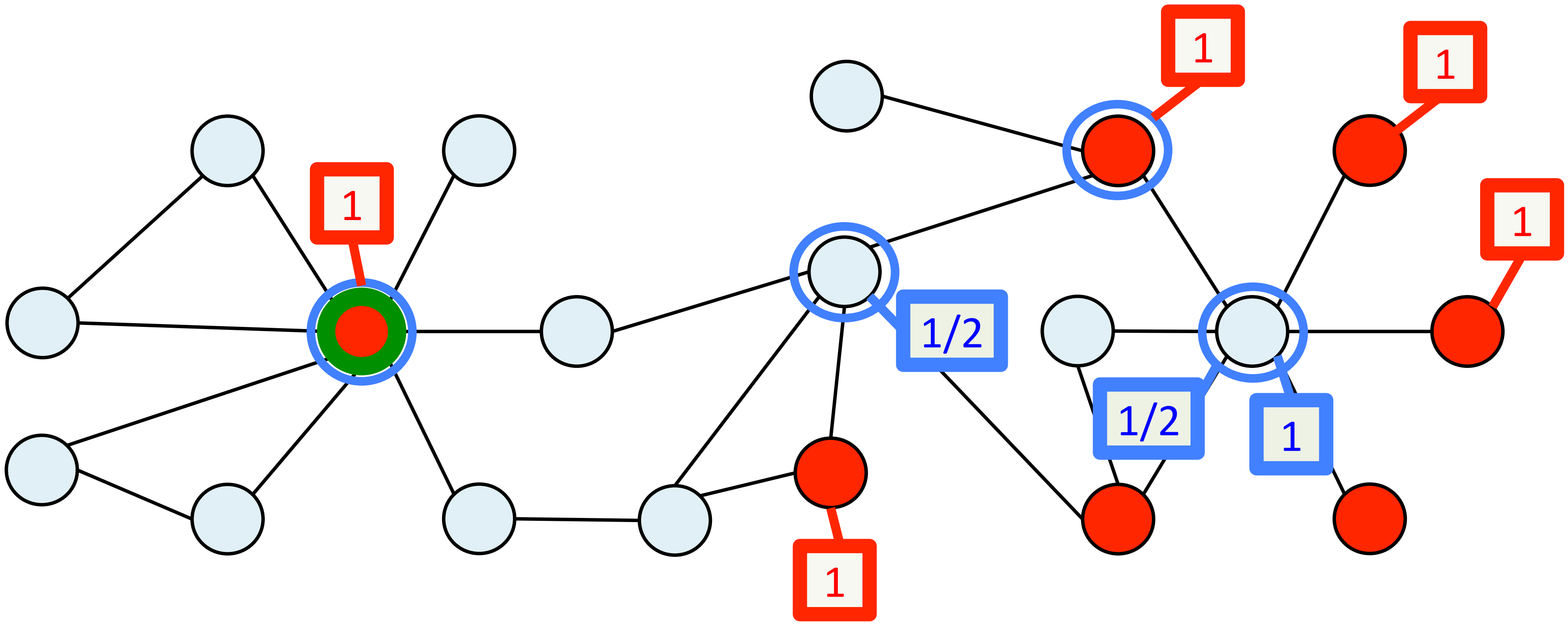

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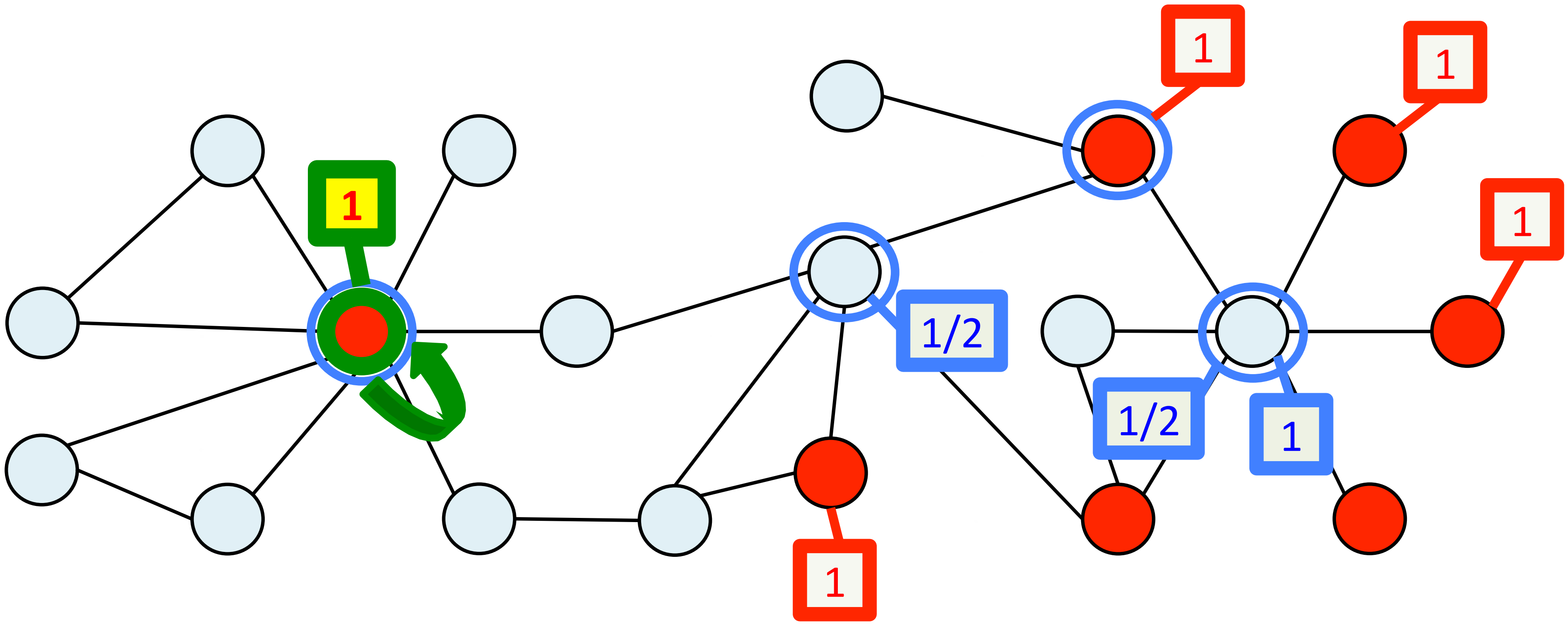
D – a maximal independent set with no reducible coronas (algorithm output)

D^* – a minimum dominating set (optimum solution)

$$\frac{|D|}{|D^*|} \leq \ ??$$



4.888...-approximation

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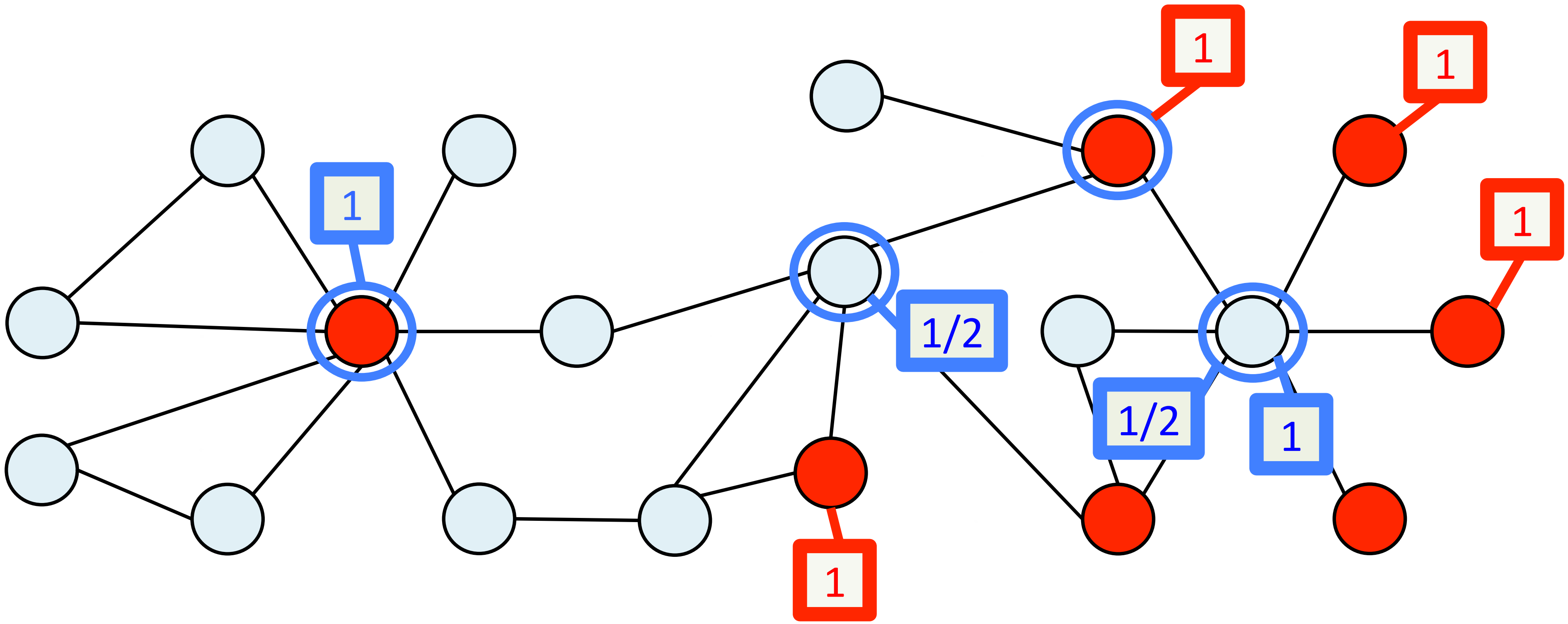

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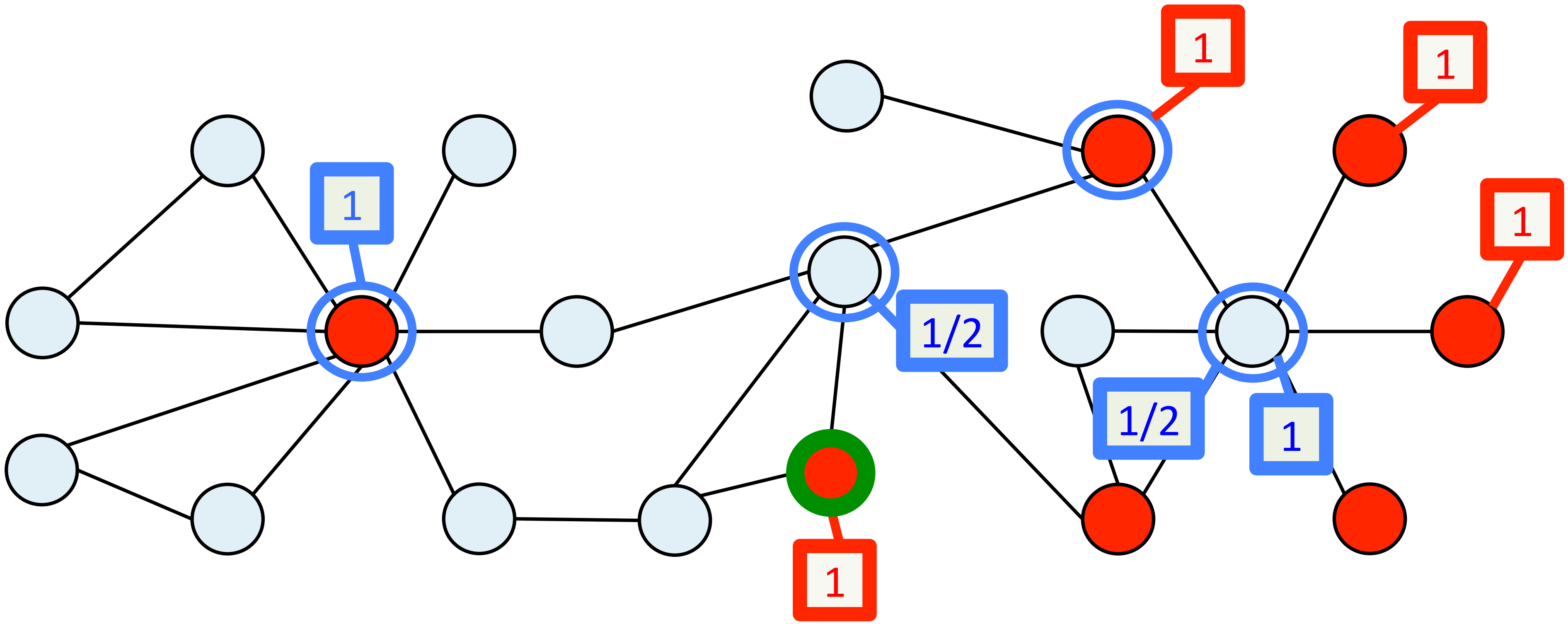

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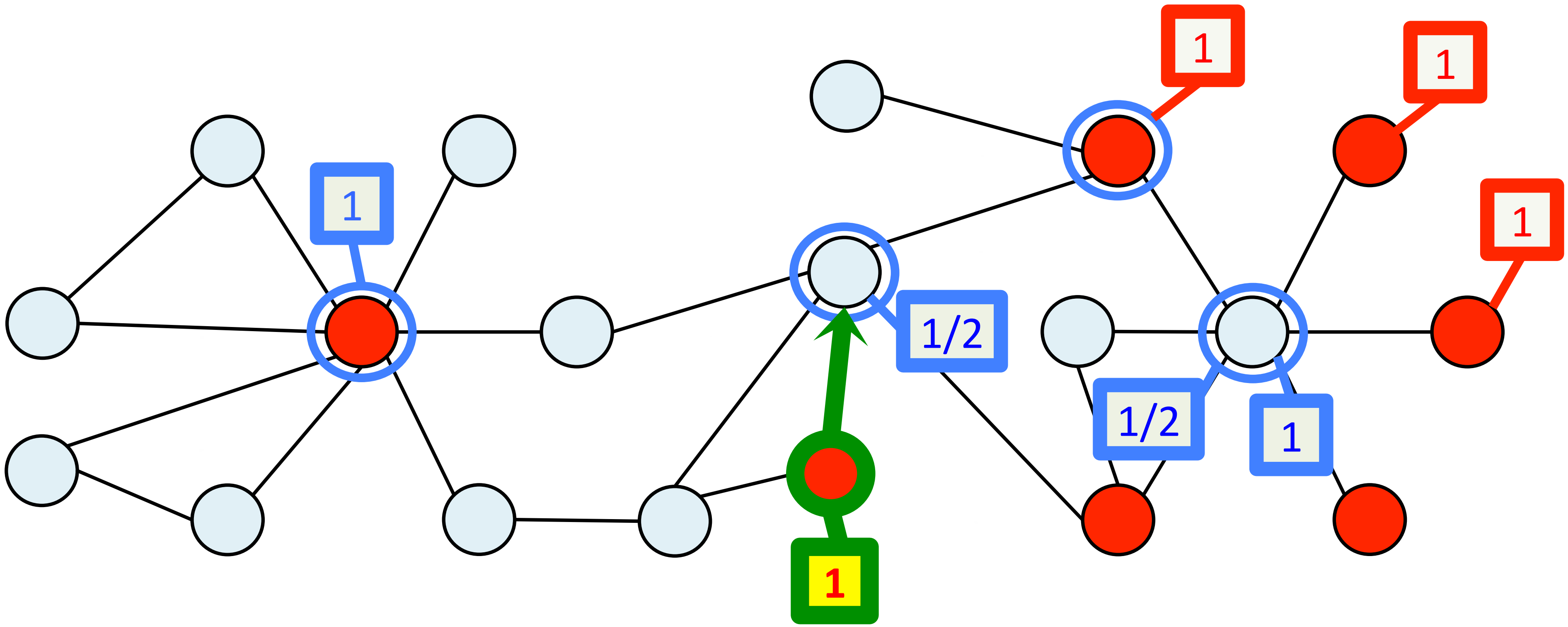

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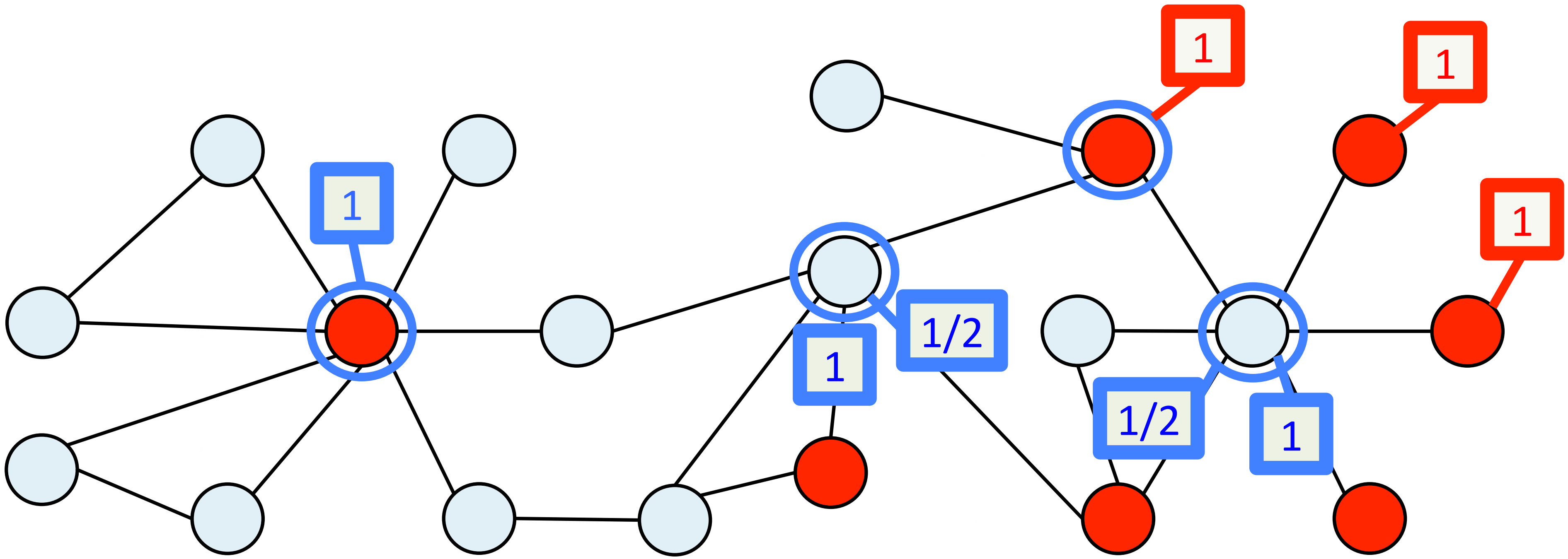

