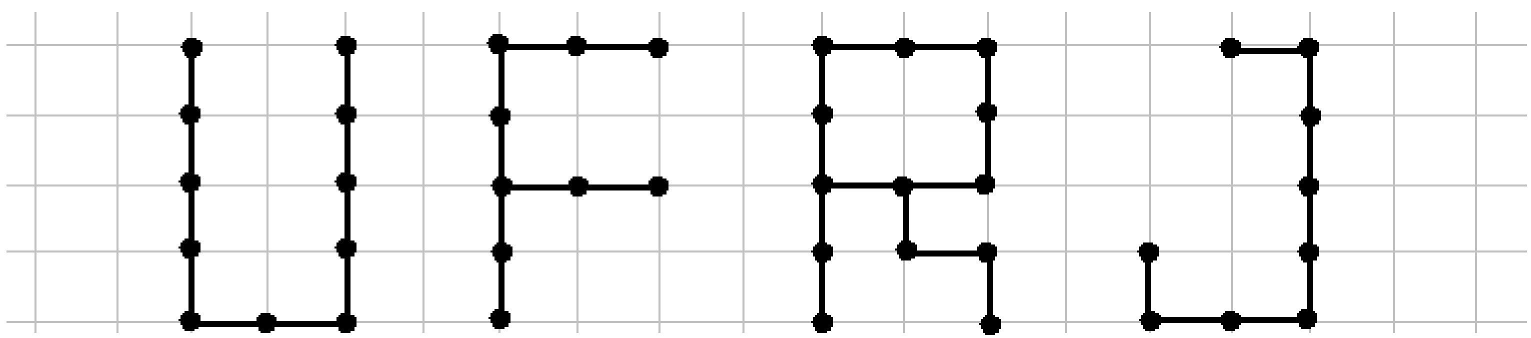
Complexity dichotomy on degree-constrained VLSI layouts with unit-length edges