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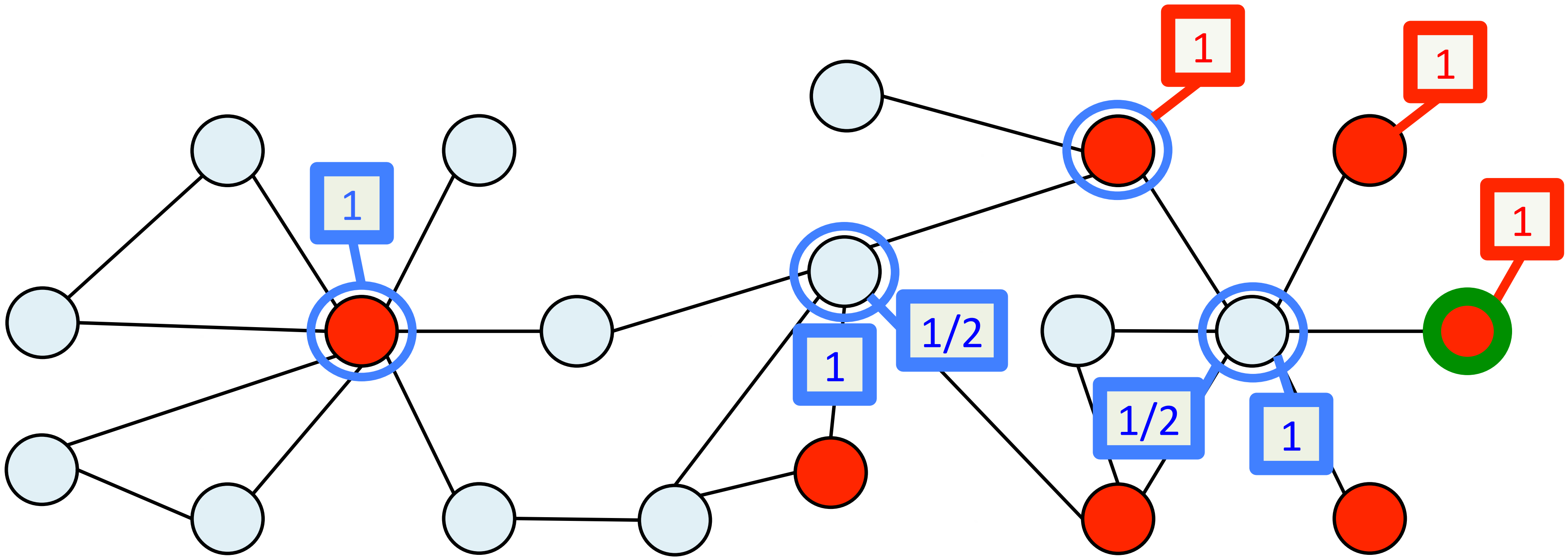

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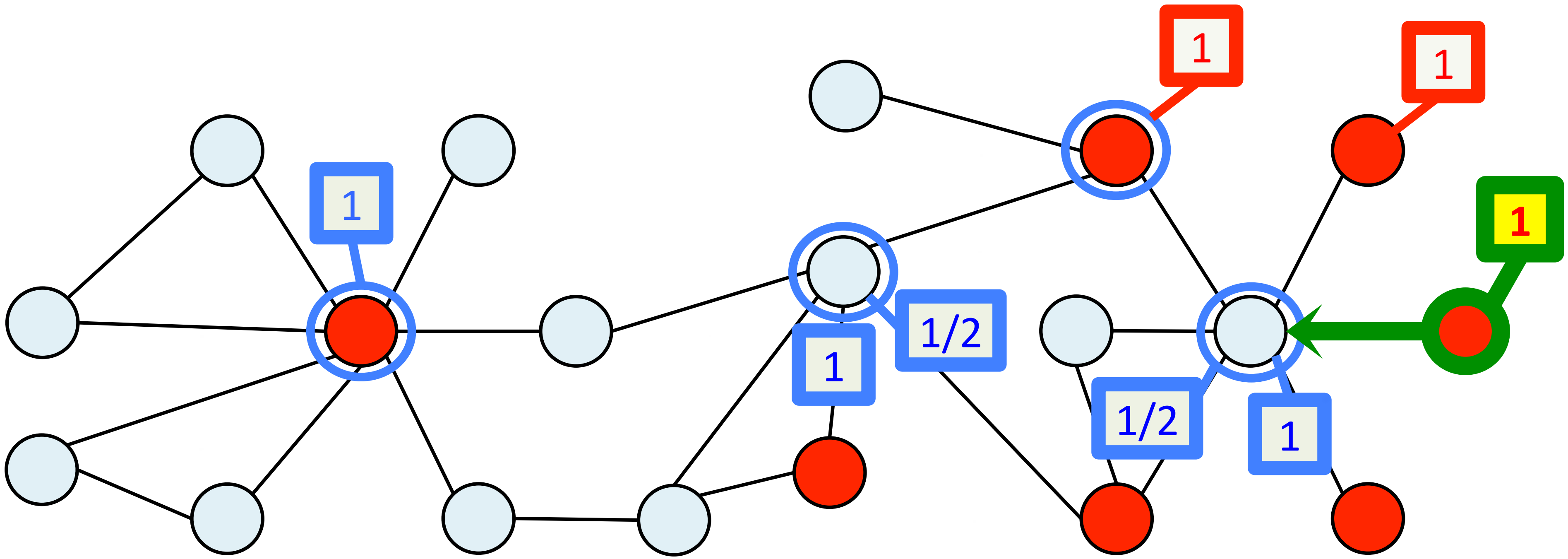

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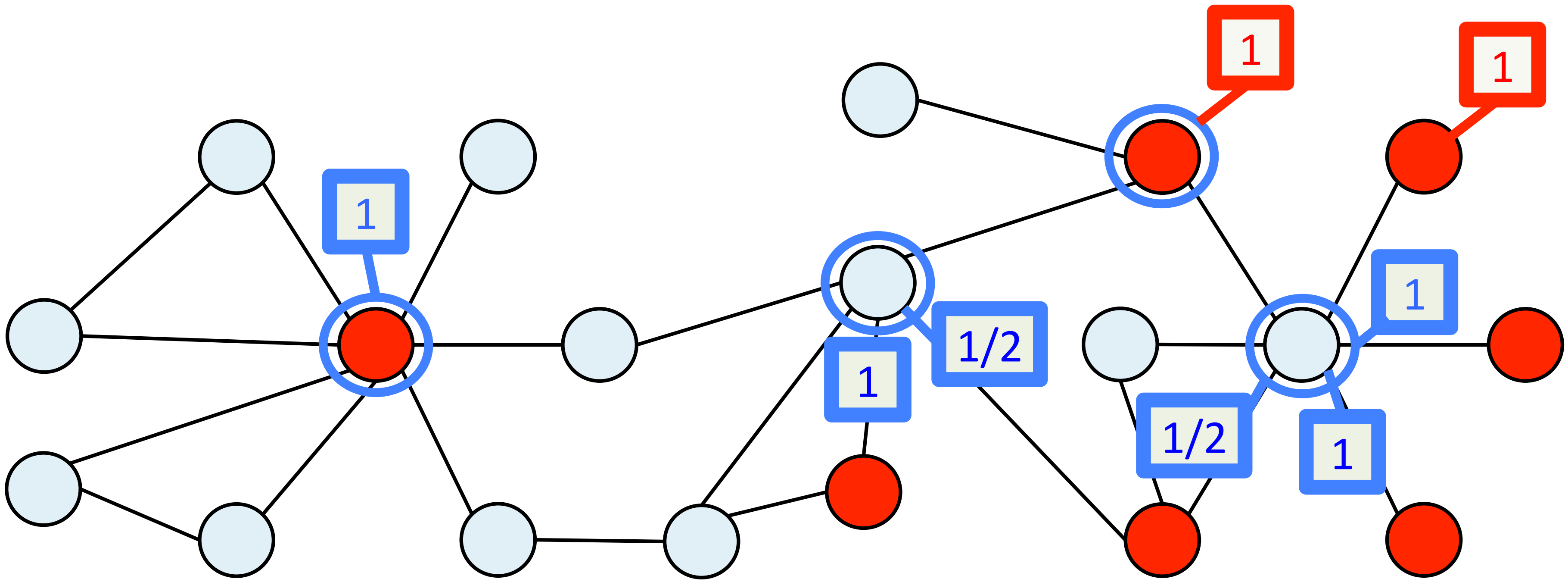

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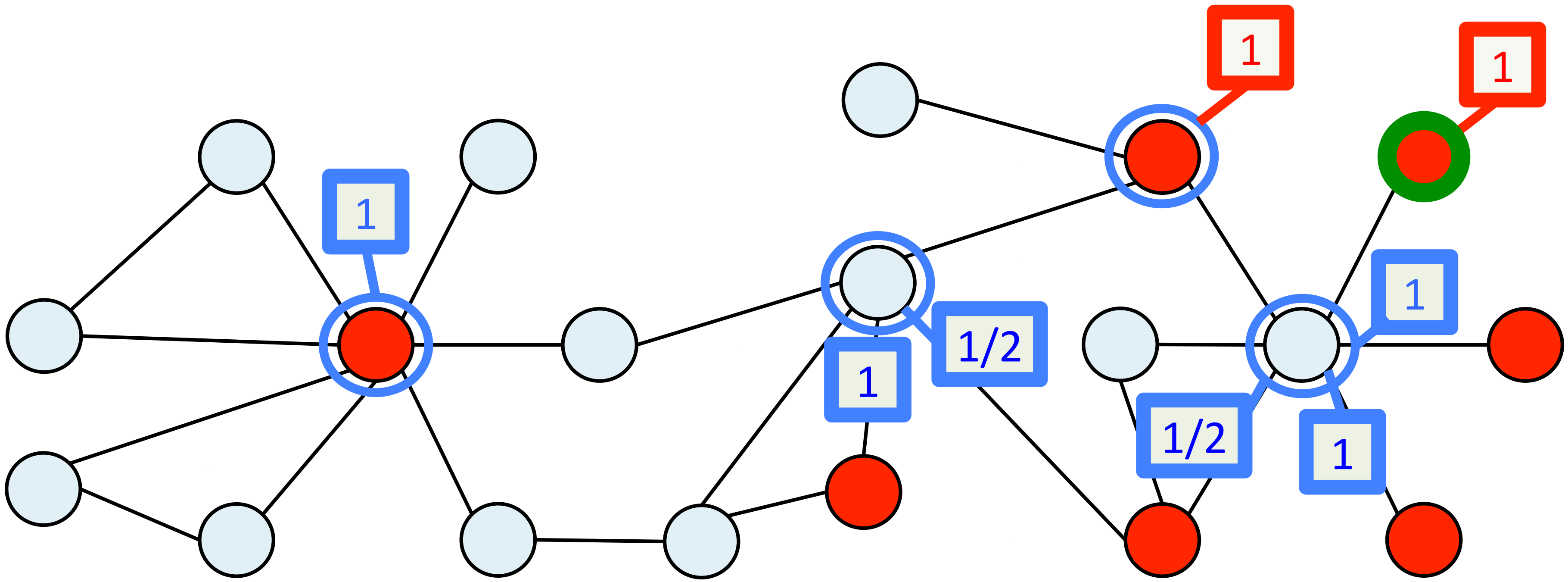

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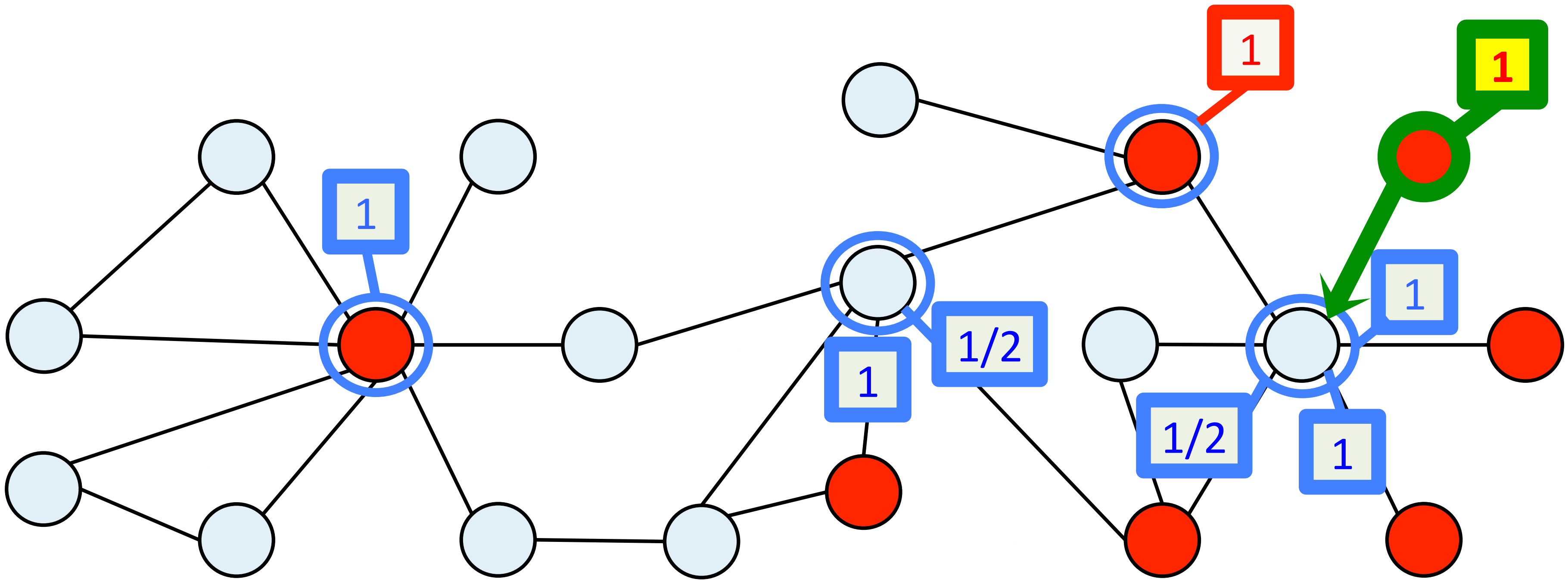

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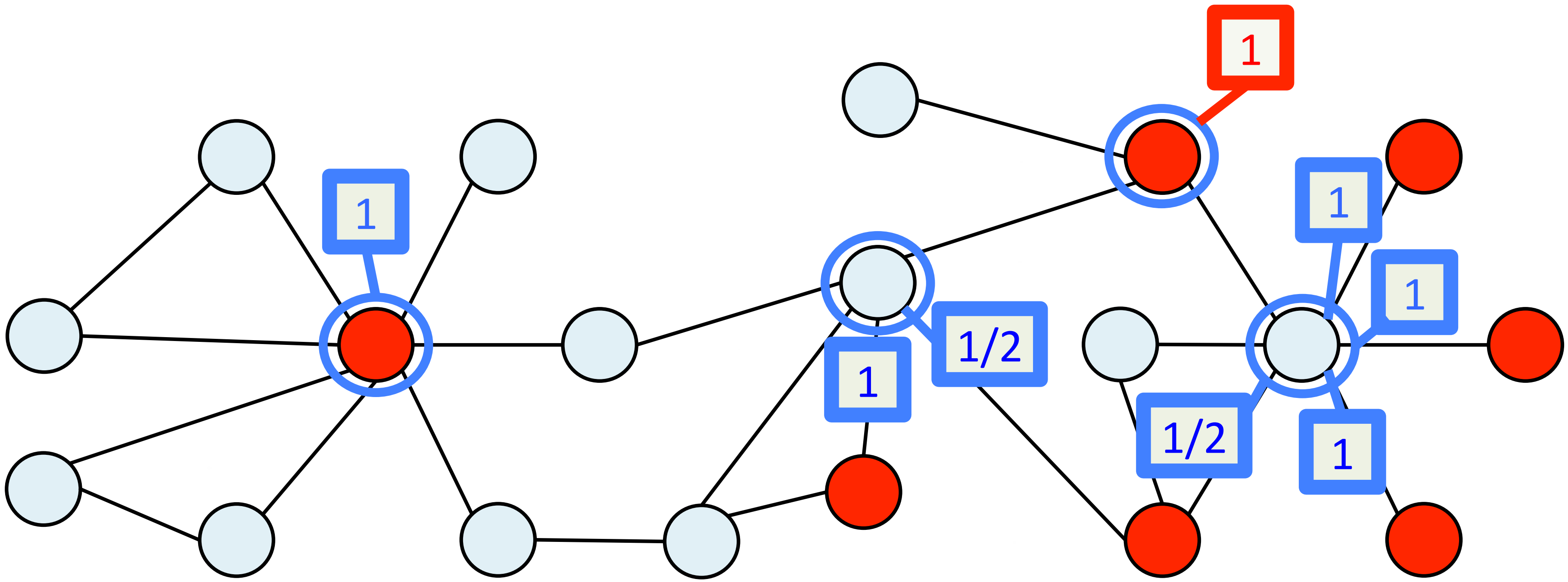

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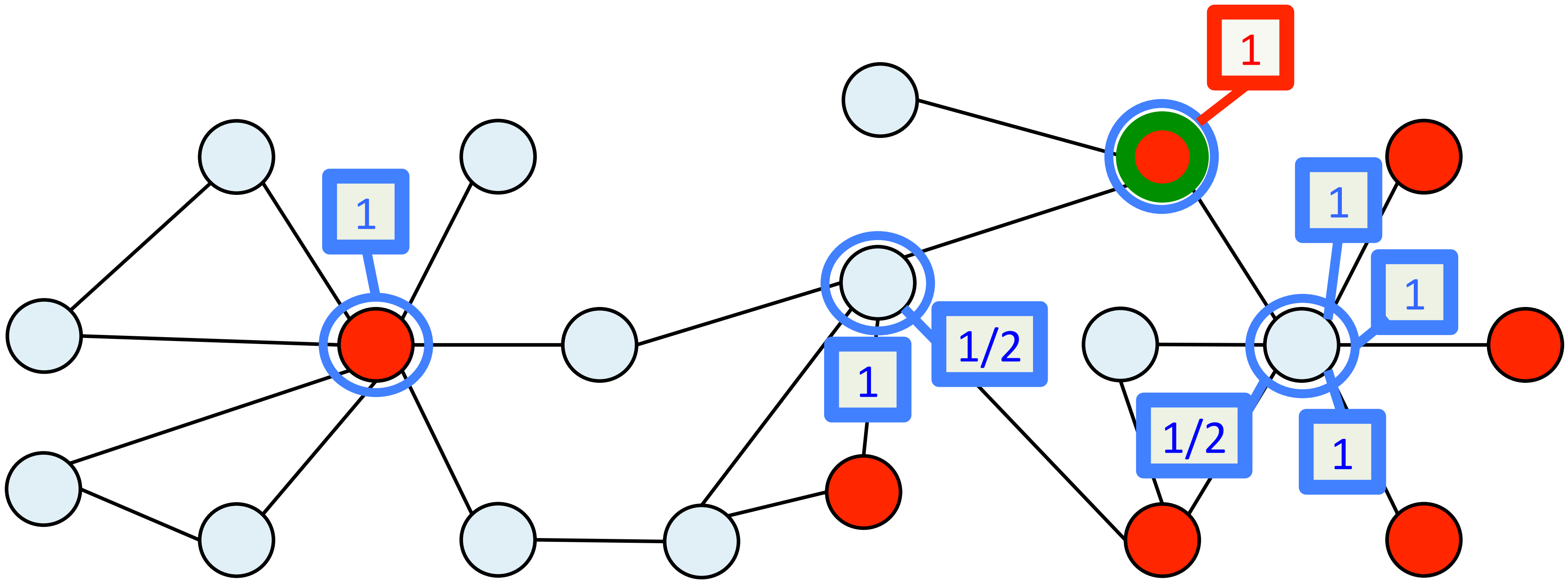

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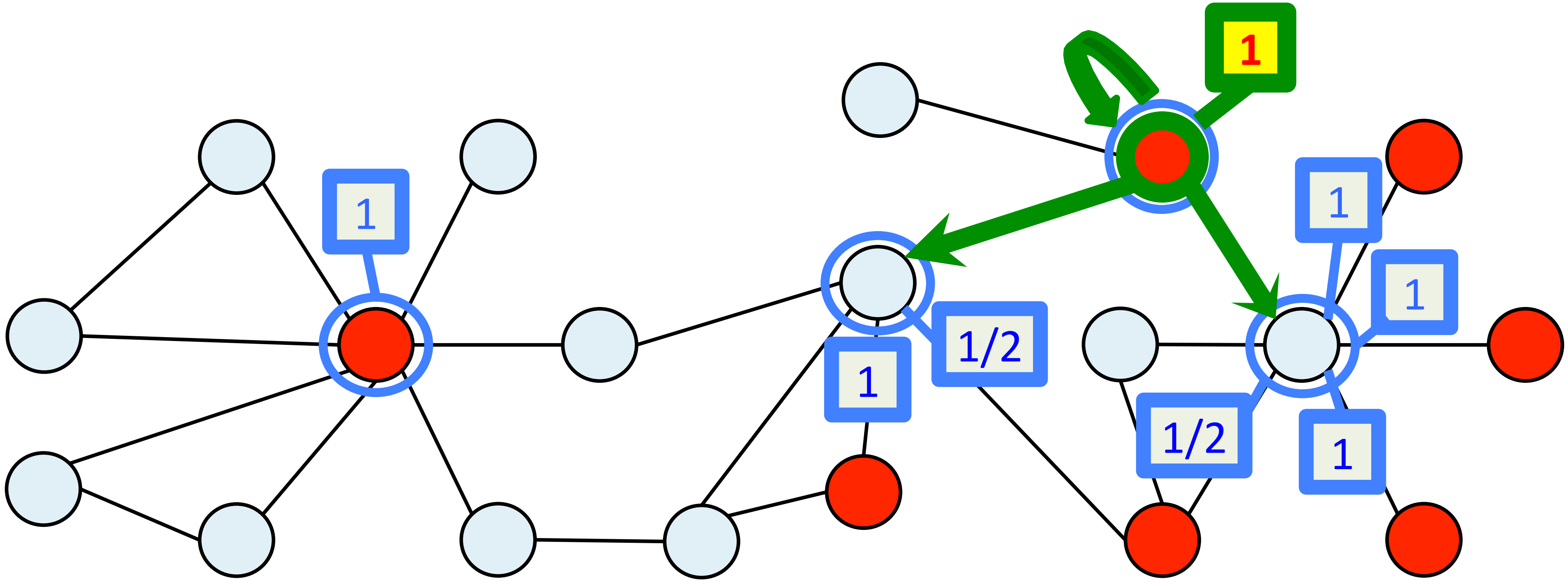

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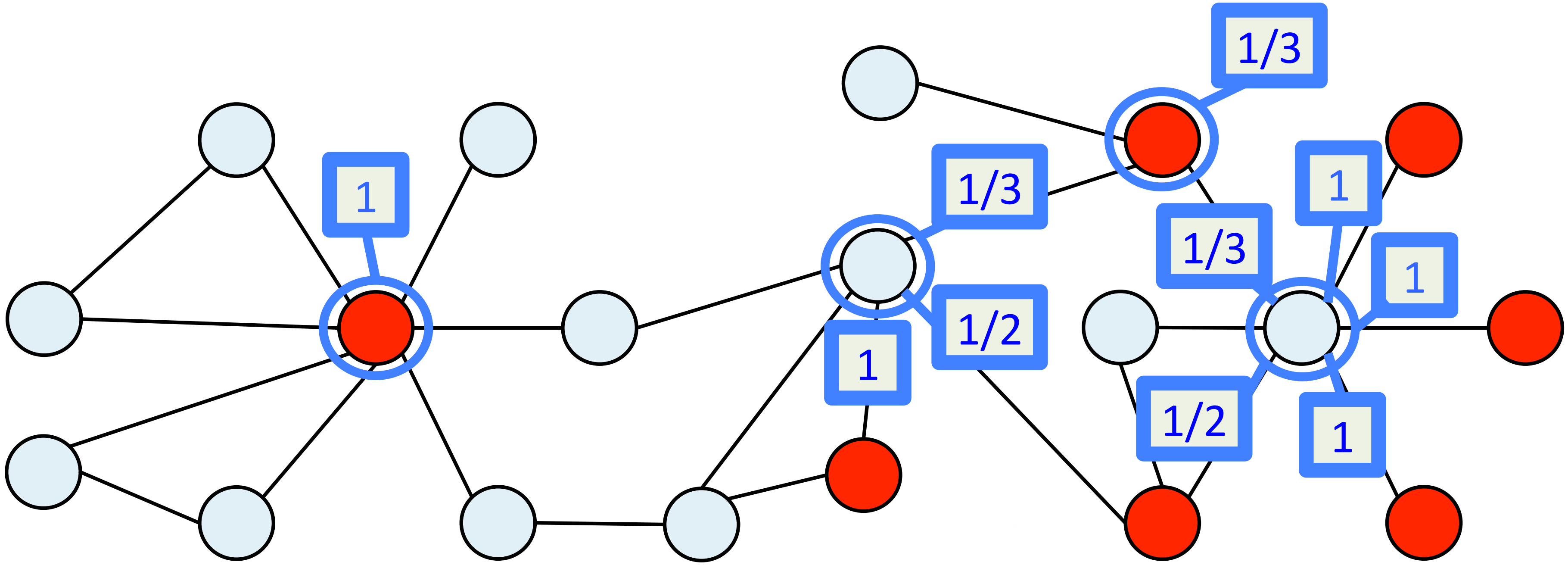

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D^* – a minimum dominating set (optimum solution)

$$\frac{|D|}{|D^*|} \leq \;??$$

$$f : D^* \to (0, 5]$$

$$f(v^*) = \sum_{u \in N_D[v^*]} \frac{1}{|N_{D^*}[u]|}$$

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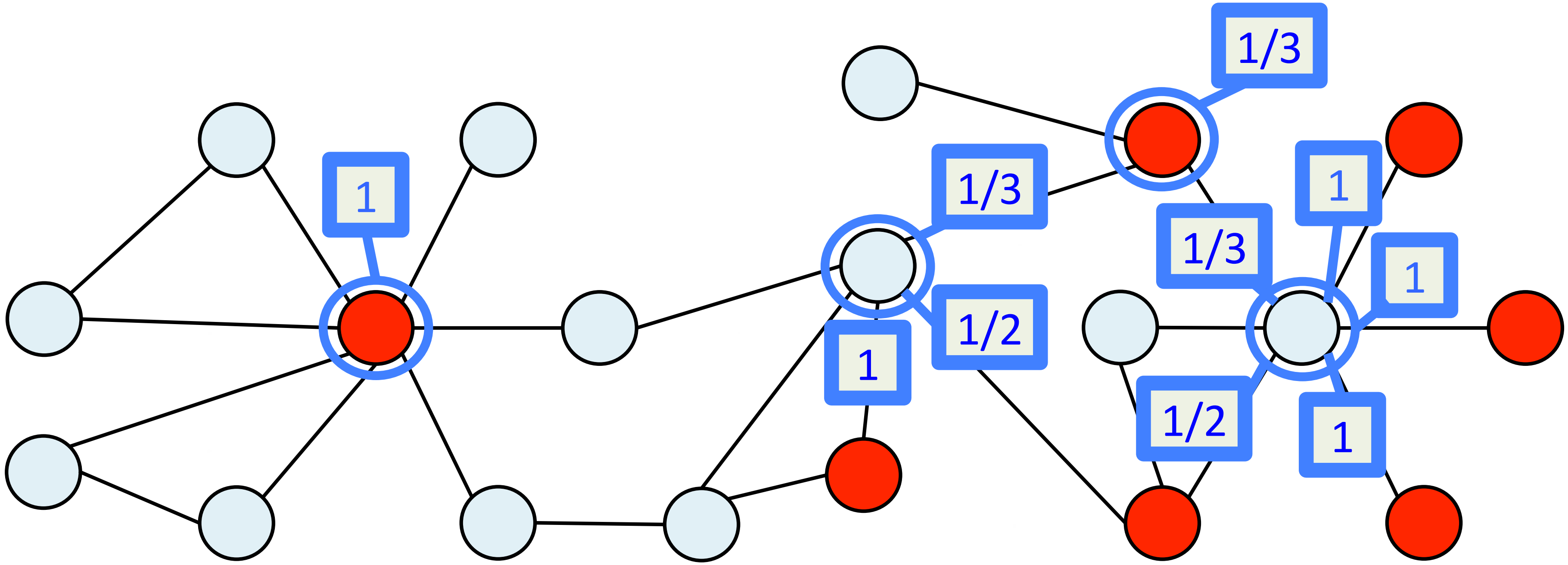

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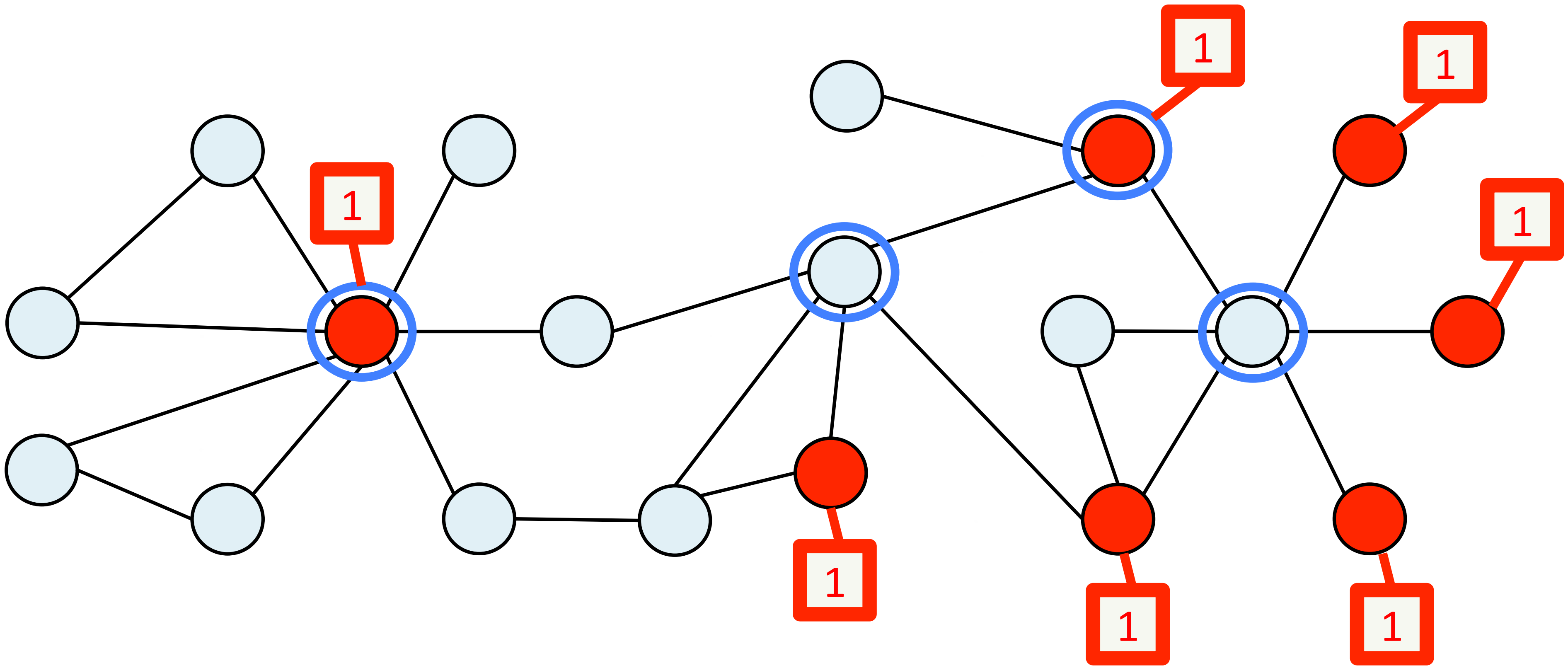

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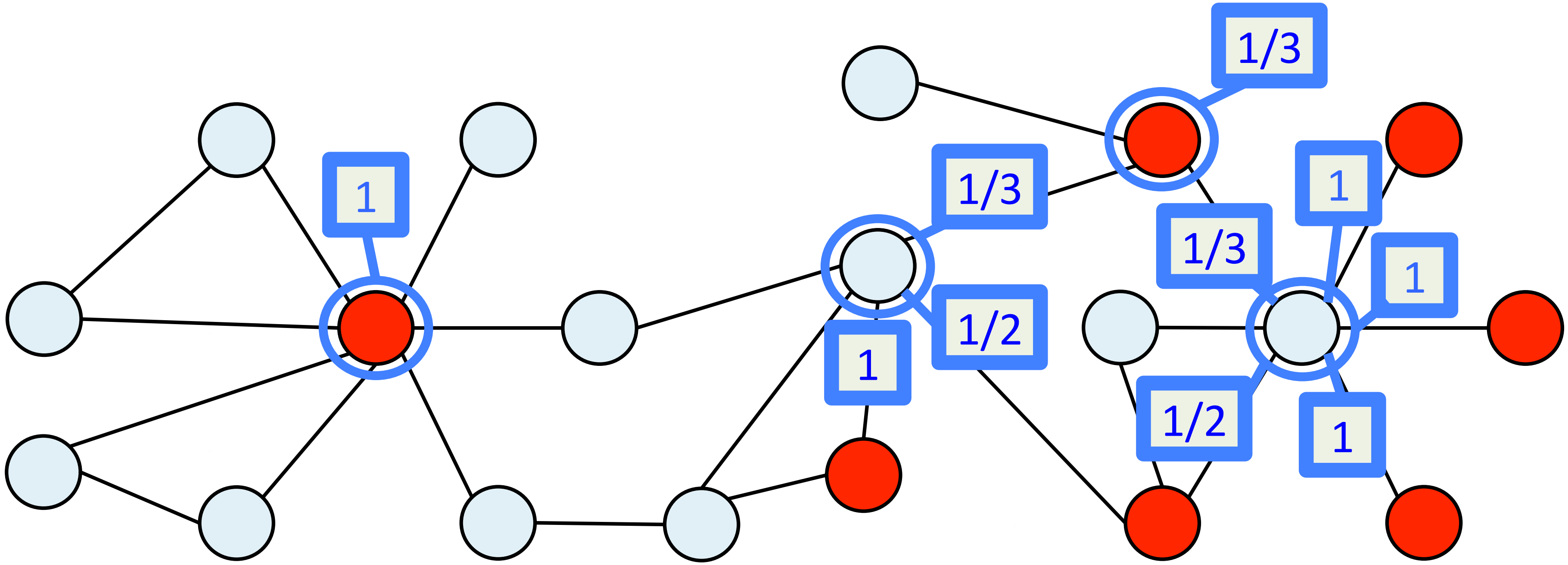

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- *D* – a maximal independent set with no reducible coronas (algorithm output)
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average of *f*(.)

4.888...-approximation

D

D^*

$$f : D^* \longrightarrow (0, 5]$$

$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f\,(.) \text{ over } D^* \leq 4{,}888\ldots}$$

4.888...-approximation

D $\qquad f : D^* \longrightarrow (0, 5]$

D^* $\qquad \dfrac{|D|}{|D^*|} = \boxed{\text{average of } f\,(.) \text{ over } D^* \leq 4{,}888\ldots}$

c^*

$4{,}888\ldots < f\,(c^*) \leq 5$

4.888...-approximation

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$4{,}888\ldots < f\,(c^*) \leq 5$ $\qquad\qquad f\,(r^*) \leq 4$

4.888...-approximation

D

D^*

$f : D^* \longrightarrow (0, 5]$

$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f\,(.) \text{ over } D^* \;\leq\; 4{,}888...}$$

$4{,}888... < f\,(c^*) \leq\; 5$ $\qquad\qquad$ $f\,(r^*) \leq 4$

$\Updownarrow$

$f\,(c^*) = \; 5$

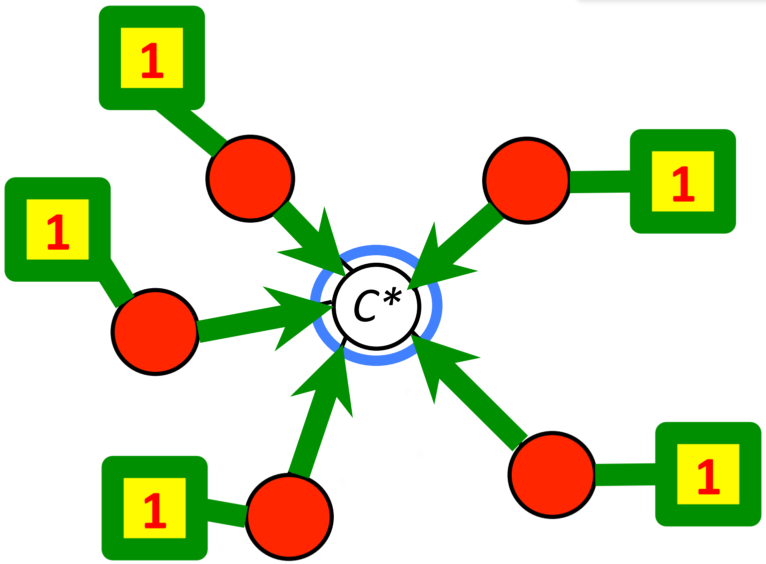
4.888...-approximation

D

D^*

$f : D^* \longrightarrow (0, 5]$

$\frac{|D|}{|D^*|} =$ average of $f(.)$ over D* $\leq$ 4,888...