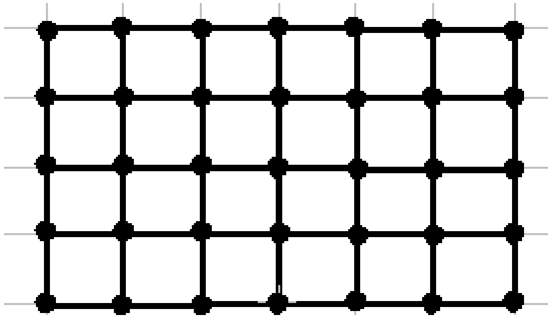


Vinícius G. P. de Sá

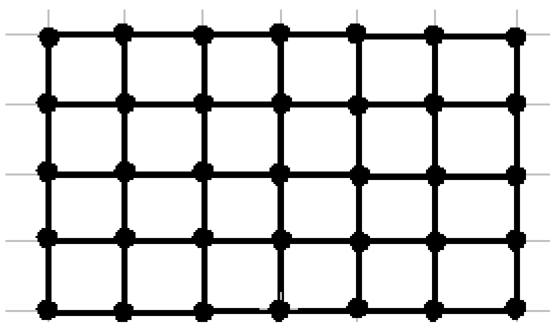
Guilherme D. da Fonseca

Raphael Machado

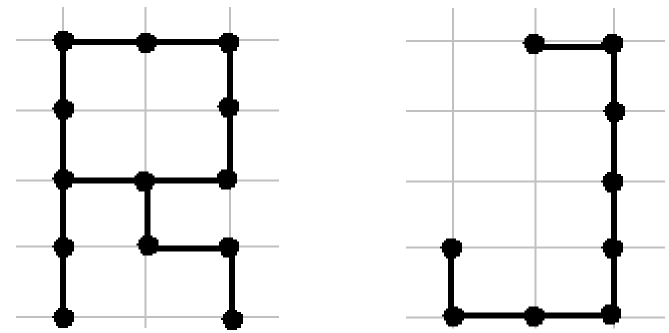
Celina M. H. de Figueiredo



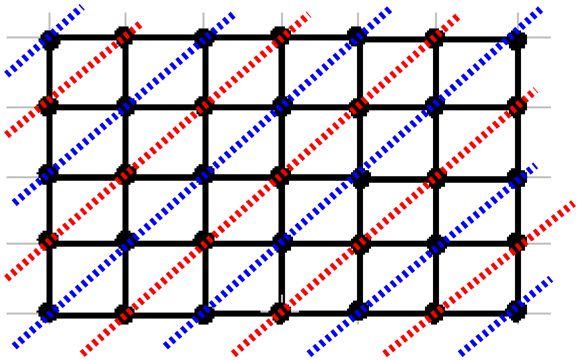
grid



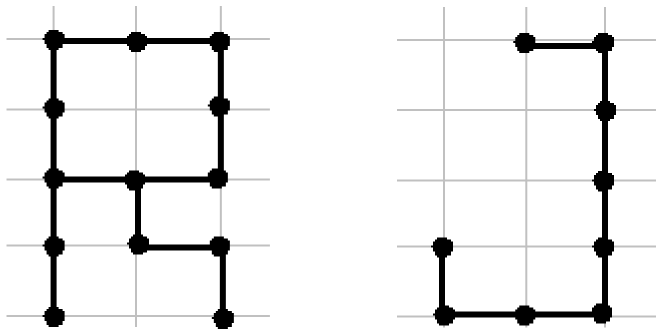
grid



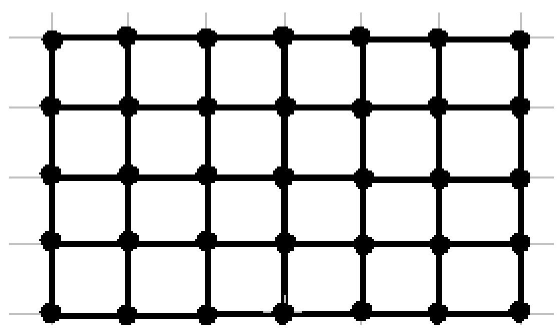
partial grids



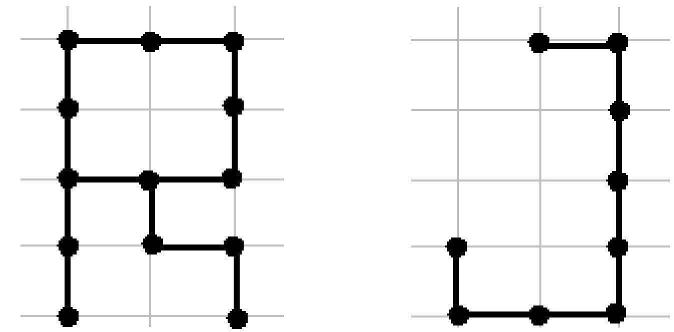
grid



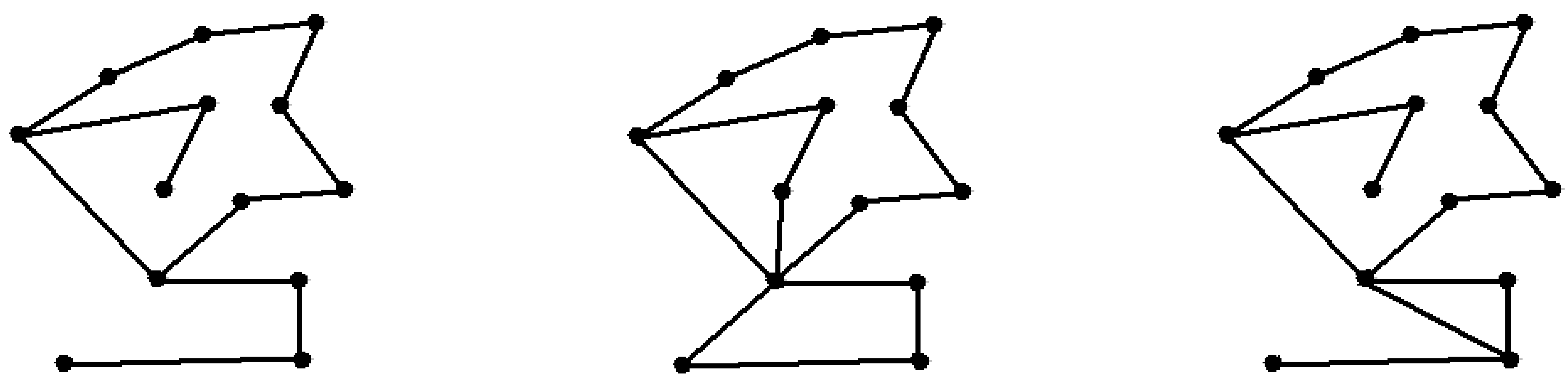
partial grids

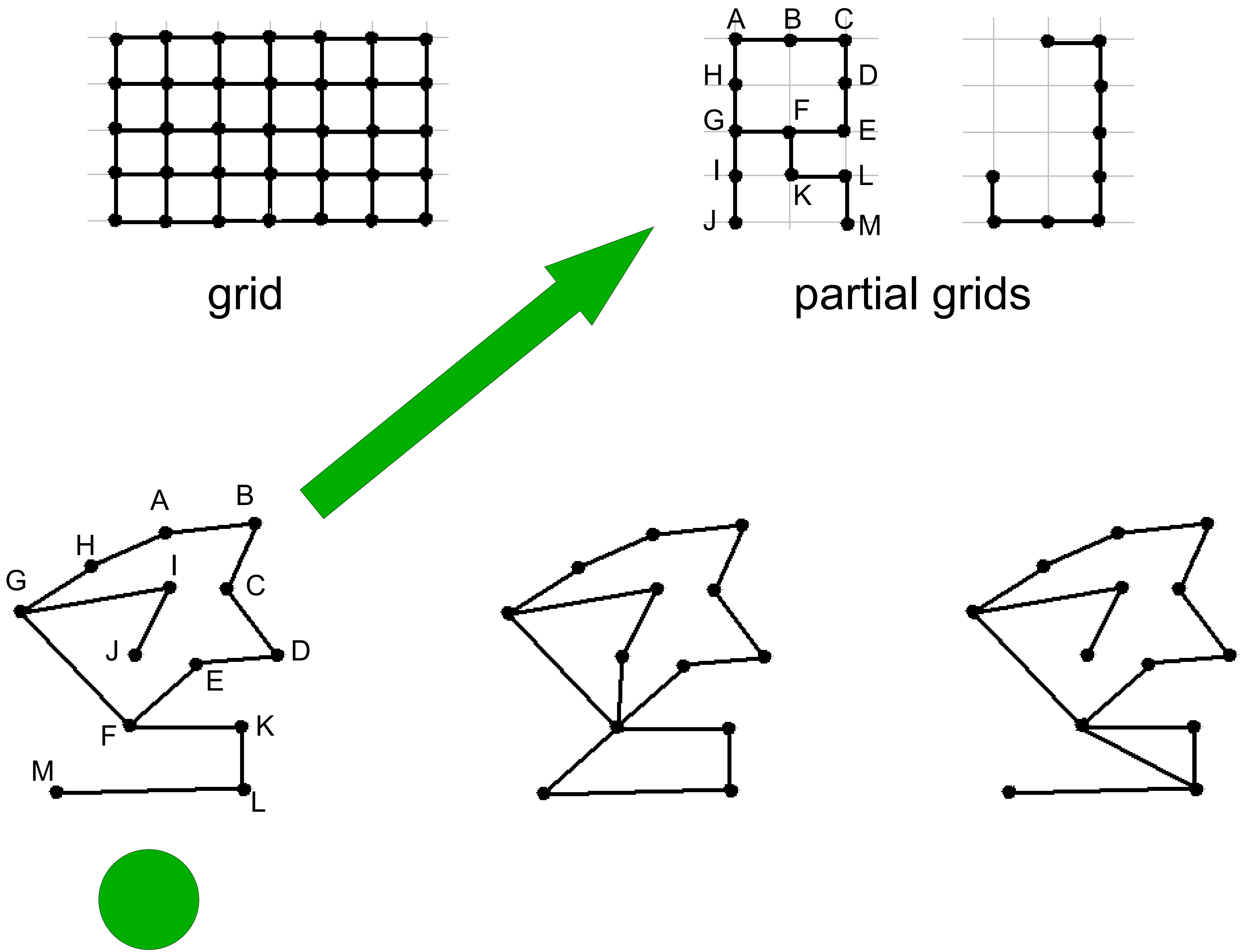


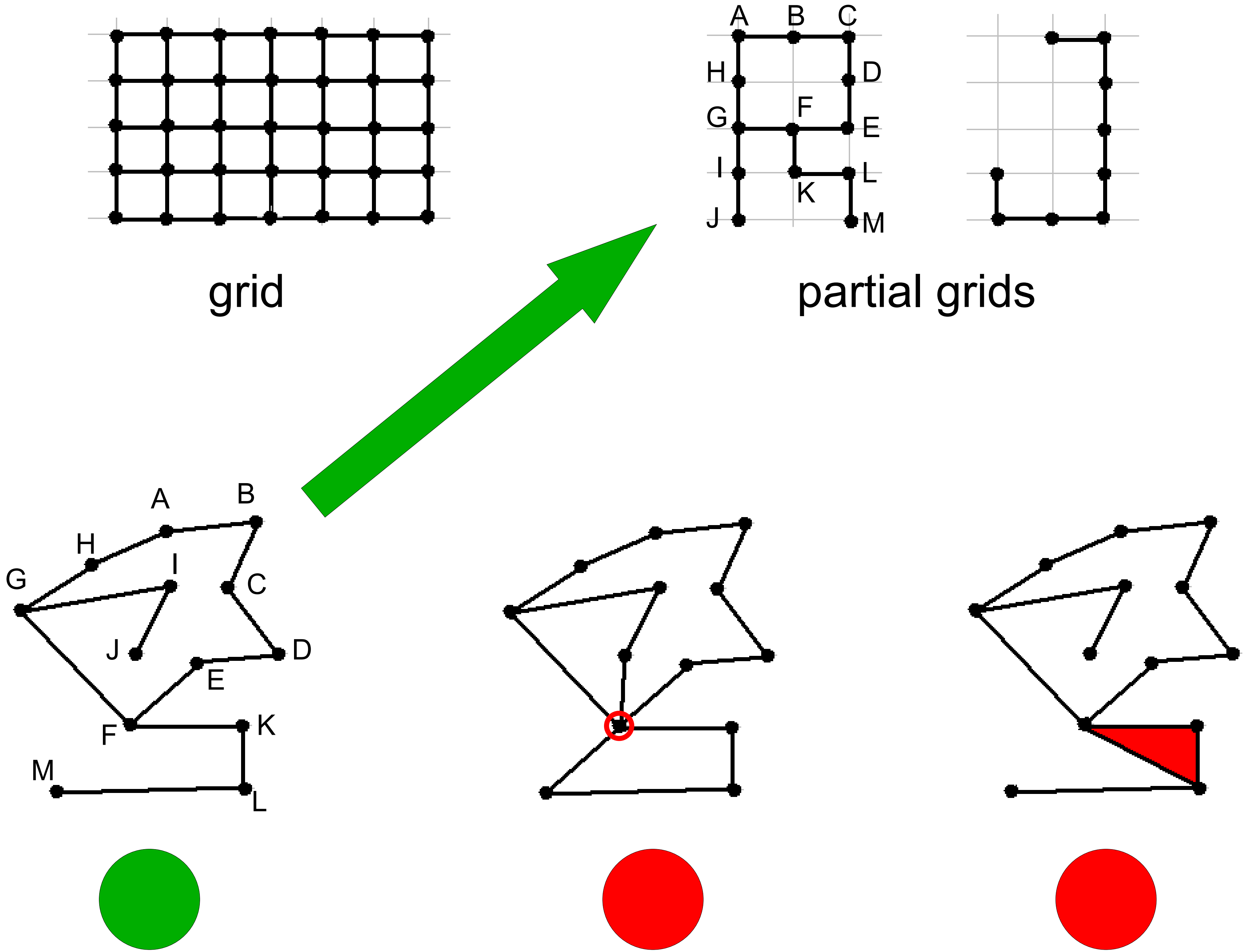
grid



partial grids




grid
partial grids
A
B
C
D
E
F
G
H
I
J
K
L
M


grid
partial grids
A B C D E F G H I J K L M

Recognizing partial grid is NP-complete.