$f(c^*) = 5$

4.888...-approximation

D $\qquad f : D^* \longrightarrow (0, 5]$

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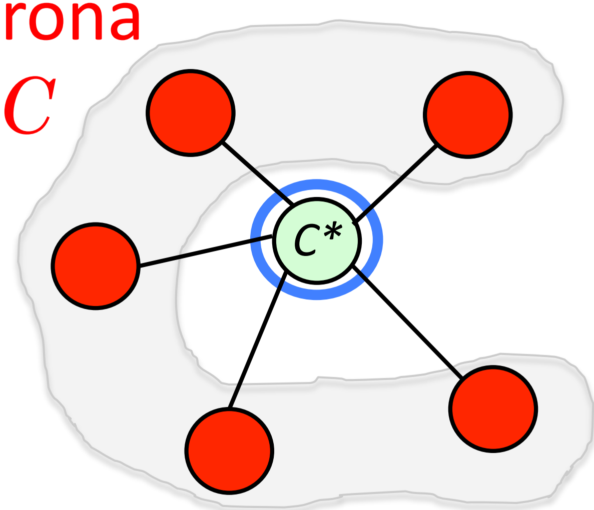
corona

C



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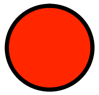
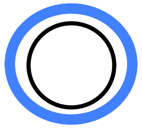
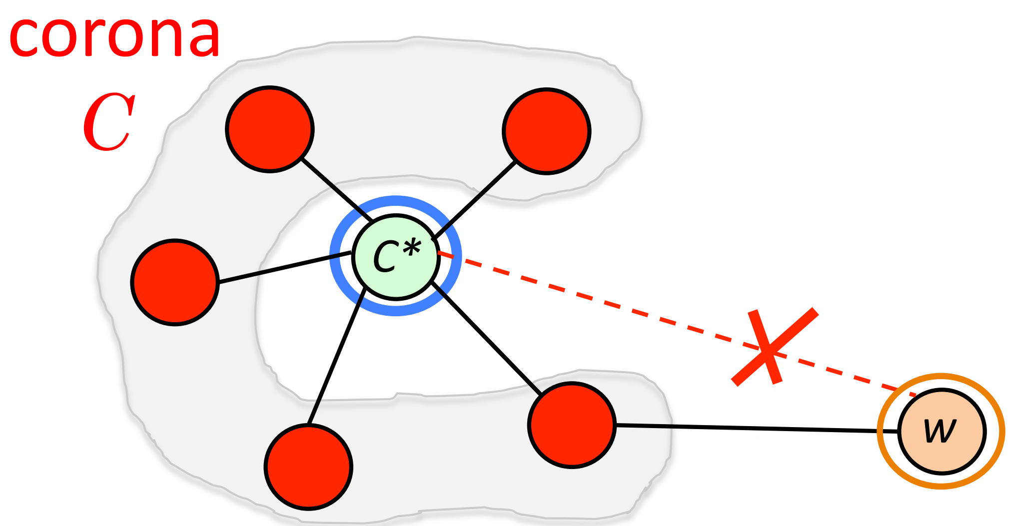
4.888...-approximation

 D $\qquad f : D^* \longrightarrow (0, 5]$

 D^* $\qquad \dfrac{|D|}{|D^*|} = \boxed{\text{average of } f\,(.) \text{ over } D^* \;\le\; 4{,}888...}$



$f\,(c^*) = 5$ $\qquad N_D[w] \subseteq C$

4.888...-approximation

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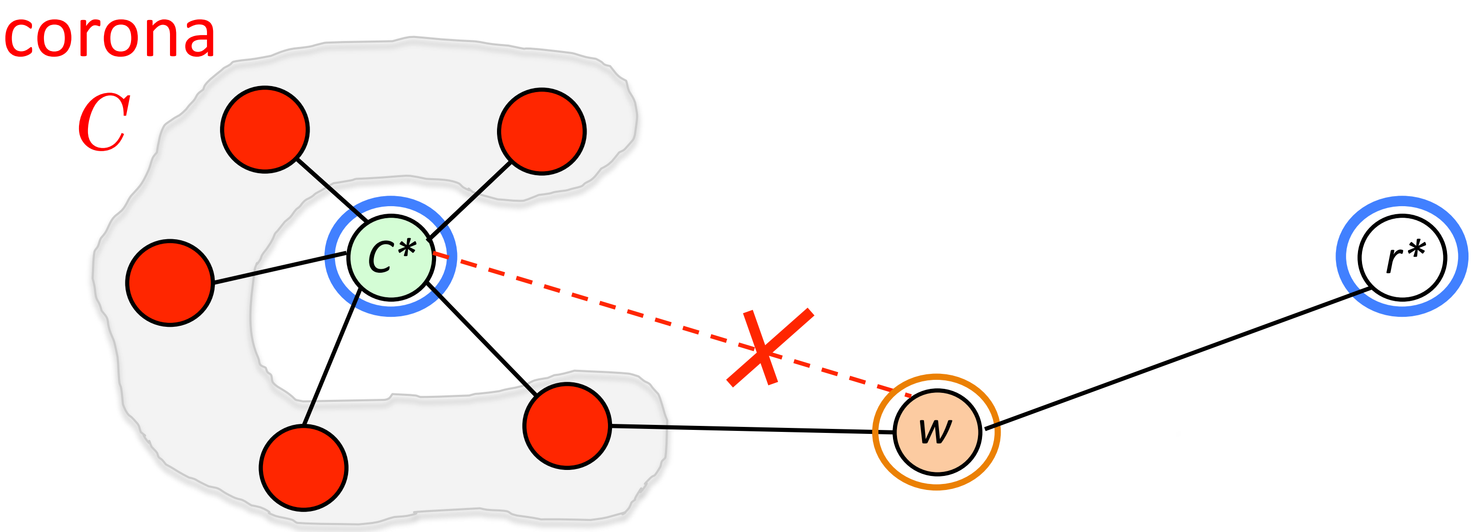

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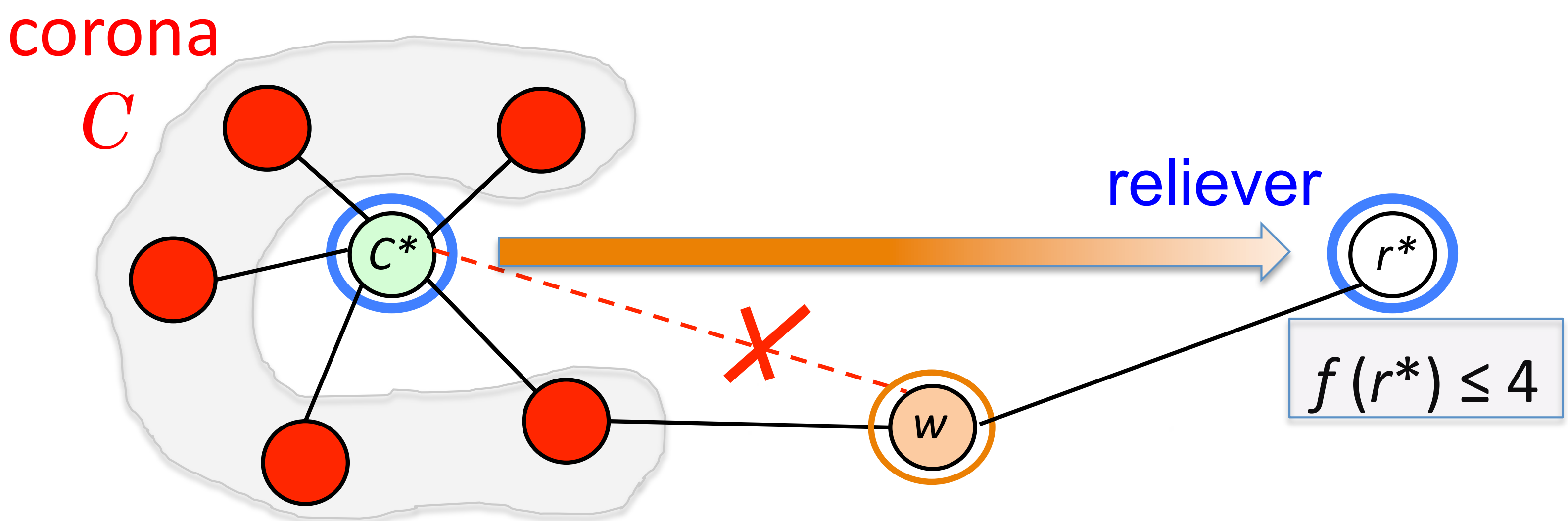

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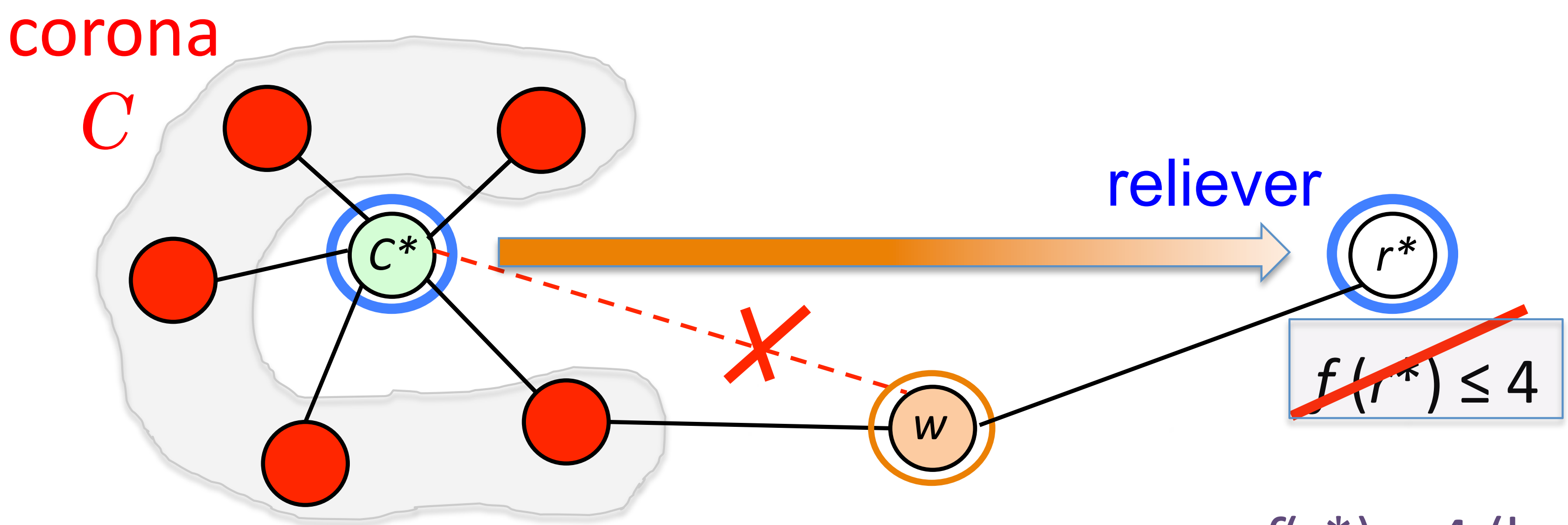

$f(r^*) > 4$ (hypothesis)

$f(c^*) = 5 \qquad N_D[w] \subseteq C$

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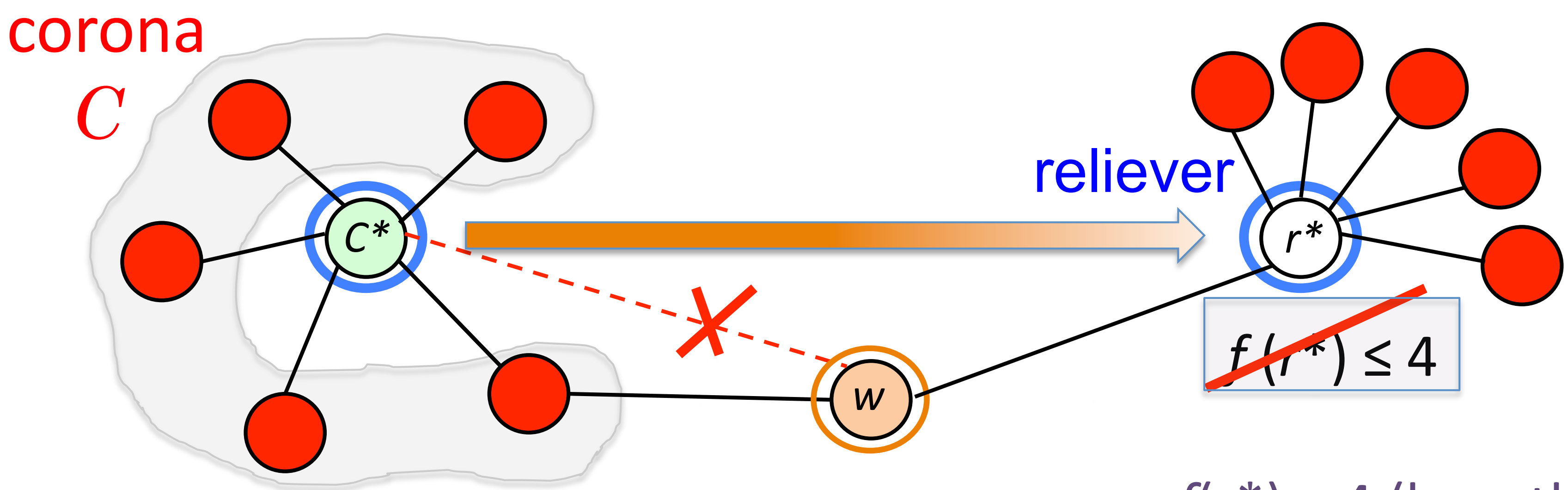

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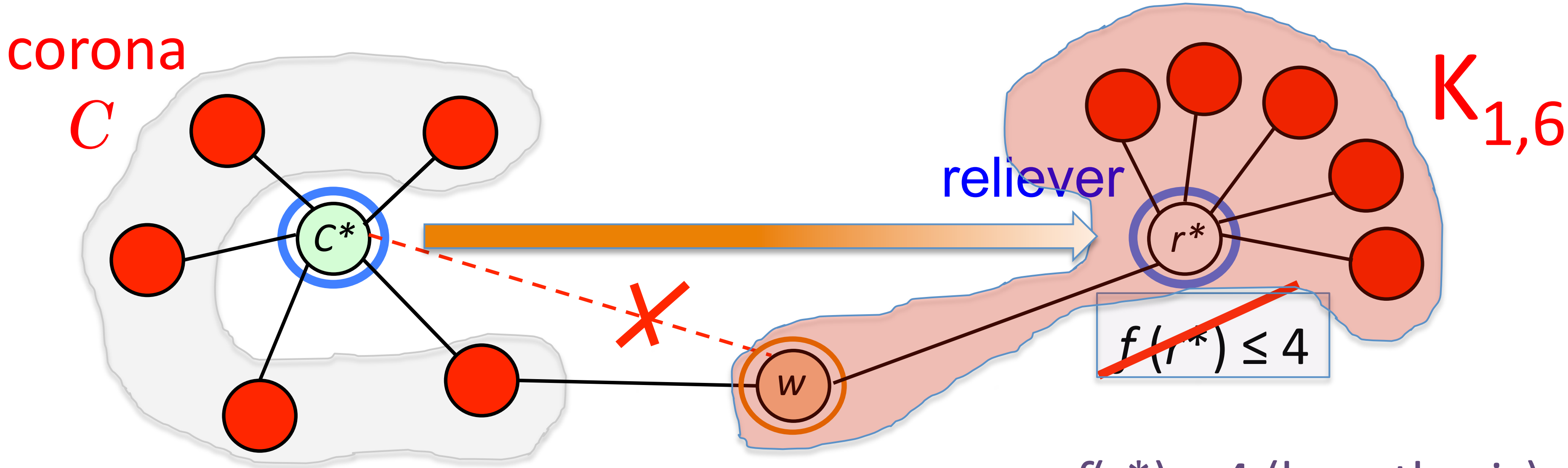

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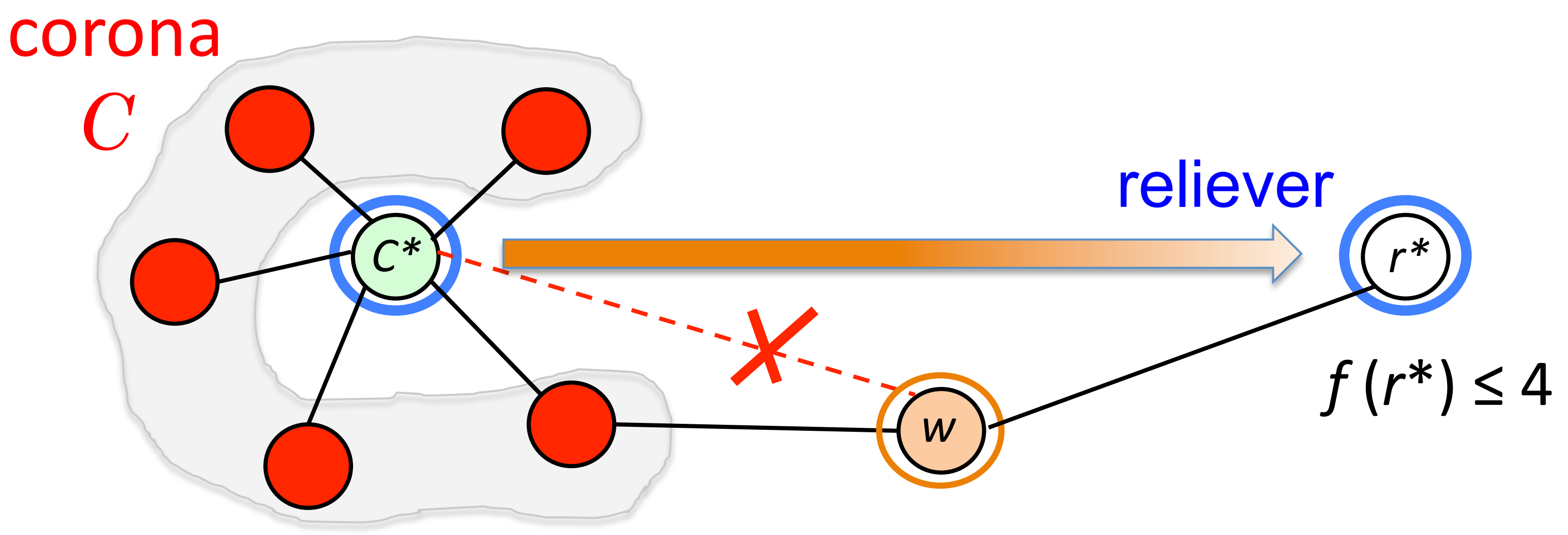

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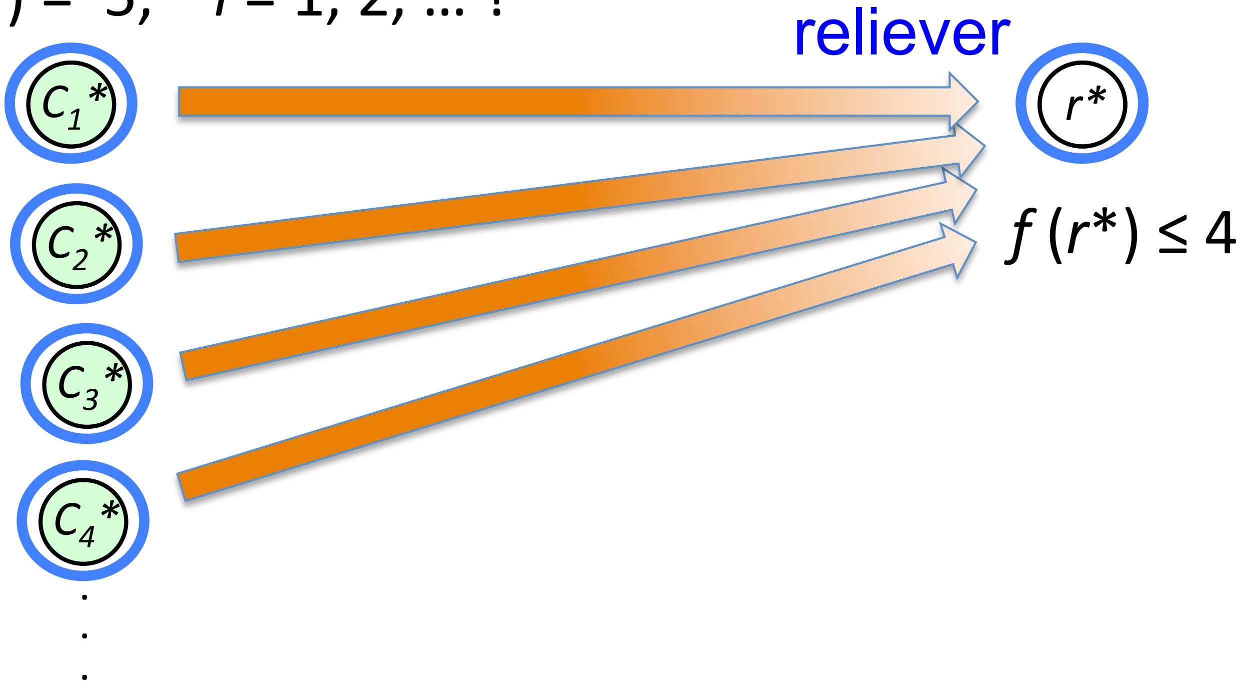
$f\,(c_i^*) = 5, \quad i = 1, 2, \ldots\ ?$