(Bhatt & Cosmadakis, IPL 1987)

Recognizing partial grids is NP-complete even for binary trees.

(Gregori, IPL 1989)

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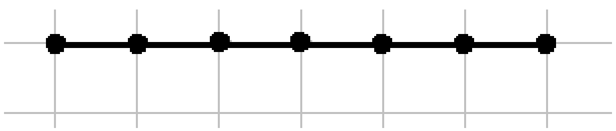
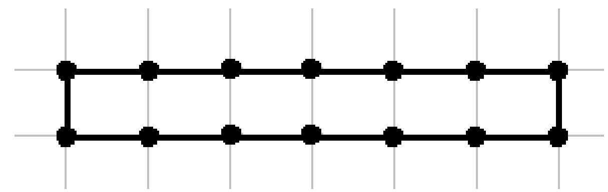
(Bhatt & Cosmadakis, IPL 1987)

Recognizing partial grids is NP-complete even for binary trees.
(Gregori, IPL 1989)

But it can be easy...



paths,



cycles,

etc.

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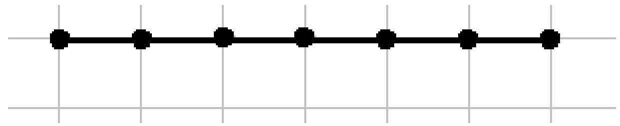
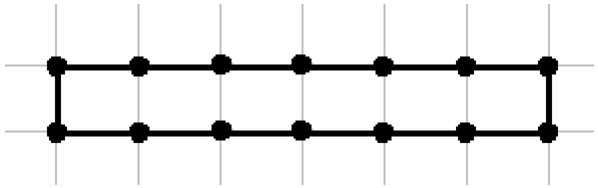
But it can be easy...



paths,



cycles,

etc.

Goal: to settle a dichotomy **P** vs. **NP-C** for the recognition problem after partitioning the input graphs according to the set of **allowed vertex degrees**.

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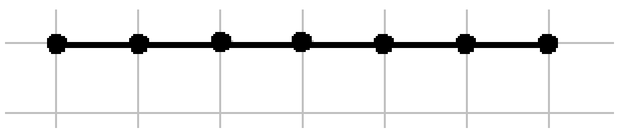
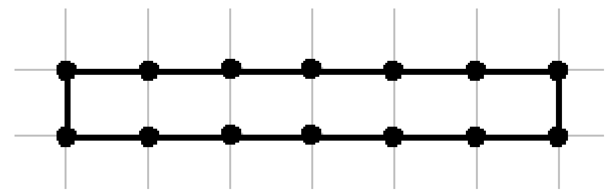
But it can be easy...



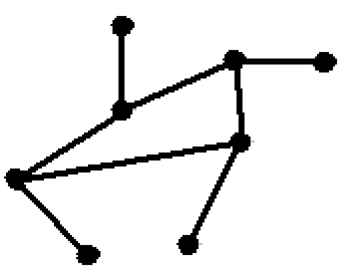
paths,



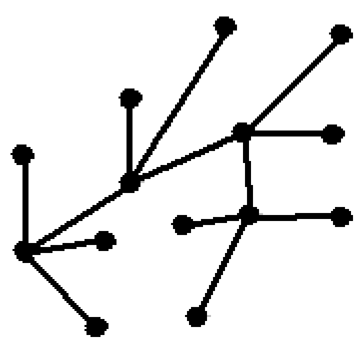
cycles,

etc.

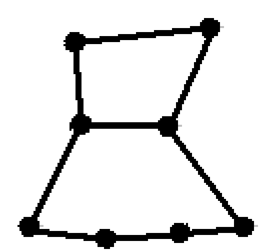
<u>Goal</u>: to settle a dichotomy **P** vs. **NP-C** for the recognition problem after partitioning the input graphs according to the set of **allowed vertex degrees**.



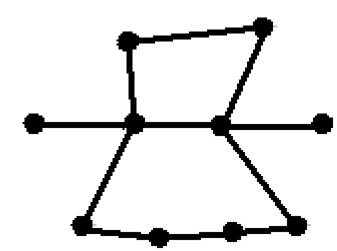
{1,3}-grafo



{1,4}-árvore



{2,3}-grafo

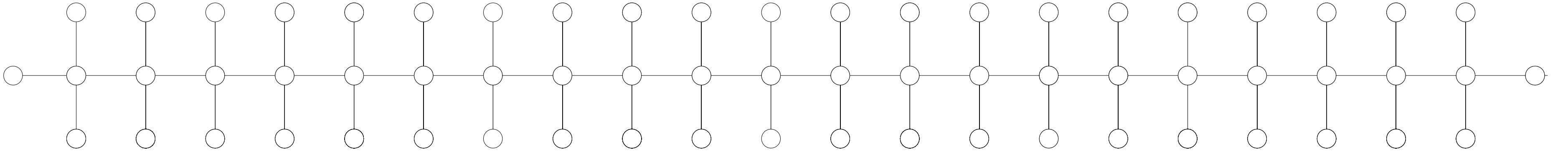


{1,2,4}-grafo

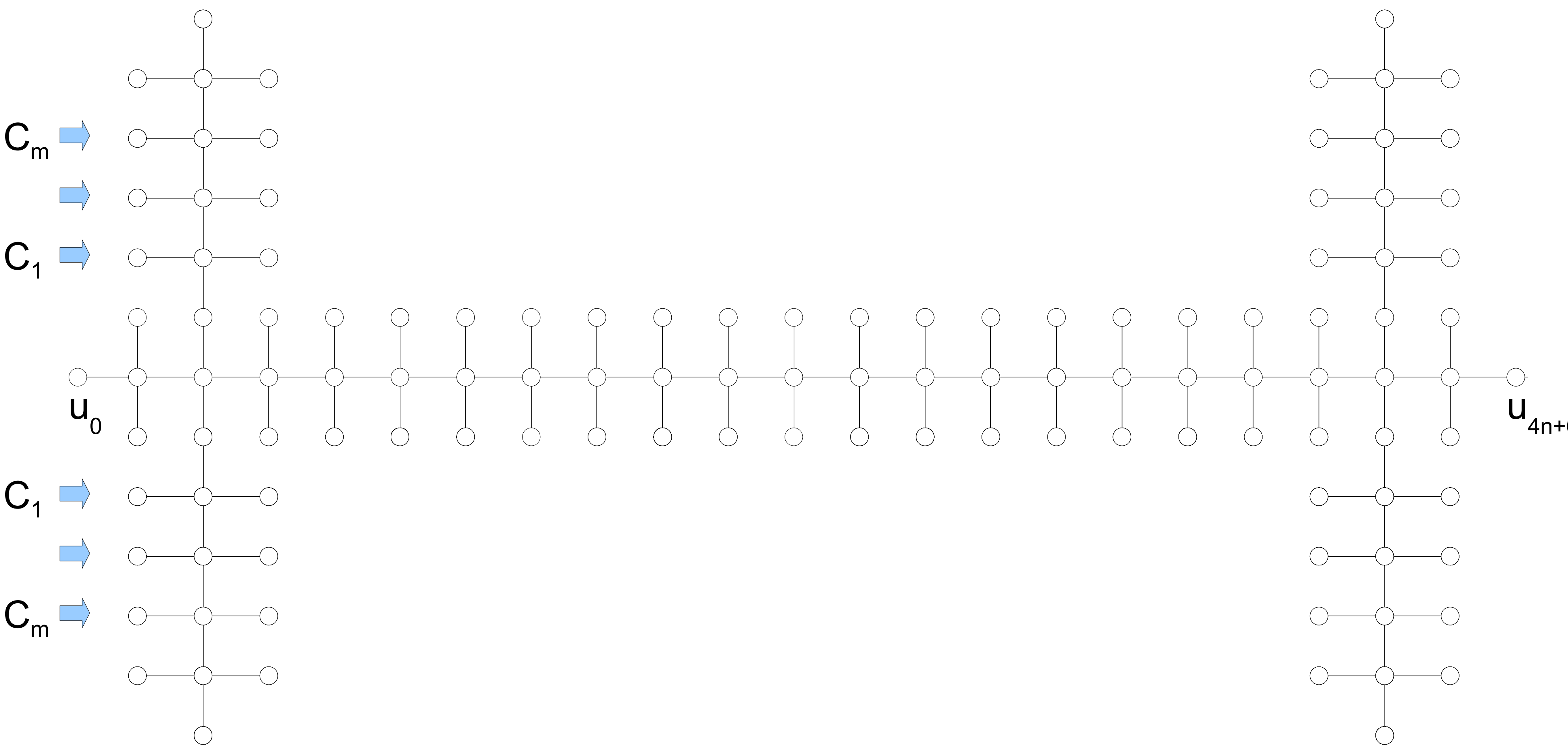
D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}				
{1,2,4}				
{1,3,4}				
{2,3,4}				
{1,2,3,4}				

D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}				
{1,2,4}				
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**		IPL '87	[Bhatt and Cosmadakis]

Spine

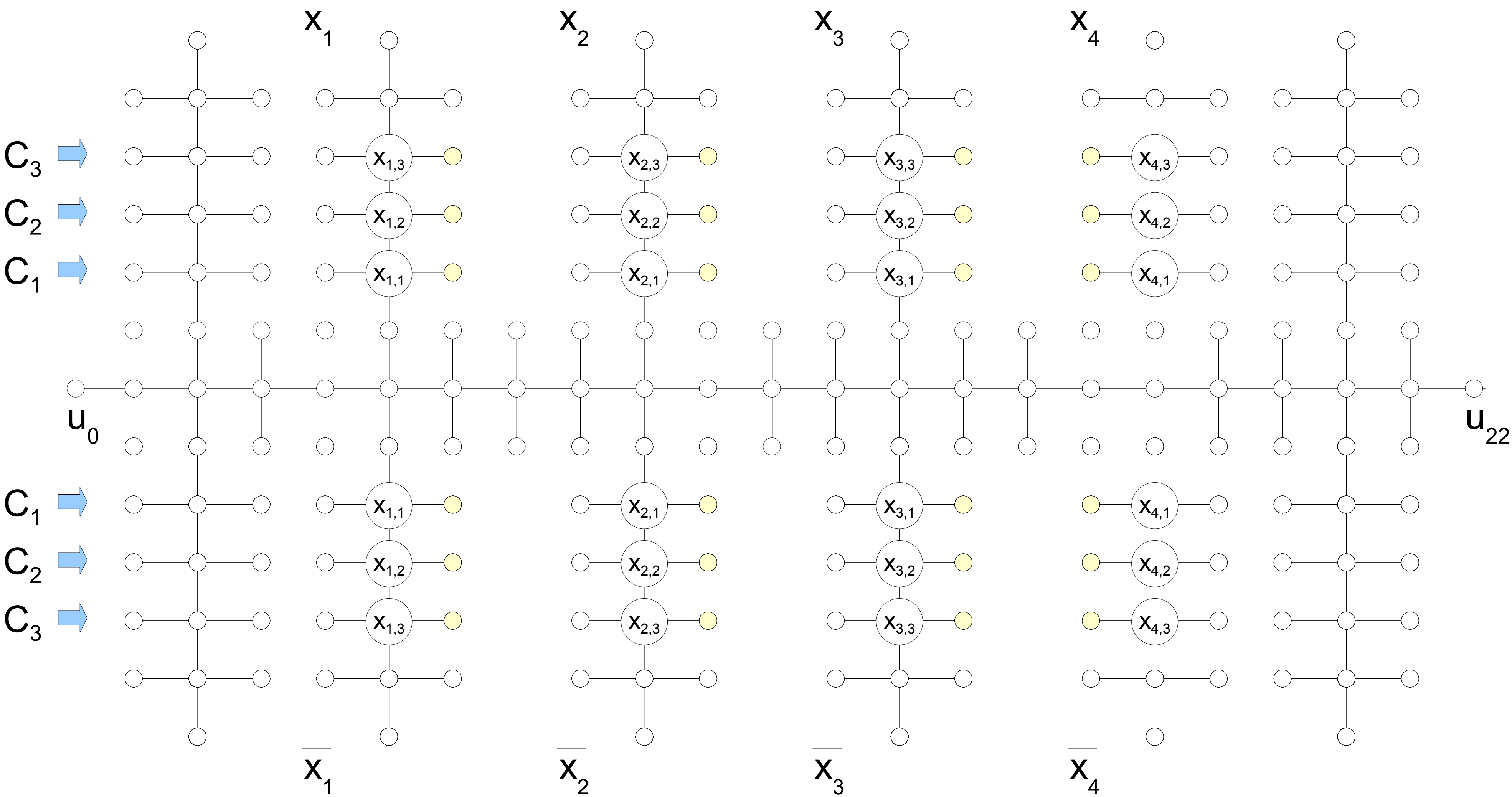


Partial Grid recognition is NP-Complete: redução of NOT-ALL-EQUAL 3-CNF SAT (IPL '87)



$$\Phi = \underbrace{(\overline{x}_2 \vee x_3 \vee \overline{x}_4)}_{C_1} \wedge \underbrace{(x_1 \vee x_2 \vee x_4)}_{C_2} \wedge \underbrace{(x_1 \vee \overline{x}_3 \vee \overline{x}_4)}_{C_3}$$