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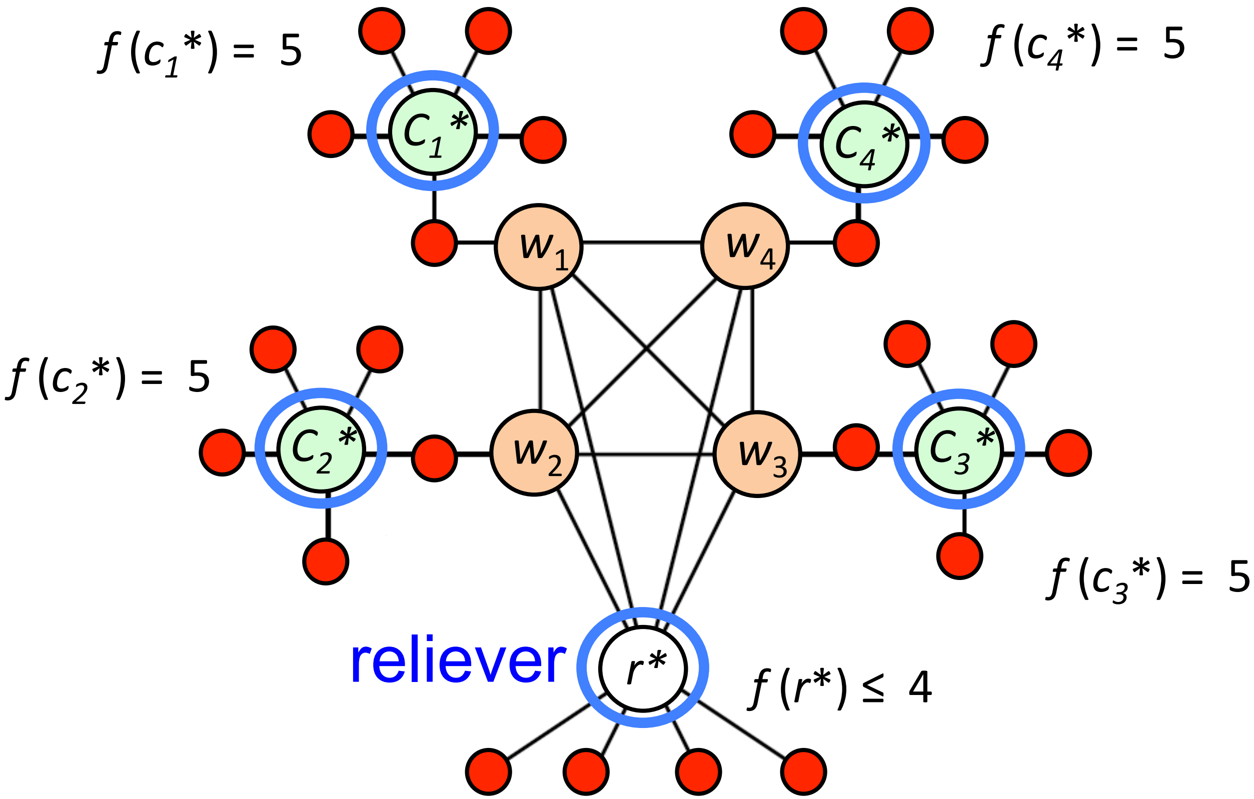

Some geometric lemmas

Lemma 1 (Pál 1921): If a set of points *P* has diameter 1, then *P* can be enclosed by a circle of radius $1/\sqrt{3}$.

Lemma 2 (Fodor 2007): The radius of the smallest circle enclosing 13 points with mutual distance ≥ 1 is $(1+\sqrt{5})/2$.

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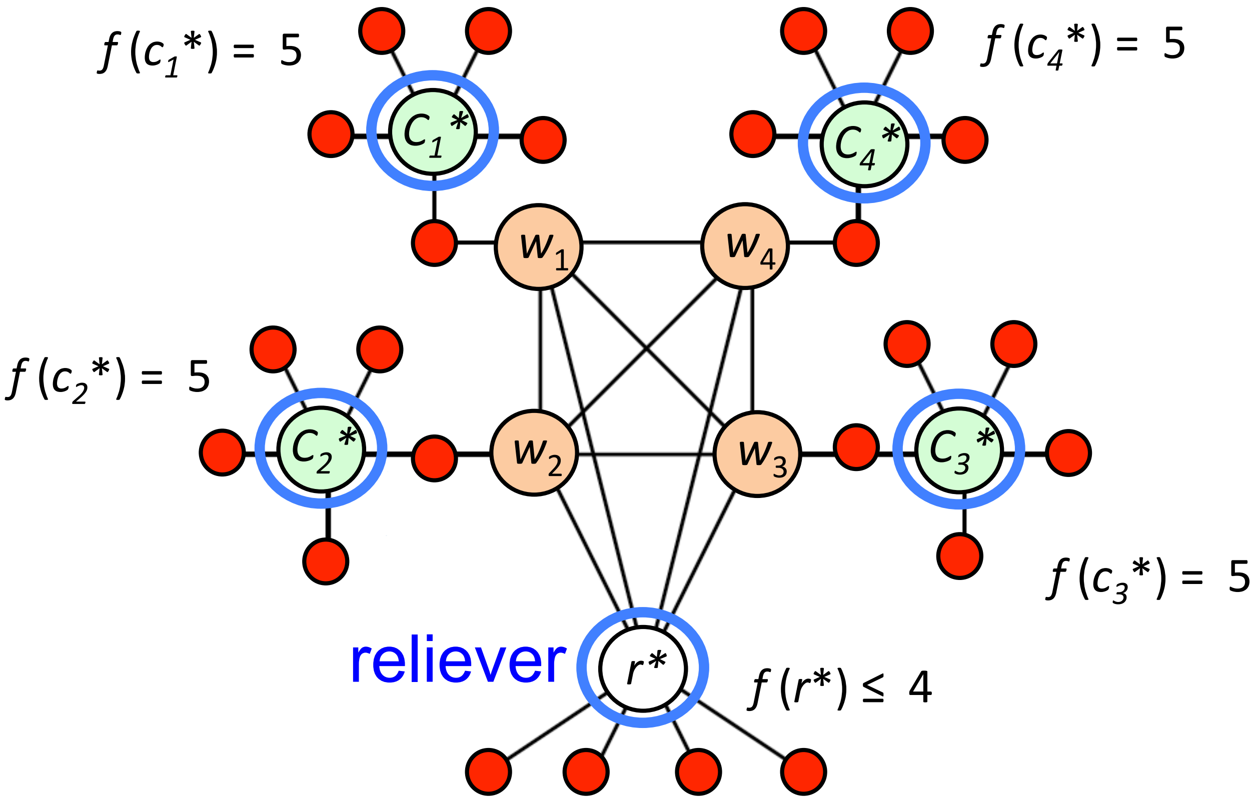
Lemma 6 (FFMS 2012): If G is a $(4,L)$-pendant unit disk graph, then $L \leq 8$.

Establishing the approximation factor



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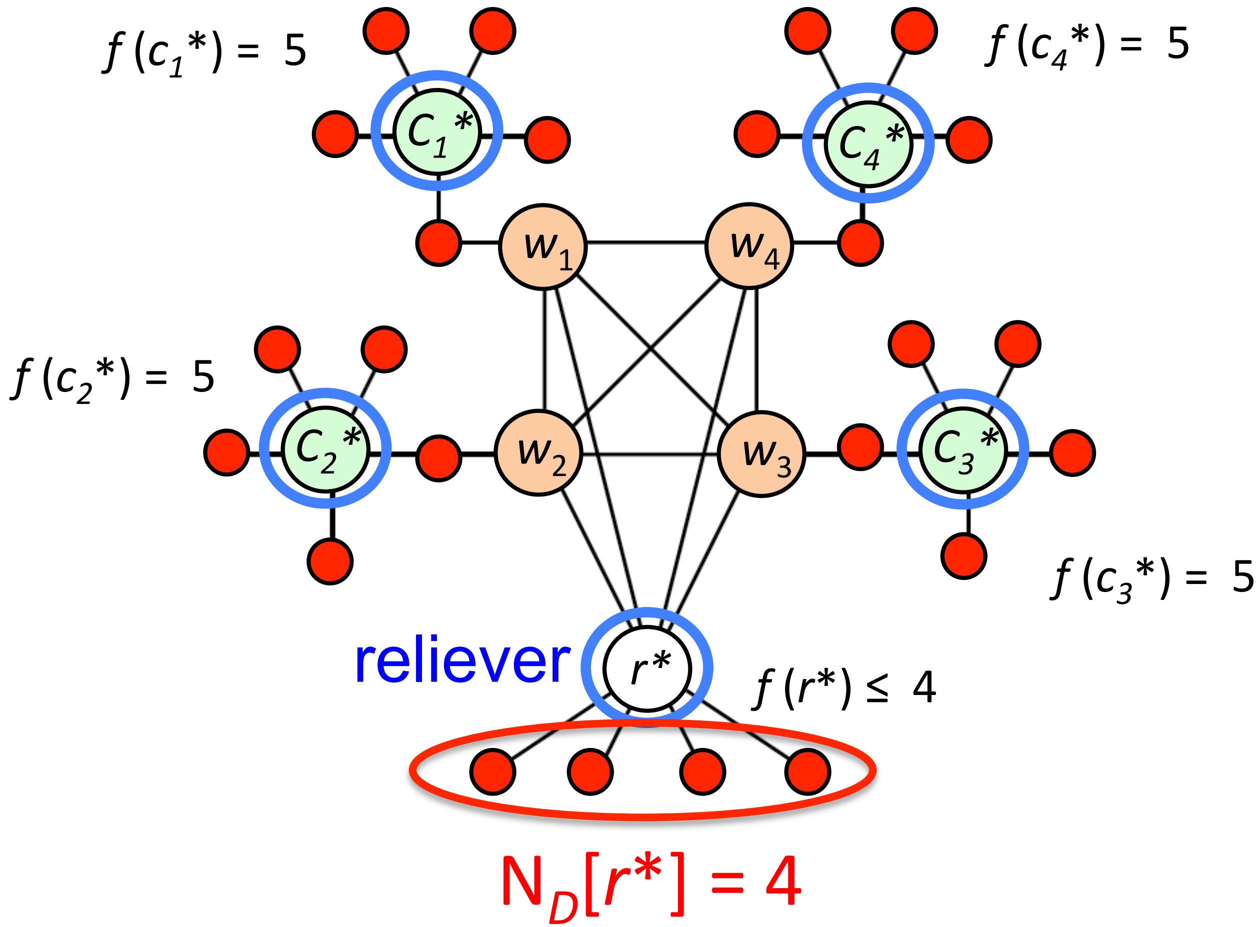

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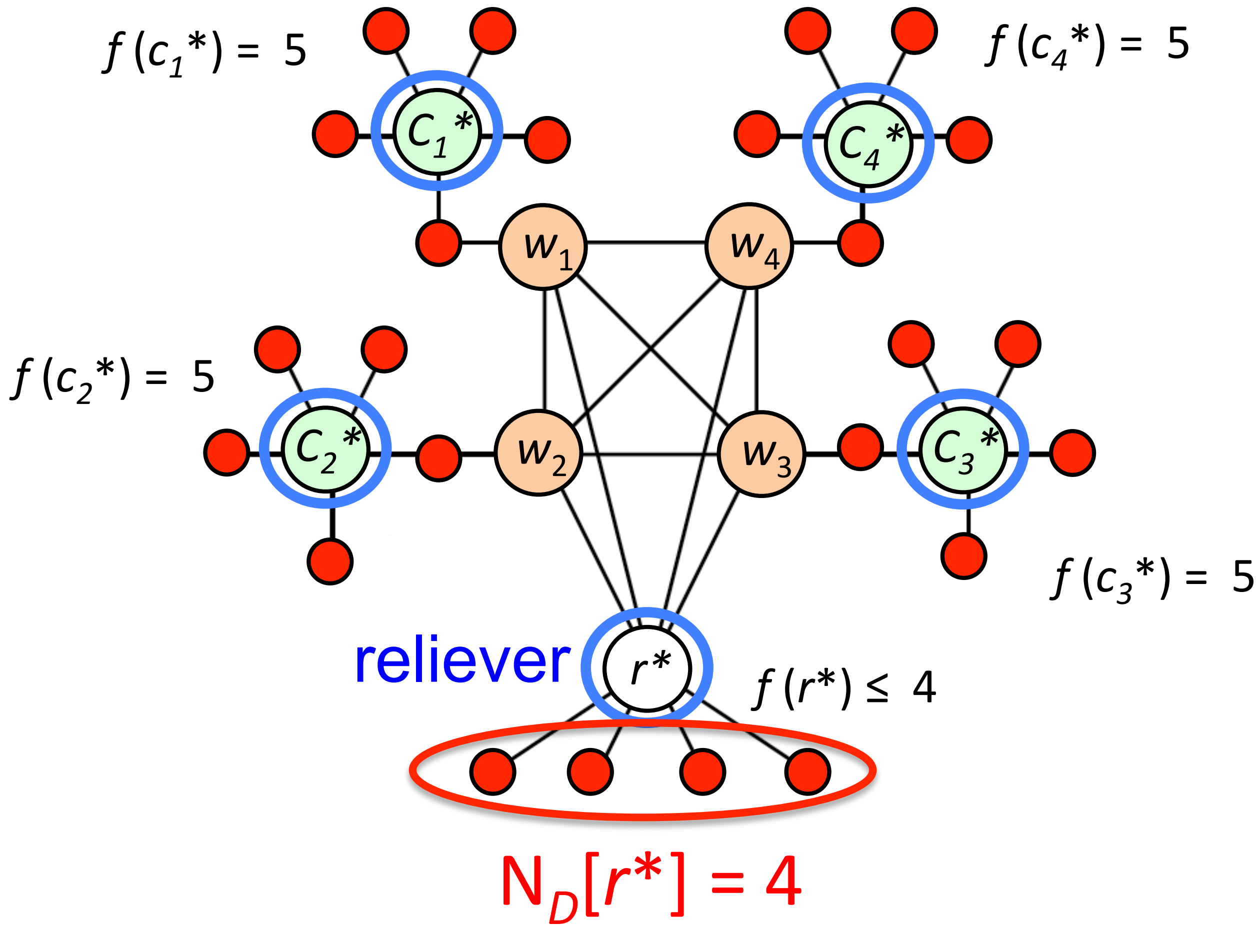

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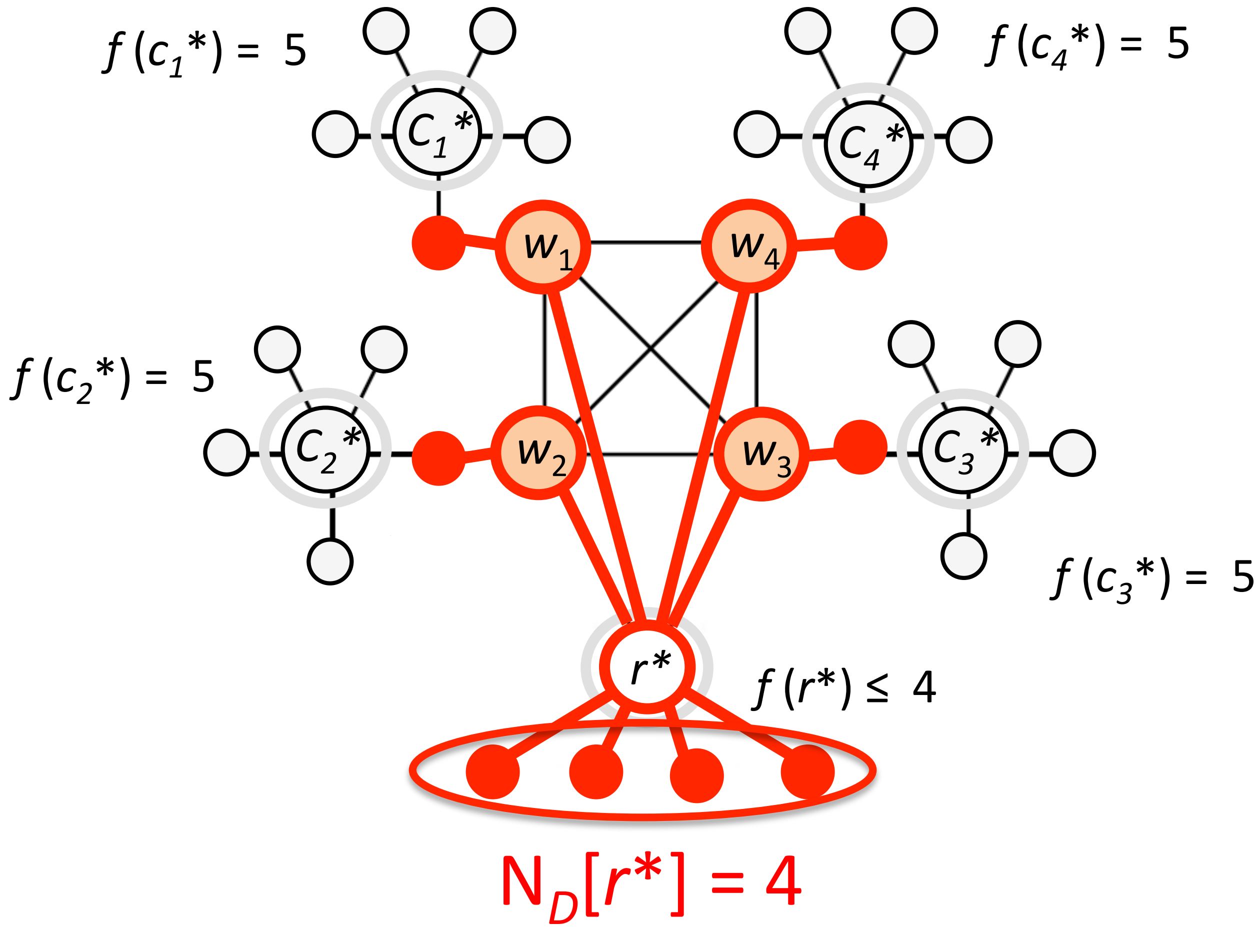

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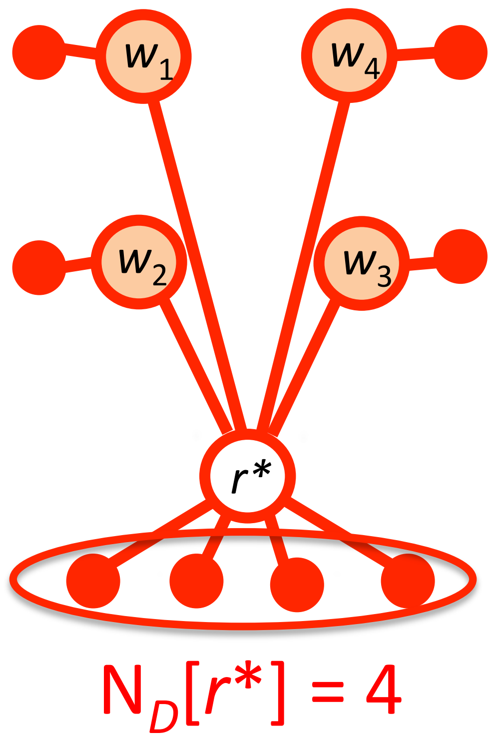