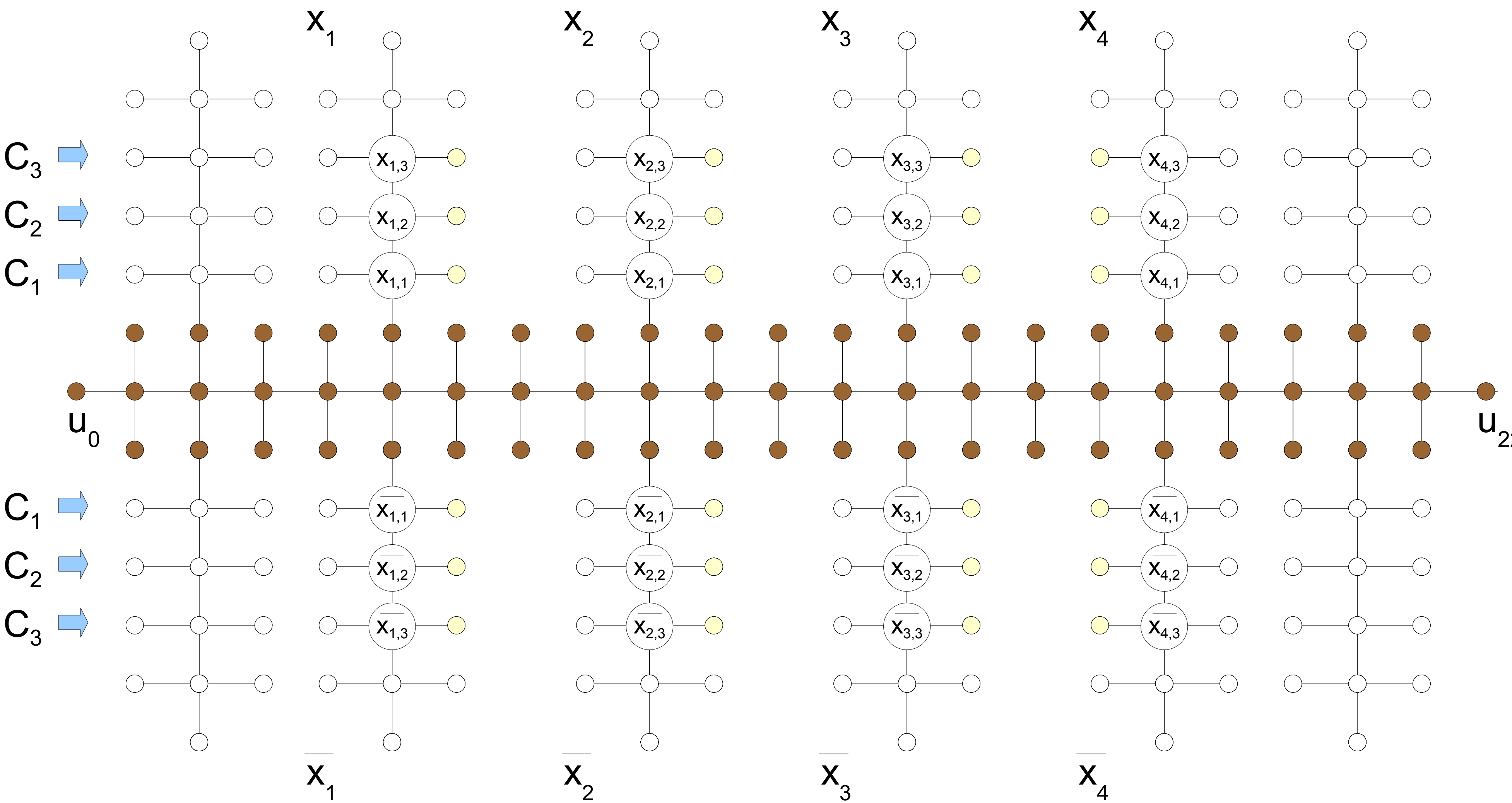
Skeleton



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

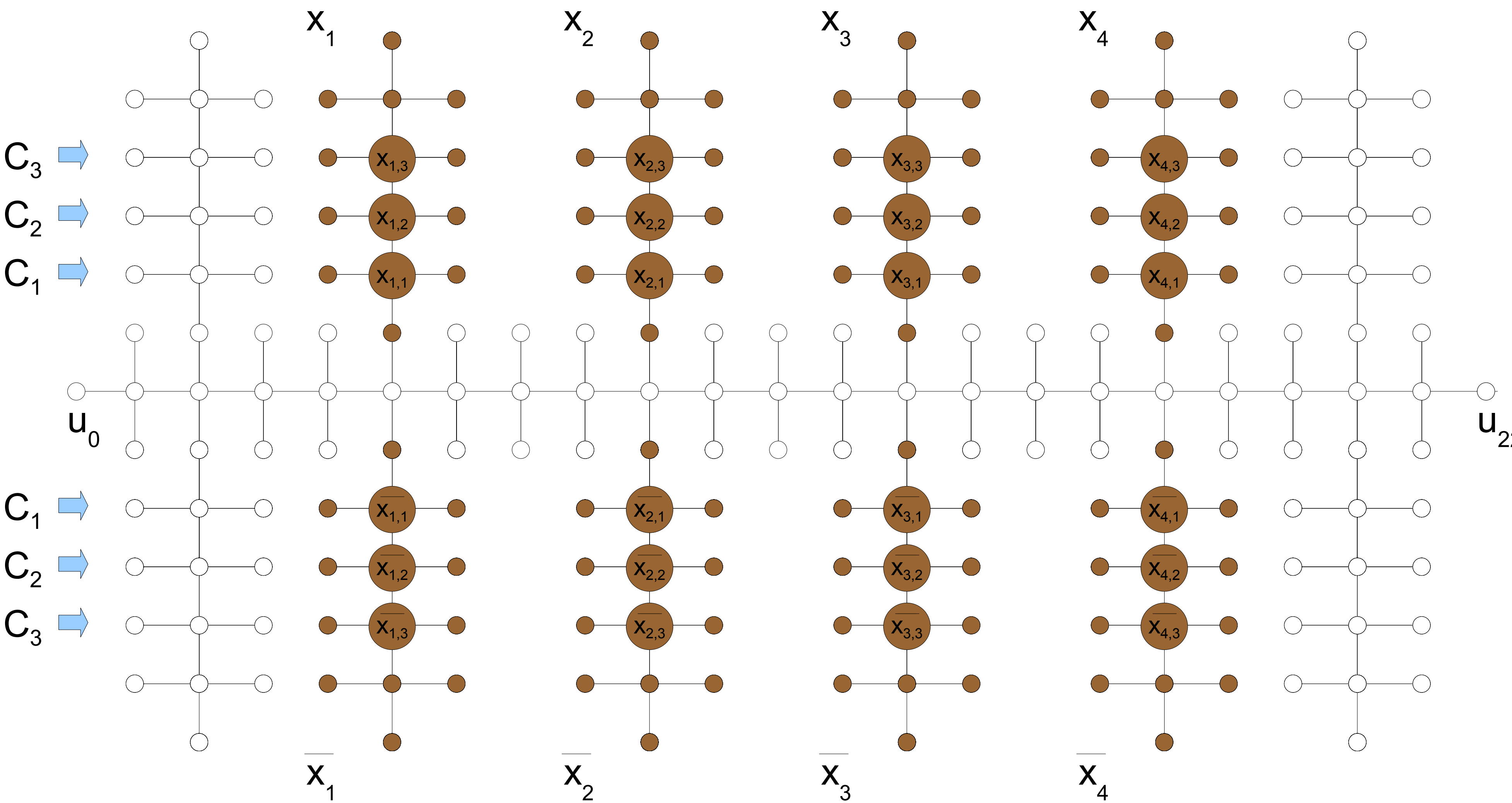
C_1 C_2 C_3

Skeleton



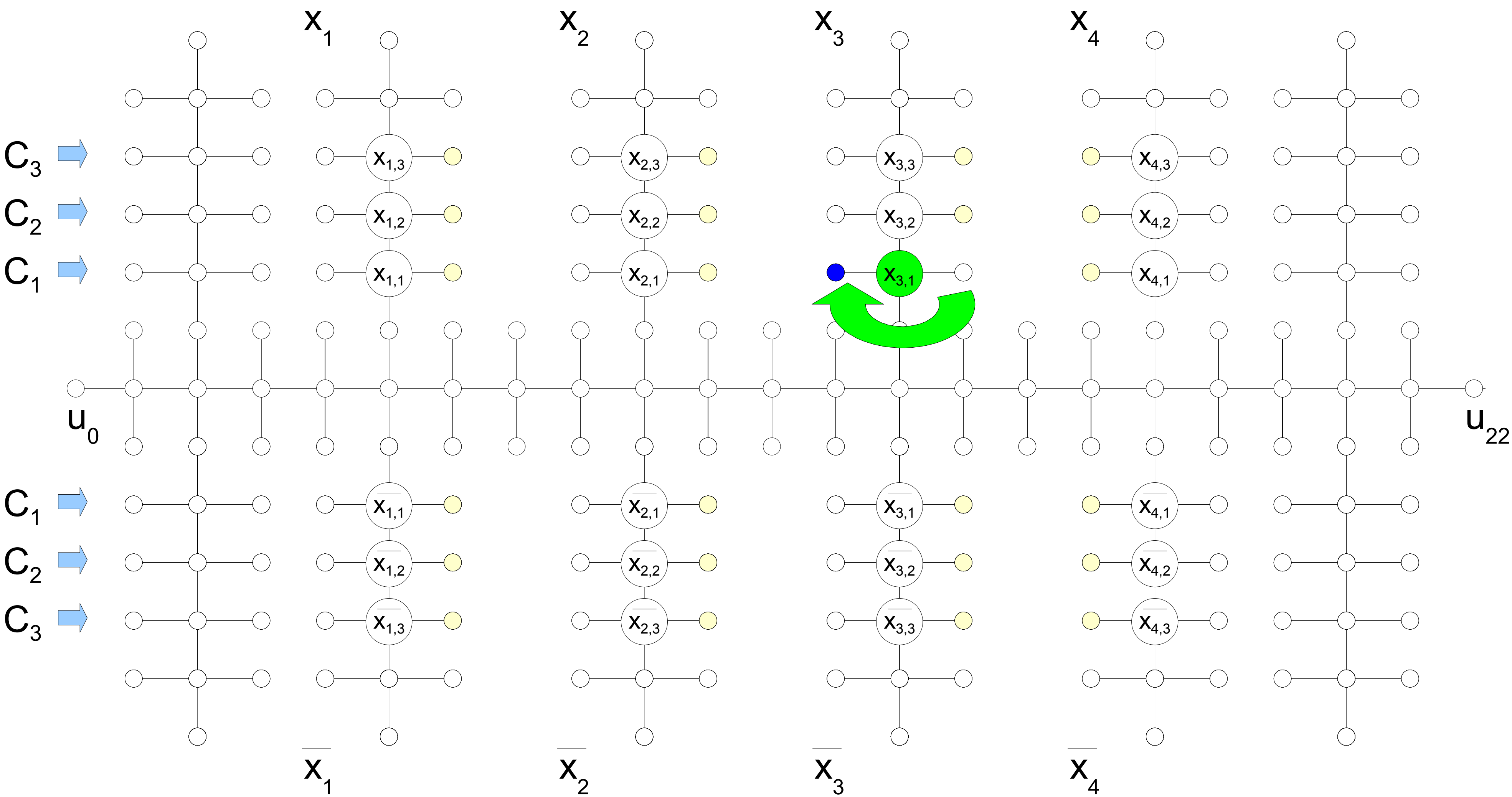
$$\Phi = \underbrace{(\overline{x_2} \vee x_3 \vee \overline{x_4})}_{C_1} \wedge \underbrace{(x_1 \vee x_2 \vee x_4)}_{C_2} \wedge \underbrace{(x_1 \vee \overline{x_3} \vee \overline{x_4})}_{C_3}$$

Skeleton



$$\Phi = \underbrace{(\bar{x}_2 \vee x_3 \vee \bar{x}_4)}_{C_1} \wedge \underbrace{(x_1 \vee x_2 \vee x_4)}_{C_2} \wedge \underbrace{(x_1 \vee \bar{x}_3 \vee \bar{x}_4)}_{C_3}$$