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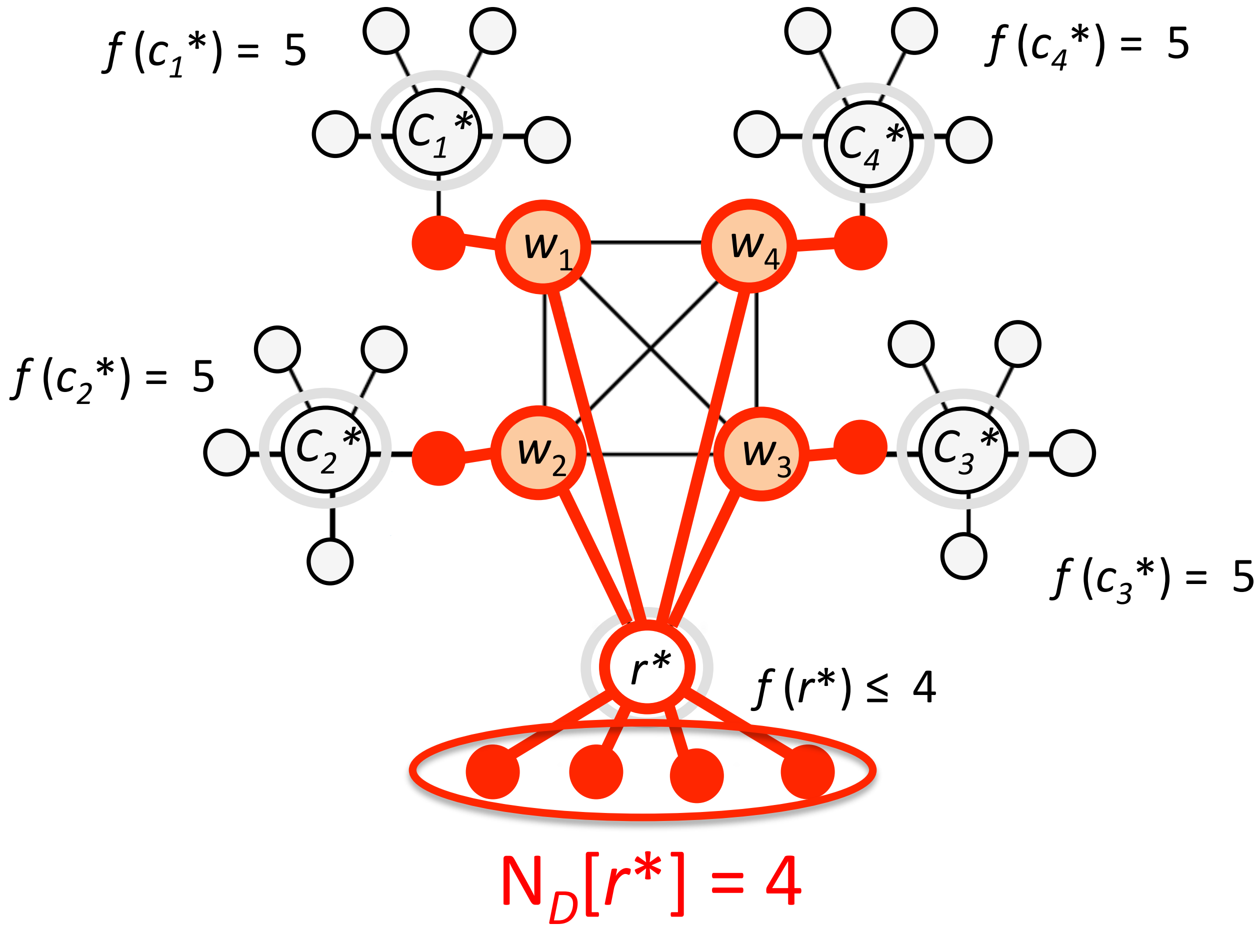

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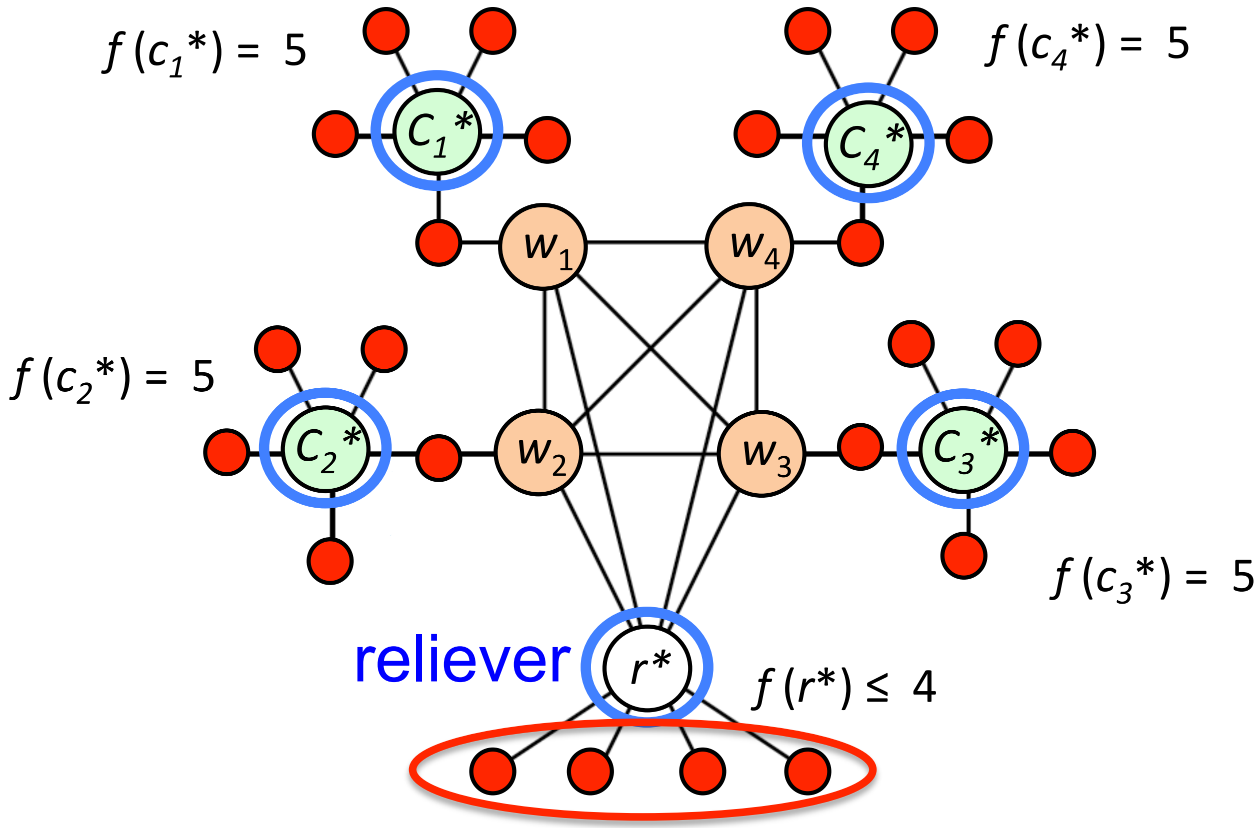
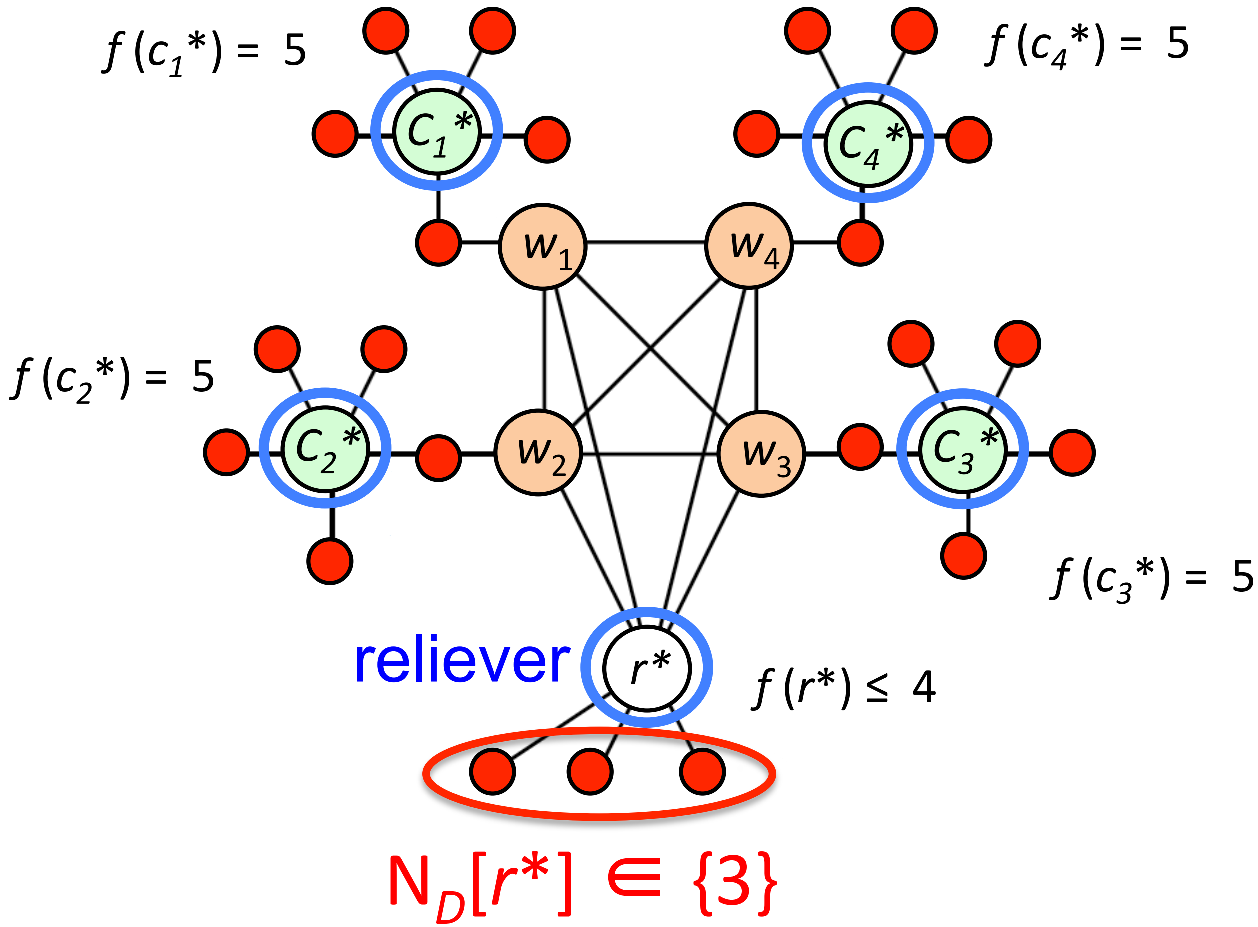


$N_D[r^*] = 4$ ⇒ at most **8** cores per reliever

Establishing the approximation factor



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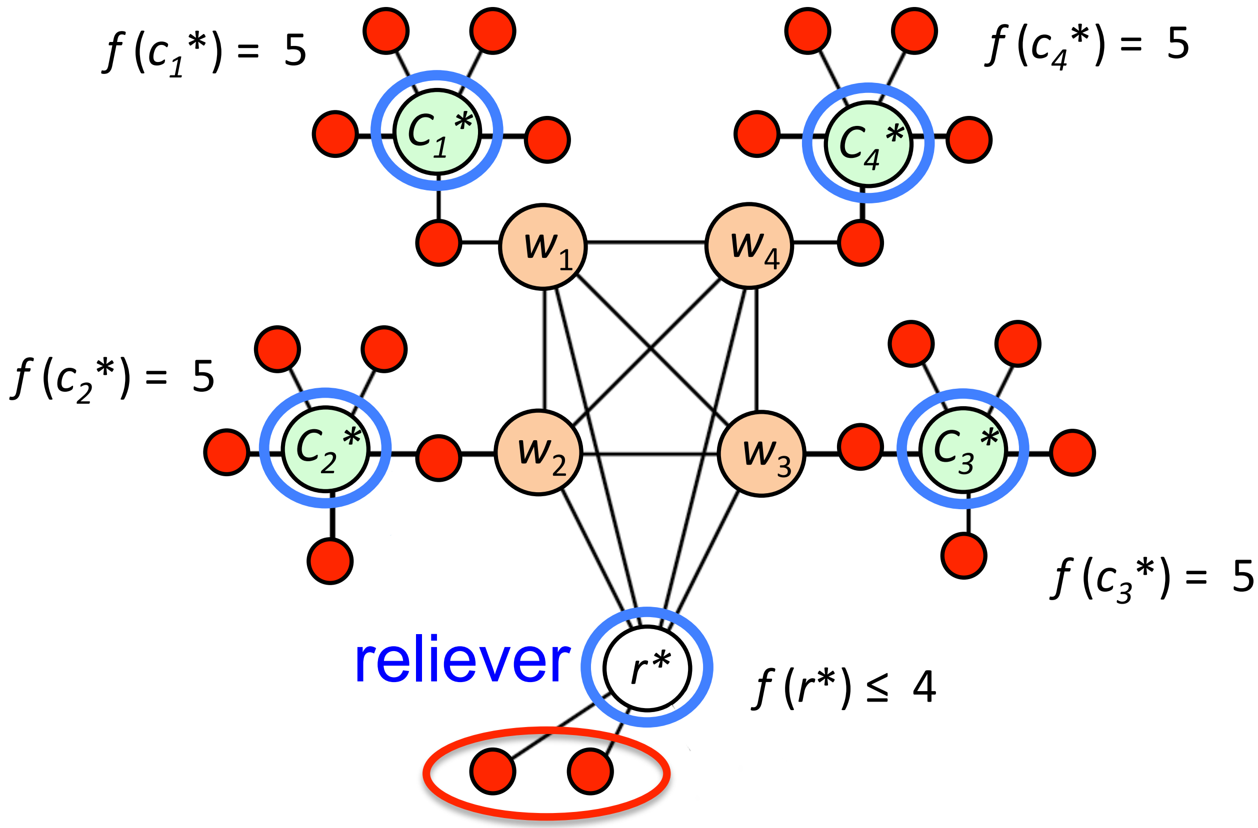
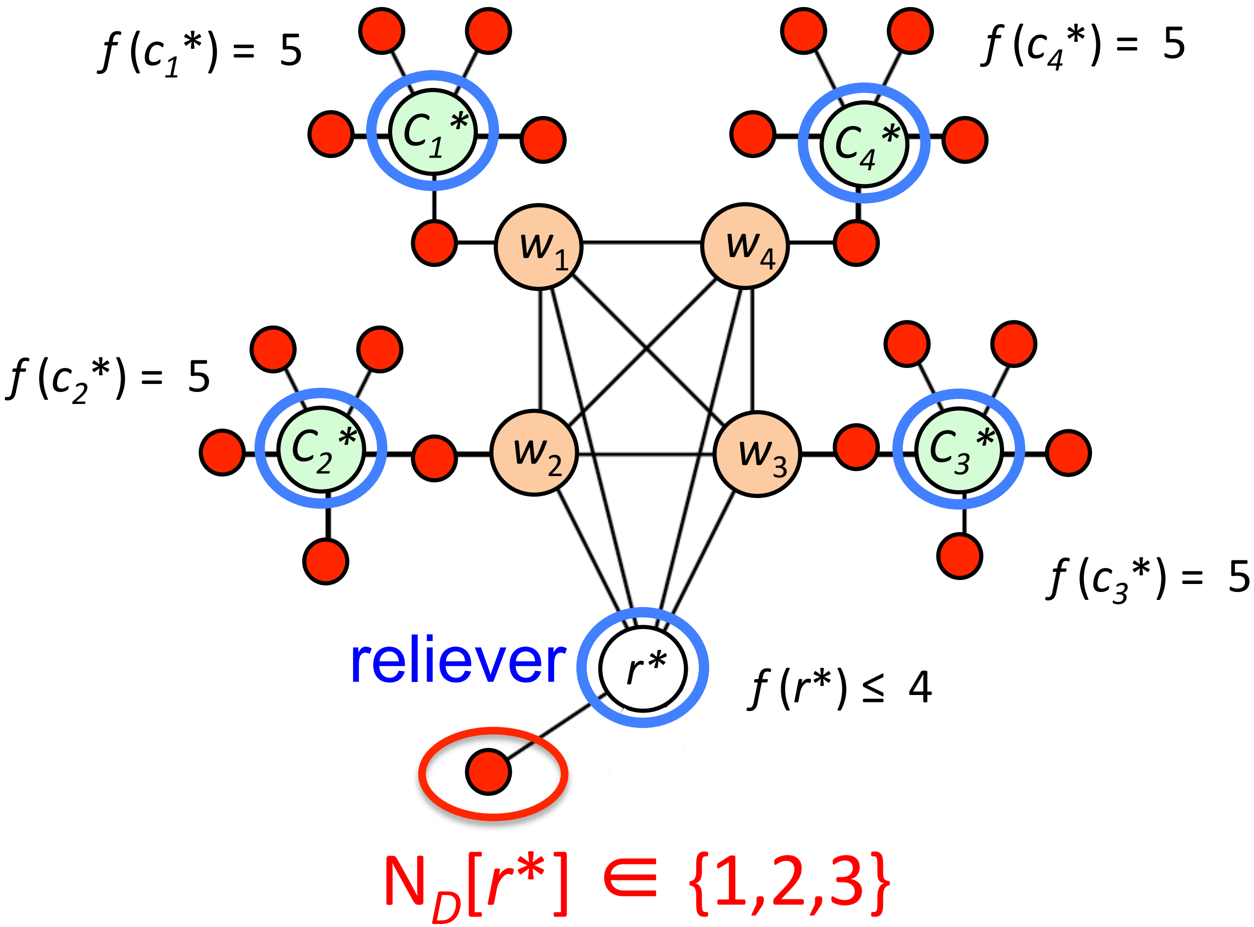


$N_D[r^*] \in \{2,3\}$

Establishing the approximation factor



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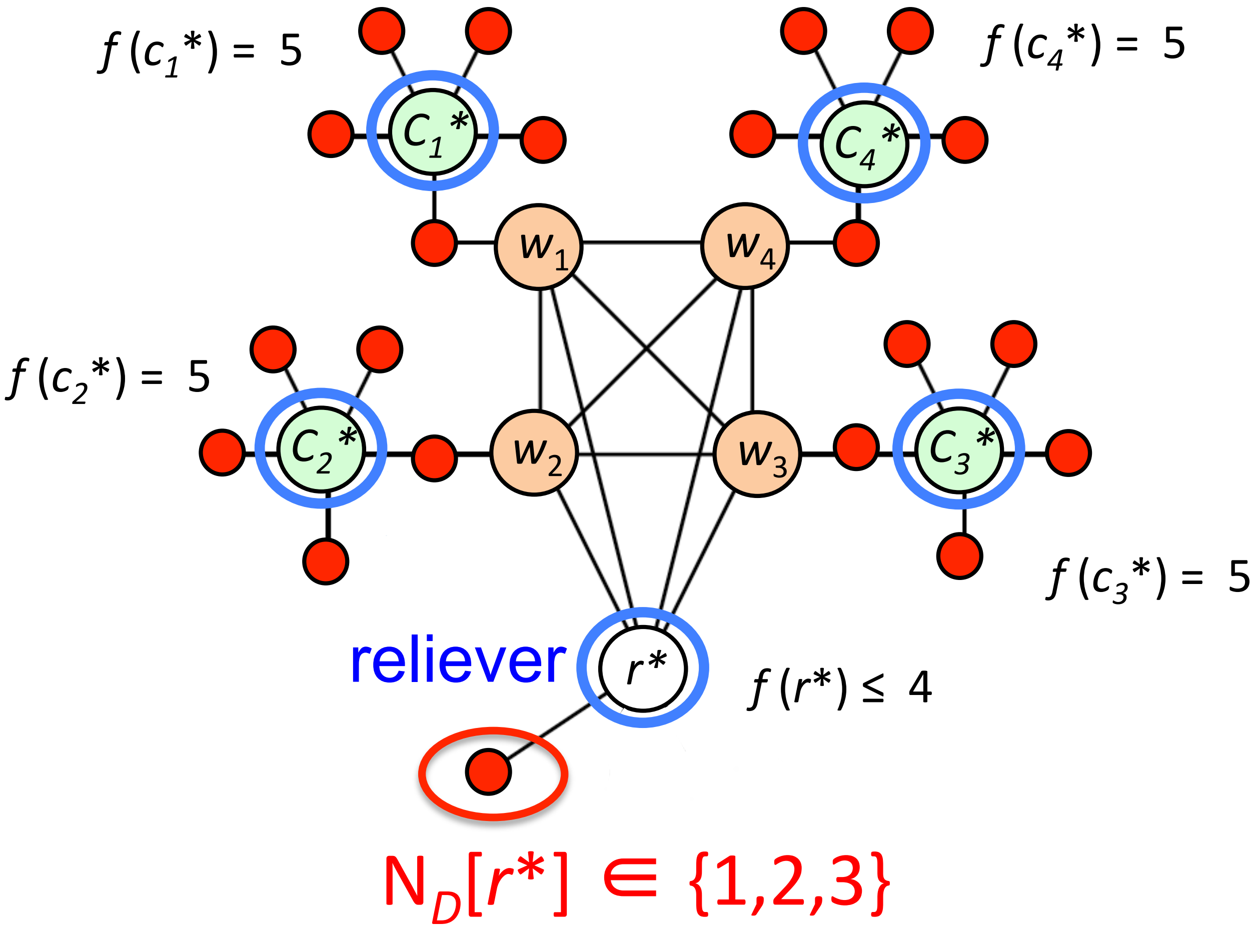
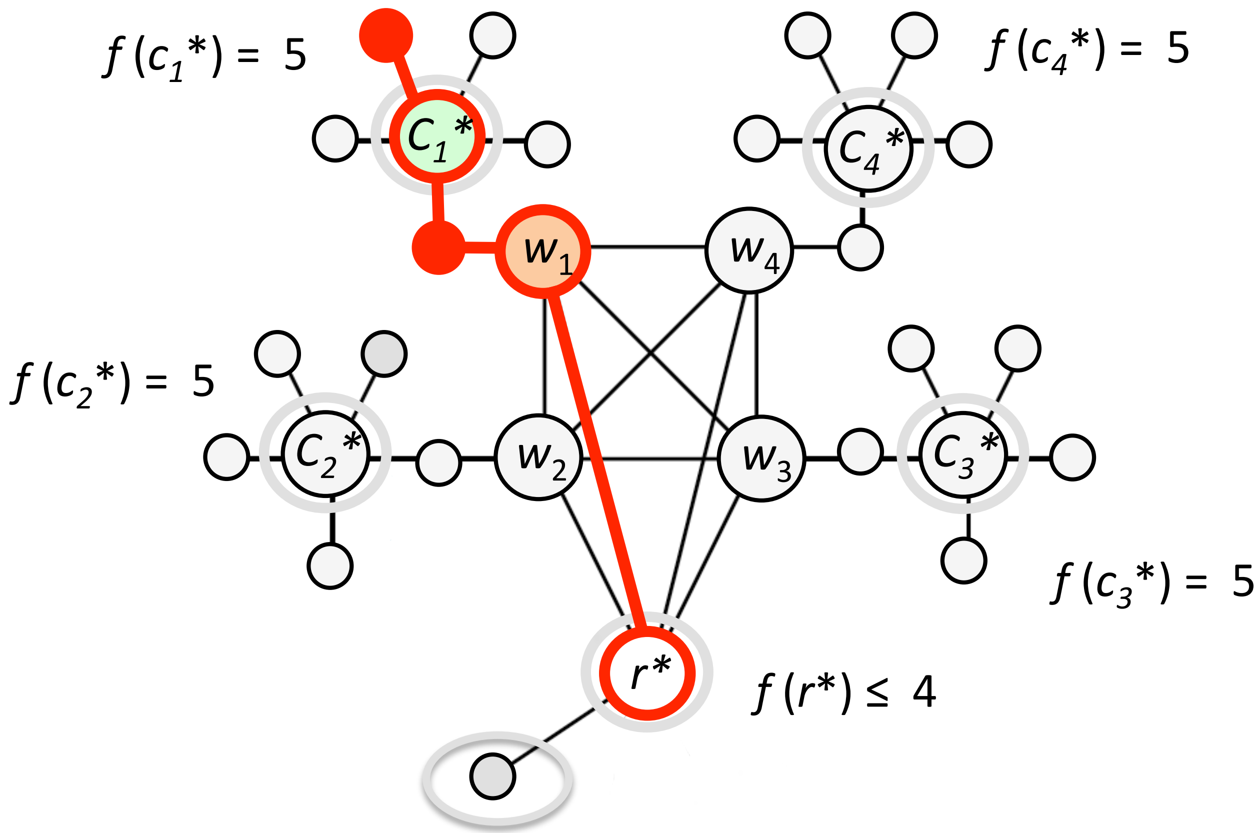
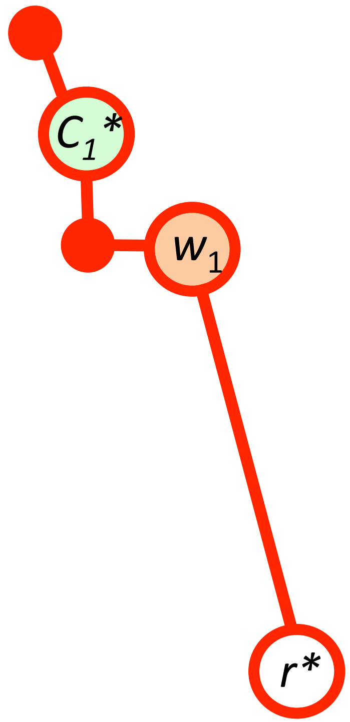


Establishing the approximation factor



$N_D[r^*] \in \{1,2,3\}$

Establishing the approximation factor



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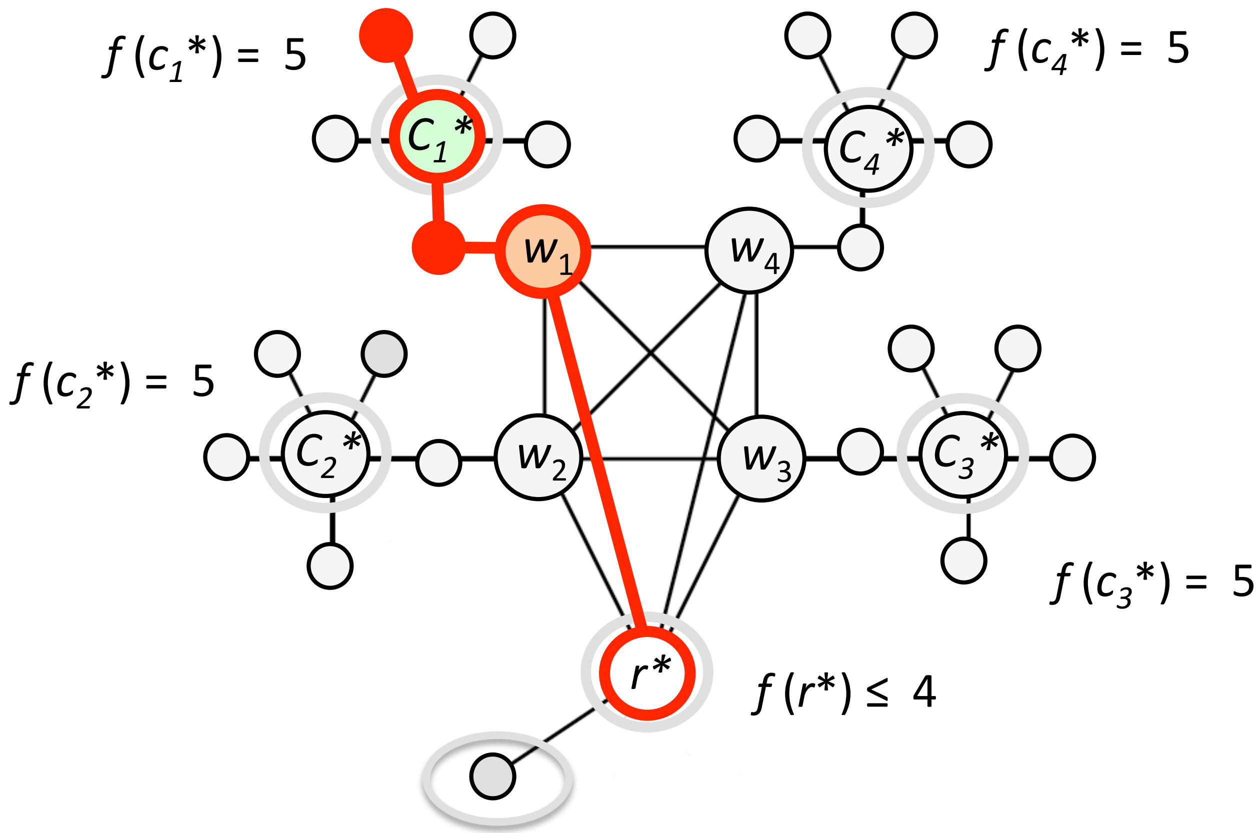
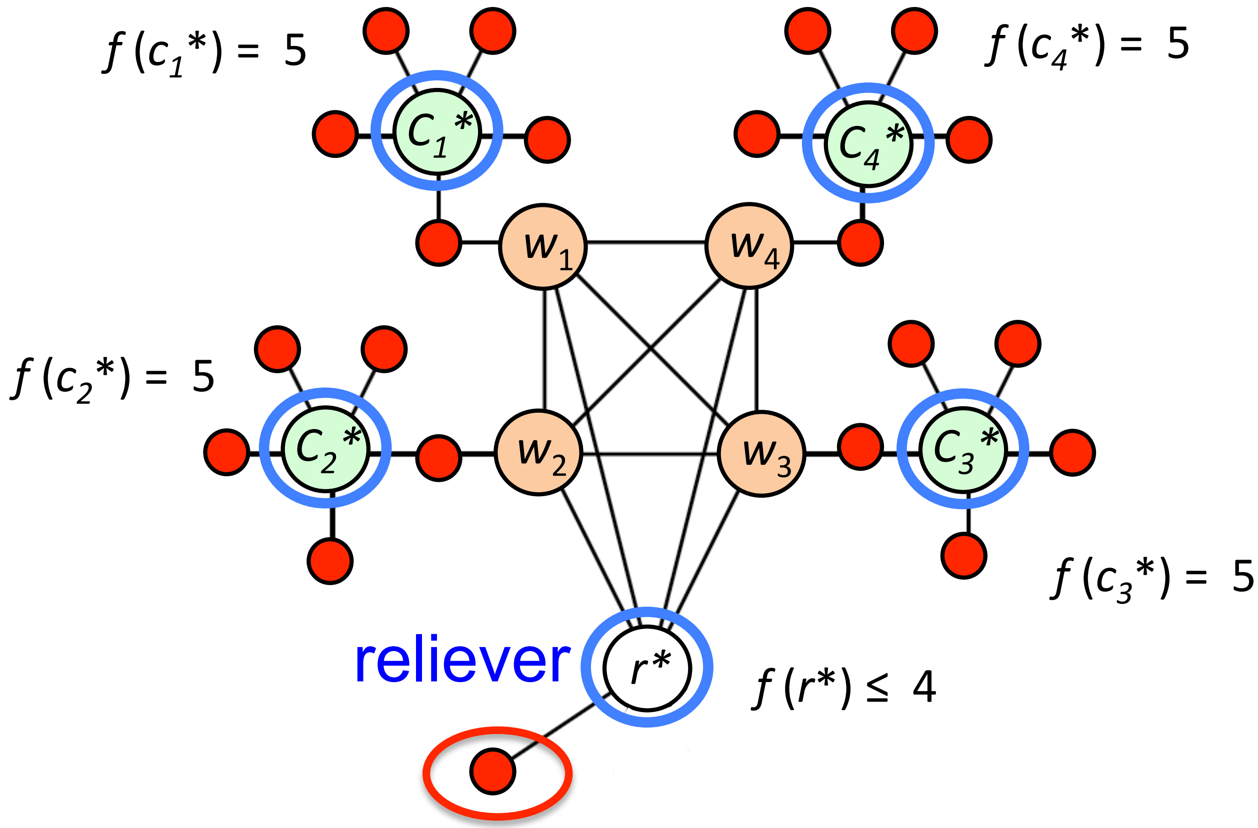
Establishing the approximation factor



$N_D[r^*] \in \{1,2,3\}$

Establishing the approximation factor



$N_D[r^*] \in \{1,2,3\}$ ⇒ at most **14** cores per reliever

Establishing the approximation factor

$\vert N_D[r^*] \vert$	Maximum number of cores c_i^* per reliever	Upper bound for $\vert D \vert / \vert D^* \vert$
1	14	$(1 \times 1 + 14 \times 5) / 15 =$ **4,7333…**
2	14	$(1 \times 2 + 14 \times 5) / 15 =$ **4,8**
3	14	$(1 \times 3 + 14 \times 5) / 15 =$ **4,8666…**
4	8	$(1 \times 4 + 8 \times 5) / 9 =$ **4,888…**

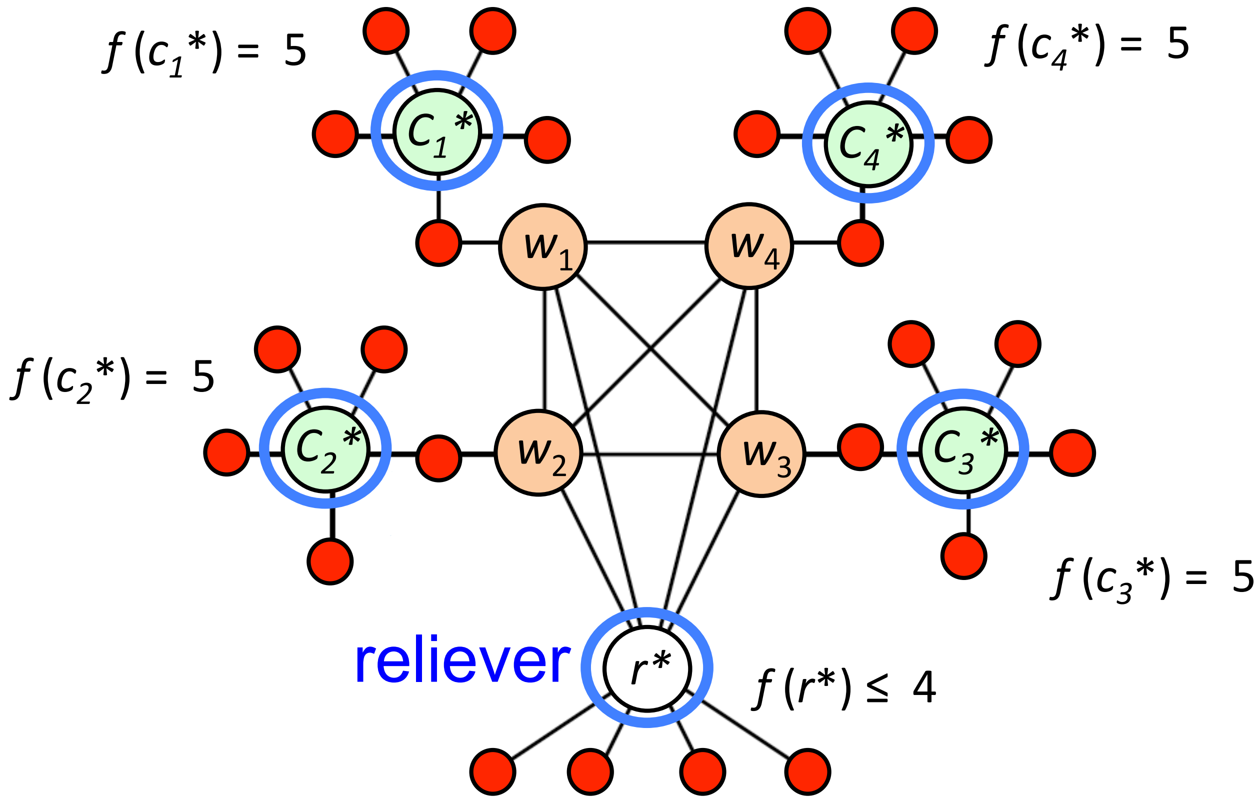
1 reliever

max # of cores

f(reliever)

f(core)

Lower bound



(**1** x 4 + 4 x 5) / 5 = **4.8**

Thank you.