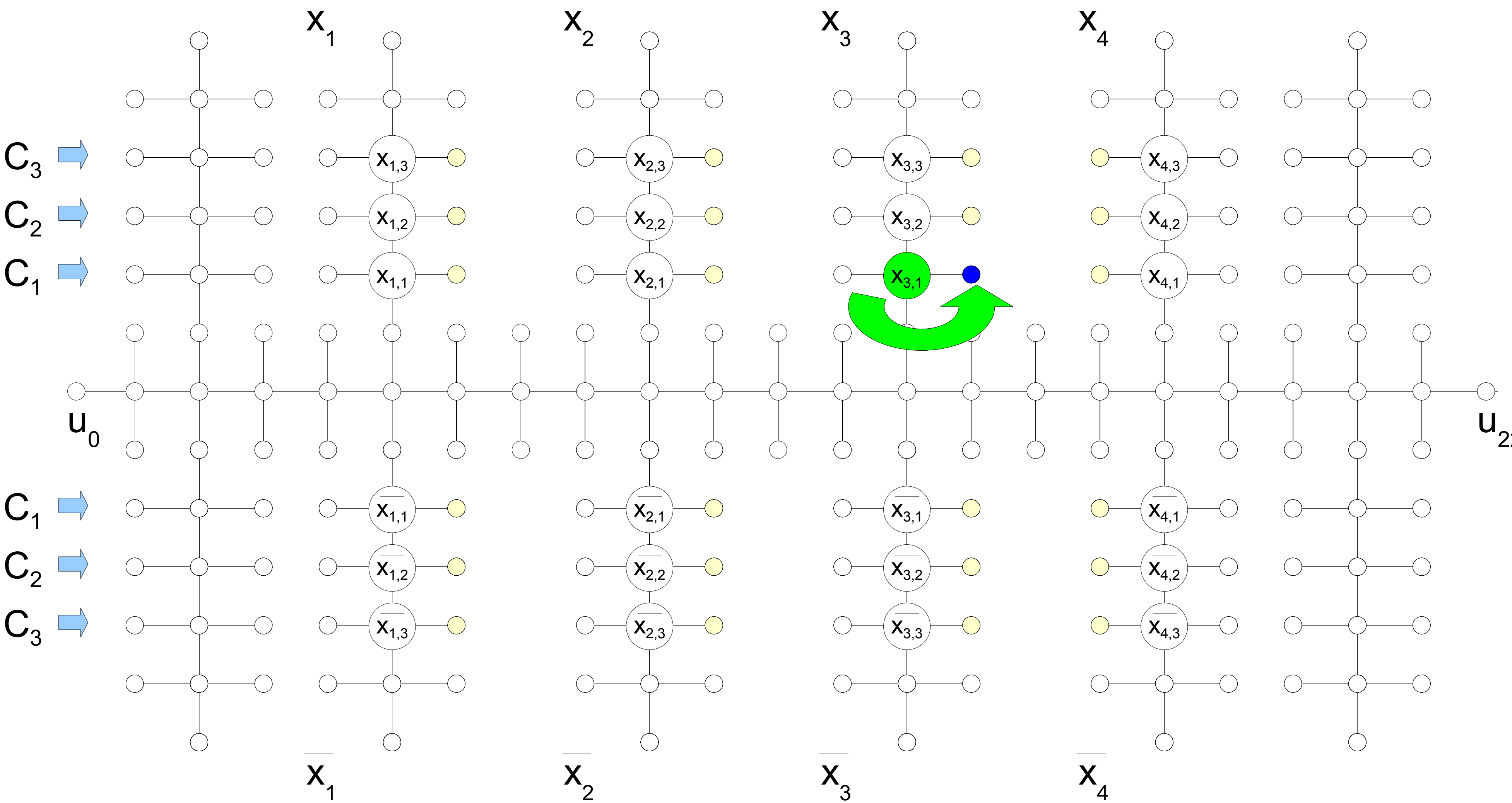
Skeleton



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

C_1 C_2 C_3

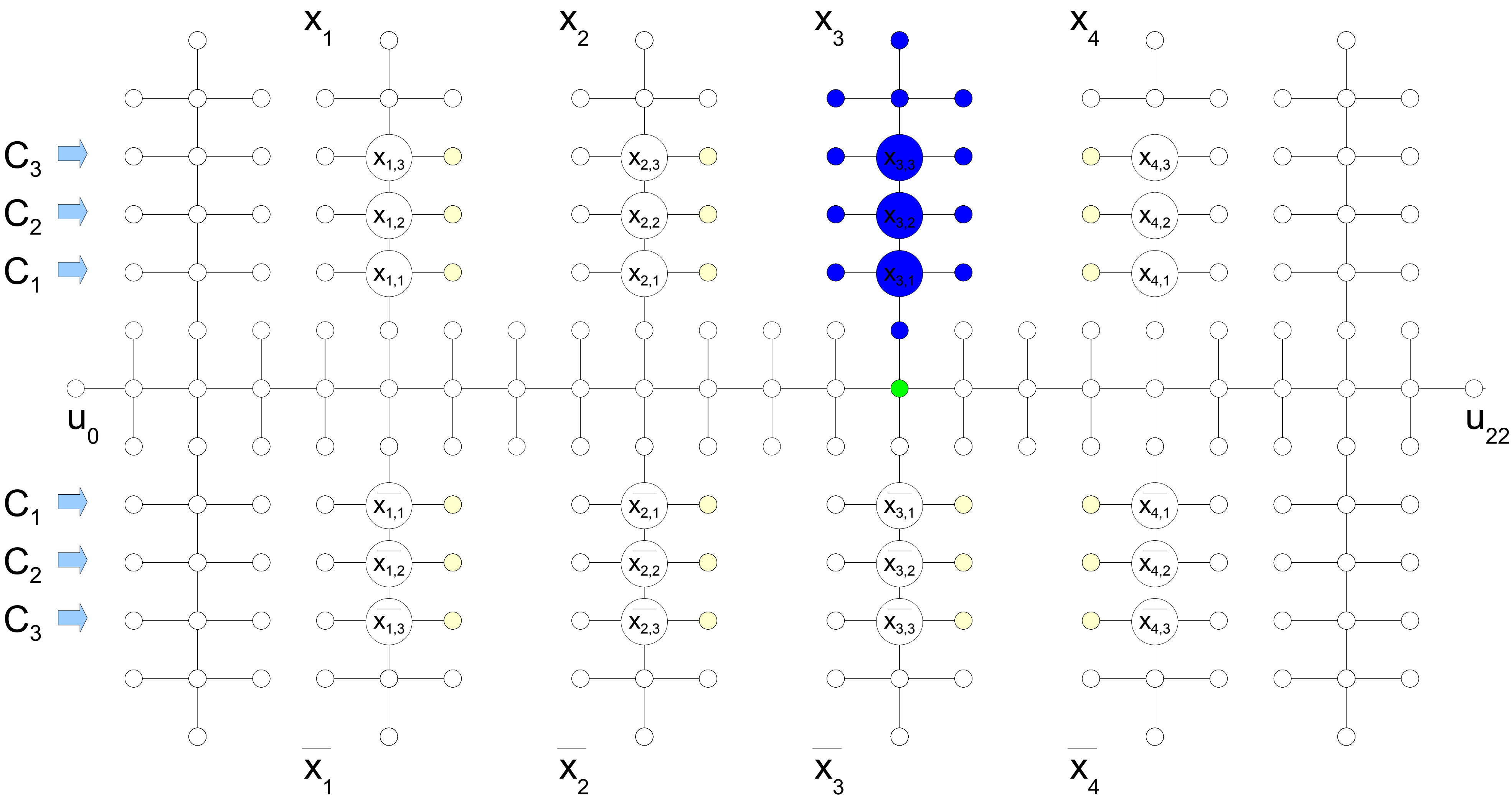
Skeleton



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

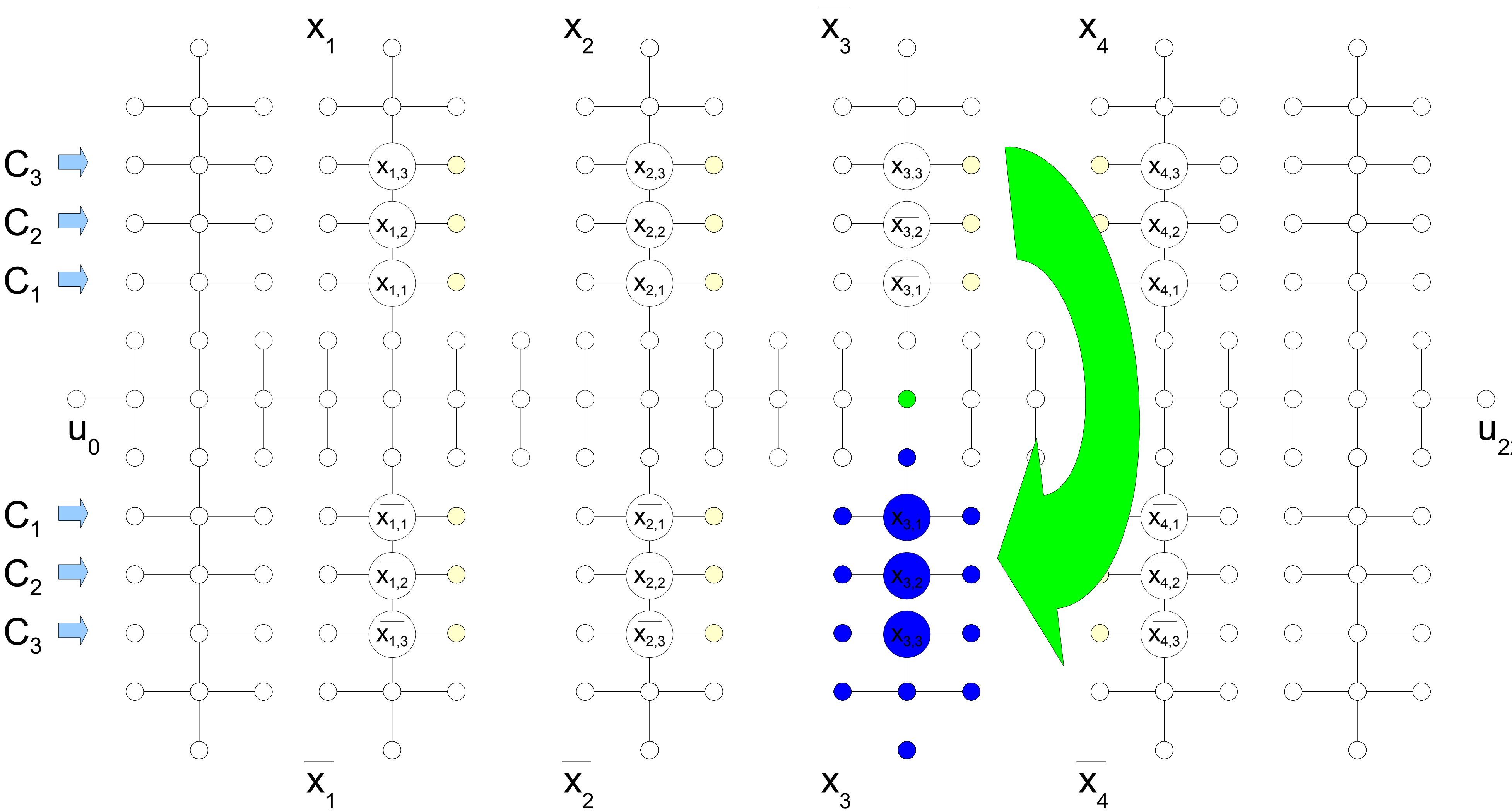
C_1 C_2 C_3

Skeleton



$$\Phi = \underbrace{(\overline{x_2} \vee x_3 \vee \overline{x_4})}_{C_1} \wedge \underbrace{(x_1 \vee x_2 \vee x_4)}_{C_2} \wedge \underbrace{(x_1 \vee \overline{x_3} \vee \overline{x_4})}_{C_3}$$