Linear-time Approximations for Dominating Sets and Independent Dominating Sets in Unit Disk Graphs

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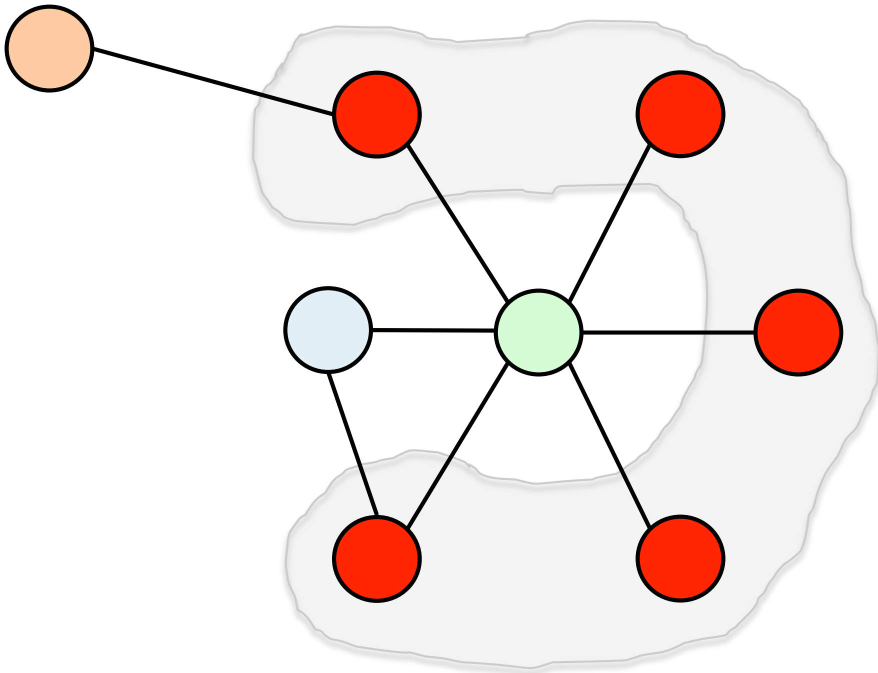
Future directions

- Improve the algorithms (partial reductions)

Irreducible corona



Future directions

- Improve the algorithms (partial reductions)

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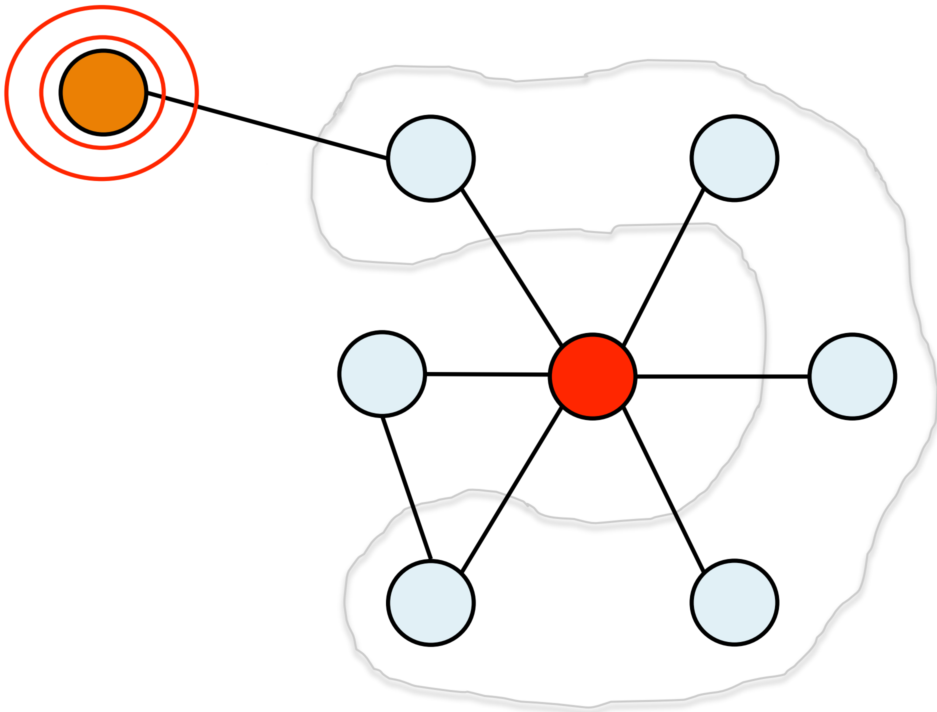
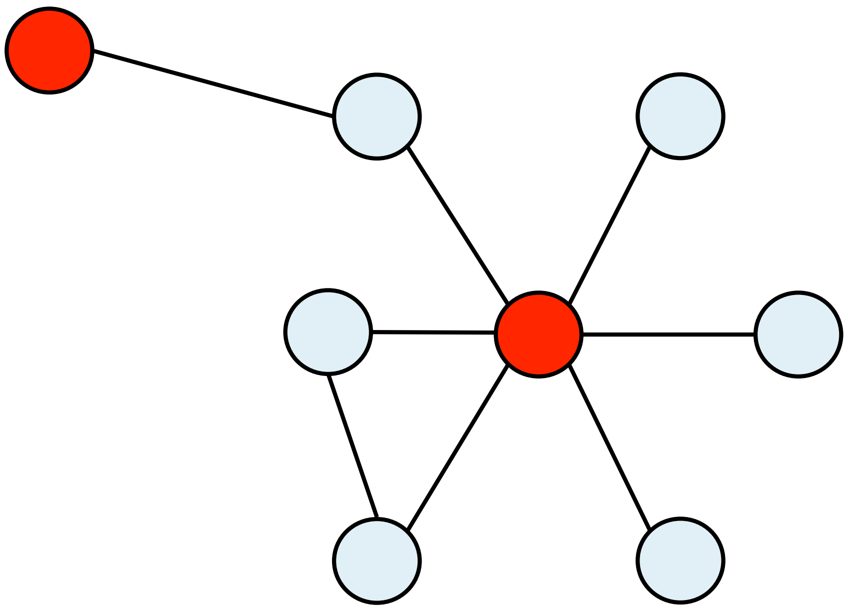


Future directions

- Improve the algorithms (partial reductions)

~~Irreducible~~ corona

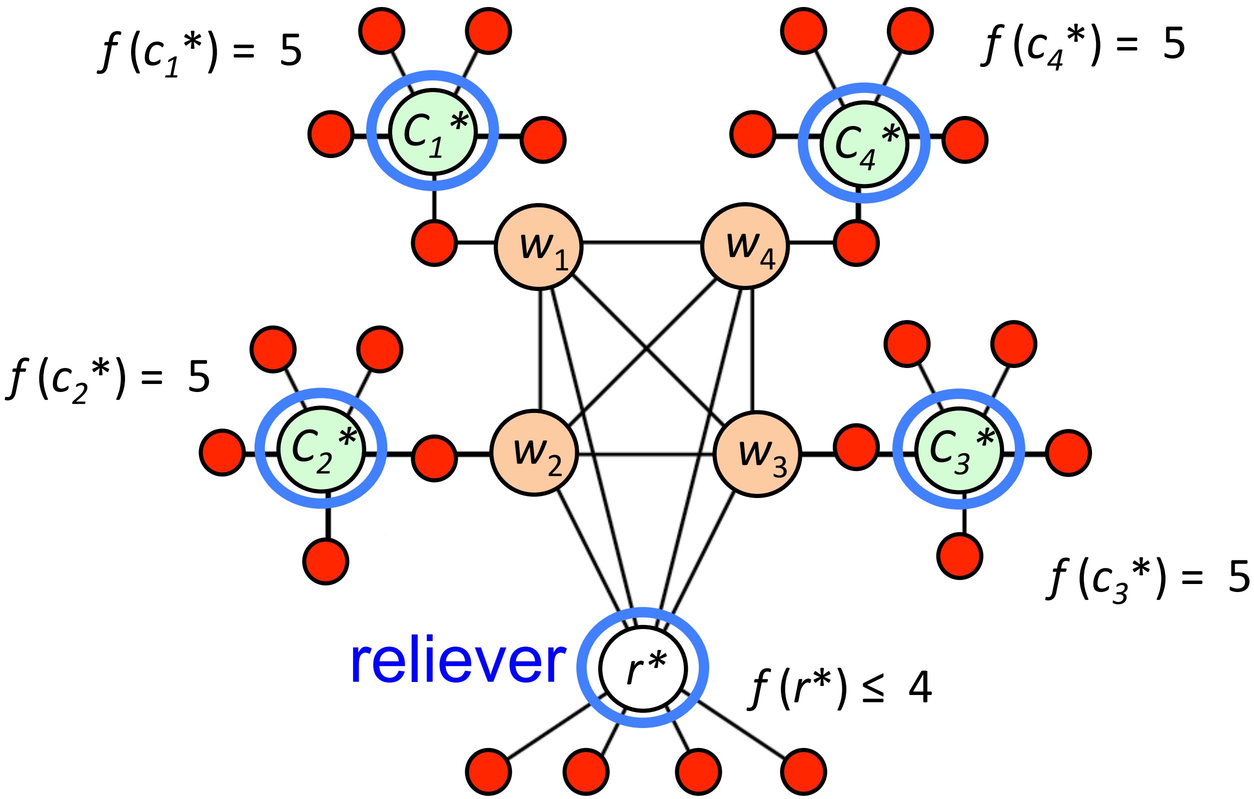


Future directions

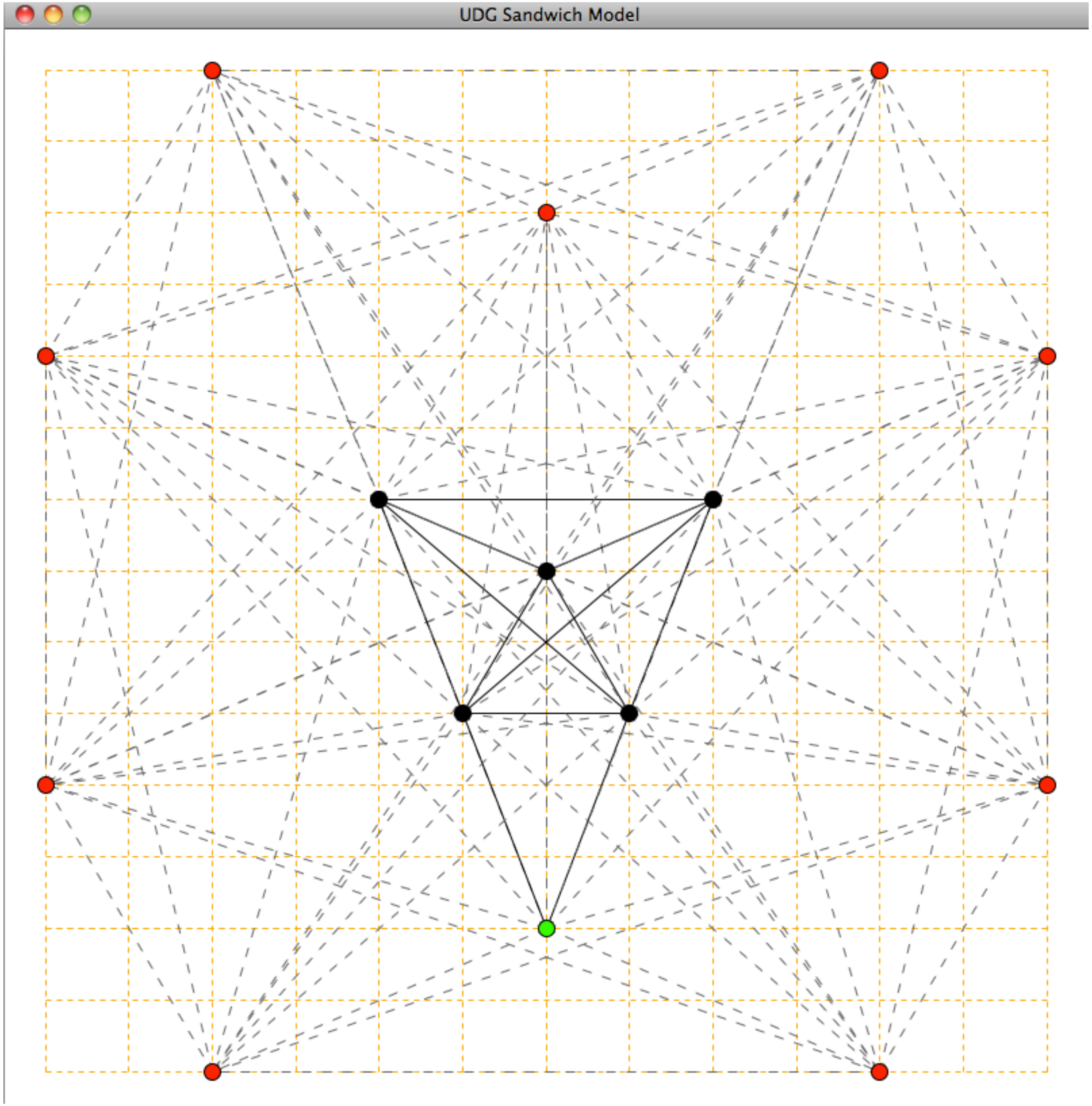
- Improve the analysis of the approximation factor (geometric/computational proofs)

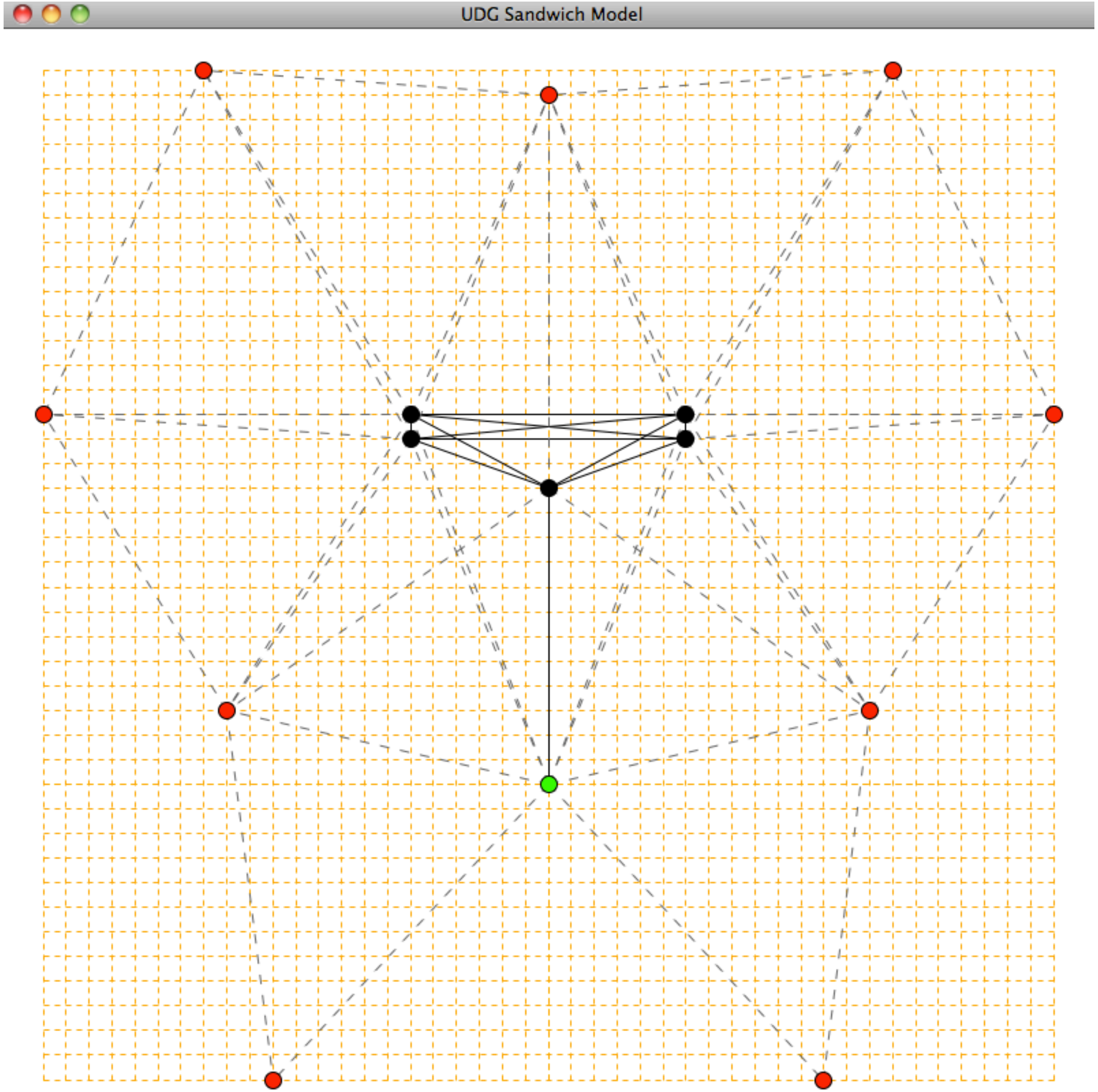
→ Show that Lemmas 5 and 6 are not tight and that some graphs that satisfy them are actually not unit disk graphs.

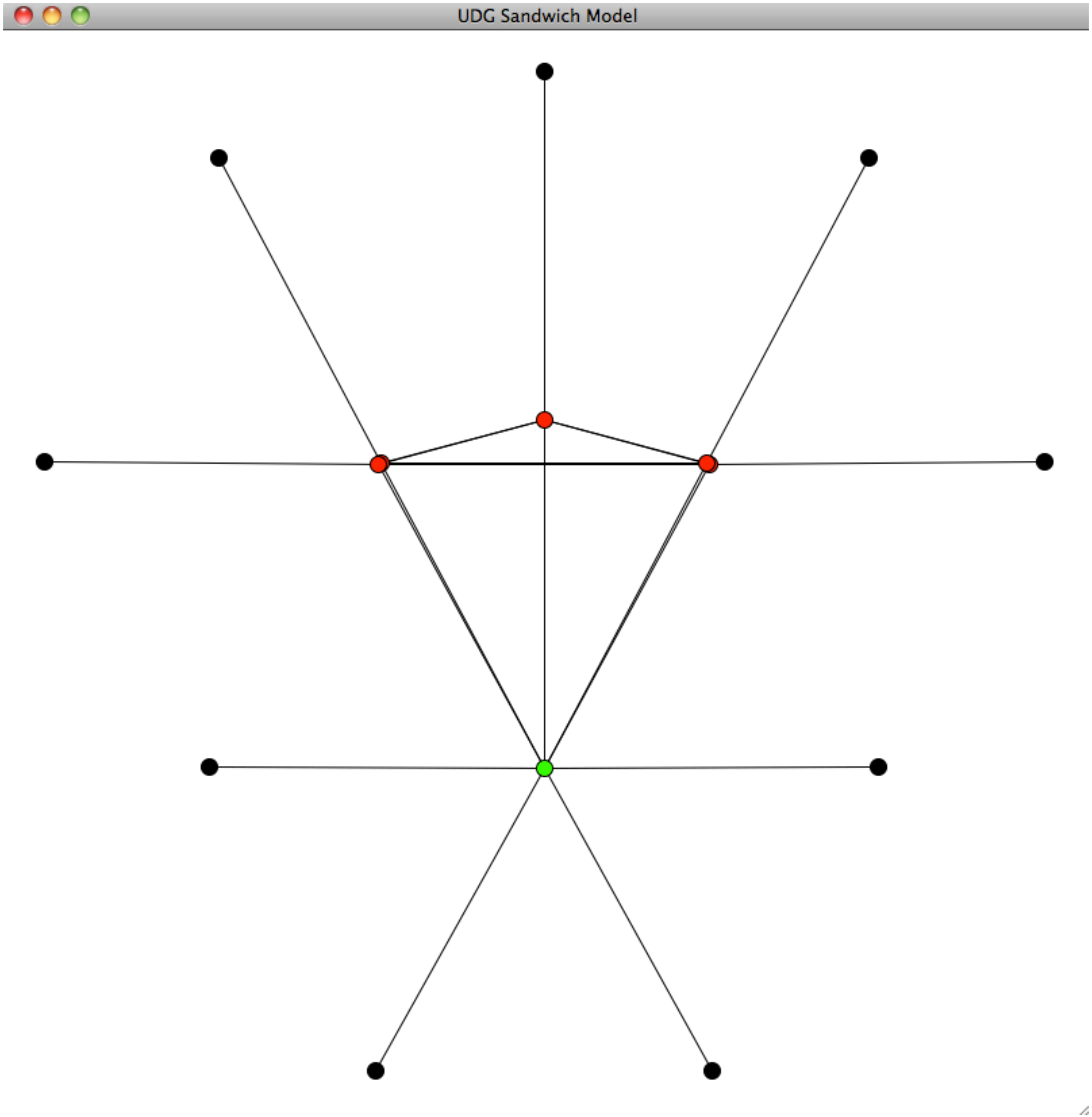
Lower bound



$(\mathbf{1} \times 4 + 4 \times 5) \;/\; 5 = \mathbf{4.8}$


UDG Sandwich Model


UDG Sandwich Model


UDG Sandwich Model

Thank you.

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(k,l)-pendant graphs

A **(k,l)-pendant** graph is a graph containing a vertex *v* with *k* pendant vertices in its open neighborhood and *l* pendant vertices in its open 2-neighborhood.

a (4,5)-pendant graph