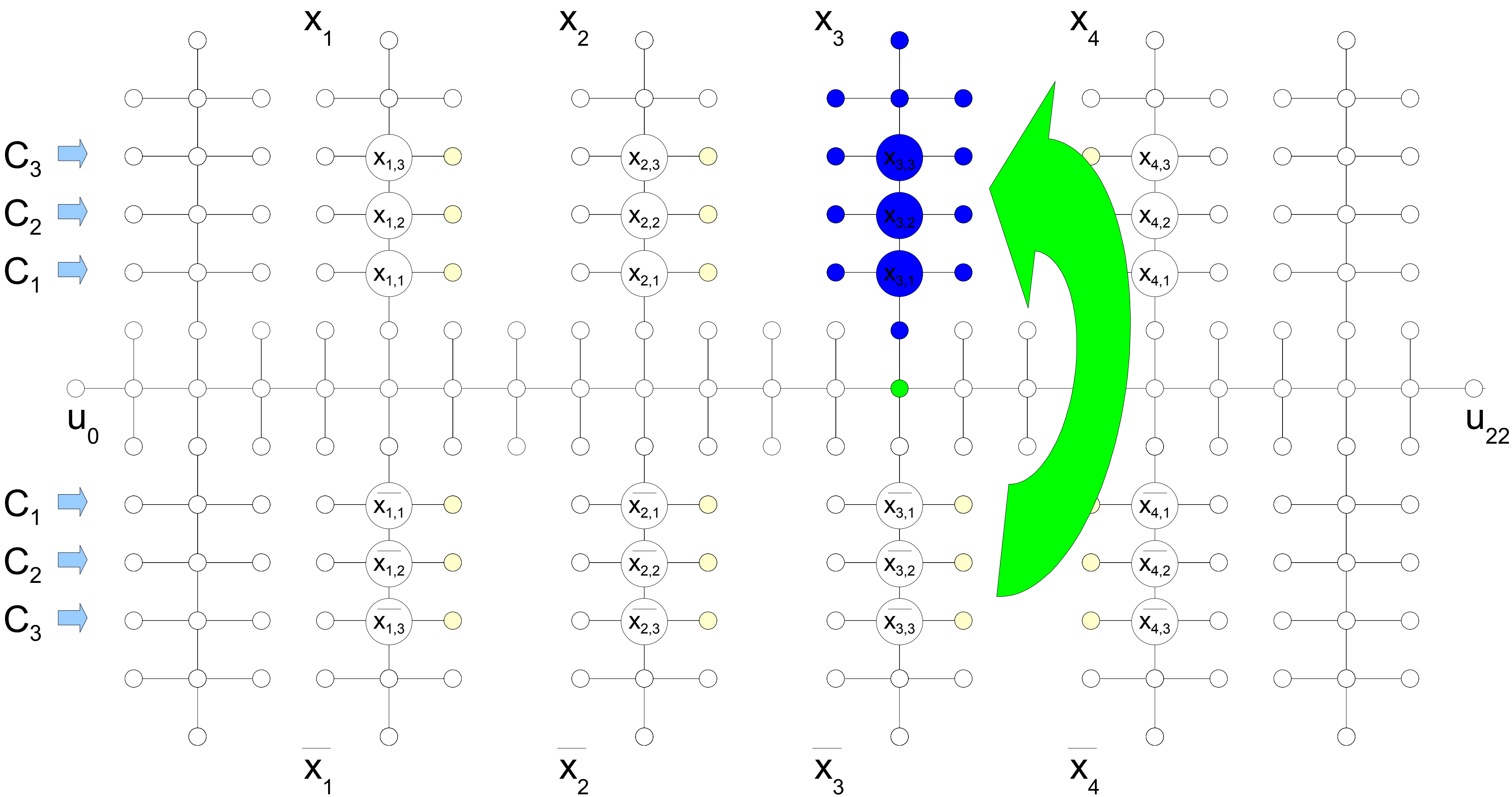
Skeleton



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

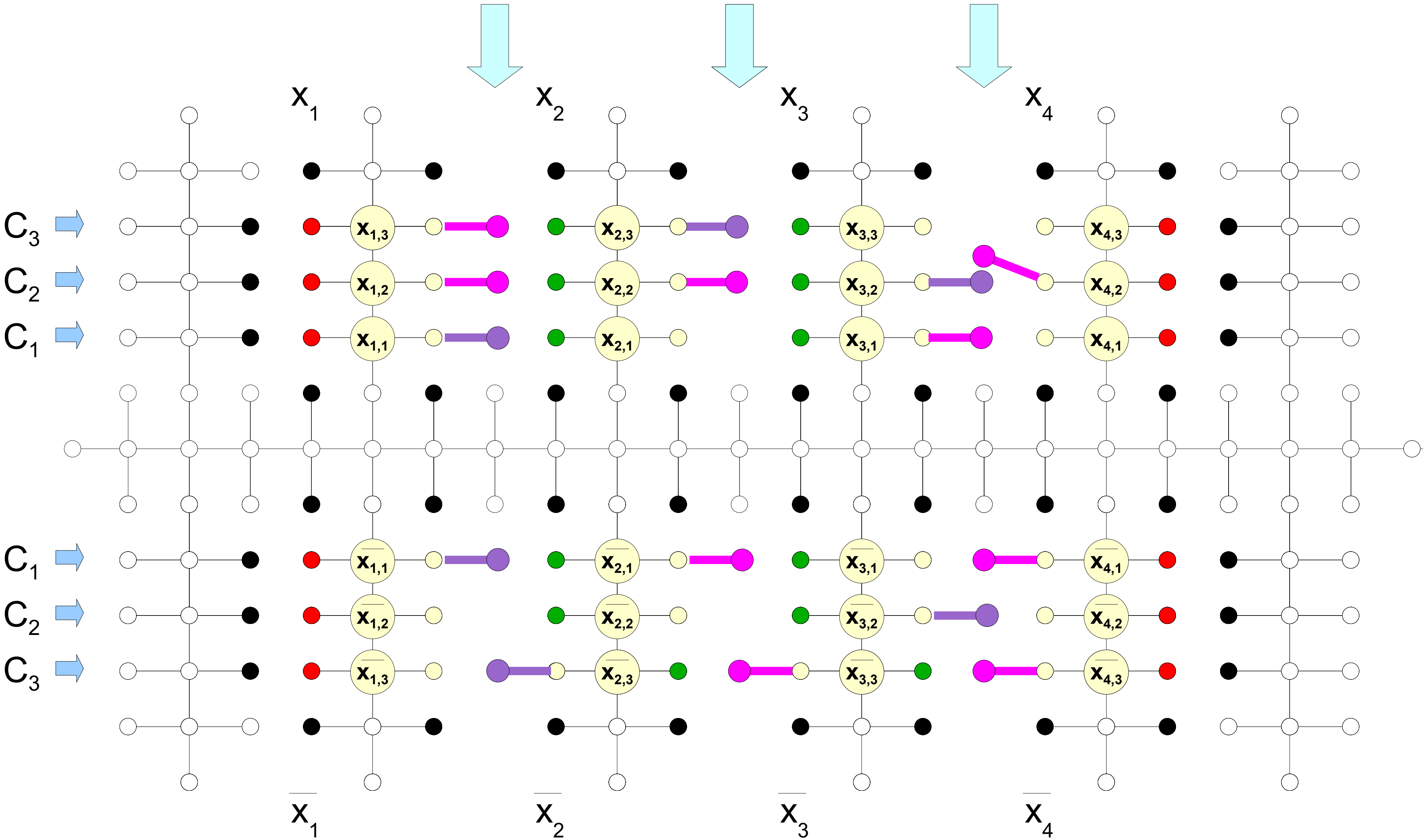
C_1 C_2 C_3

Skeleton



$$\Phi = \underbrace{(\overline{x_2} \vee x_3 \vee \overline{x_4})}_{C_1} \wedge \underbrace{(x_1 \vee x_2 \vee x_4)}_{C_2} \wedge \underbrace{(x_1 \vee \overline{x_3} \vee \overline{x_4})}_{C_3}$$

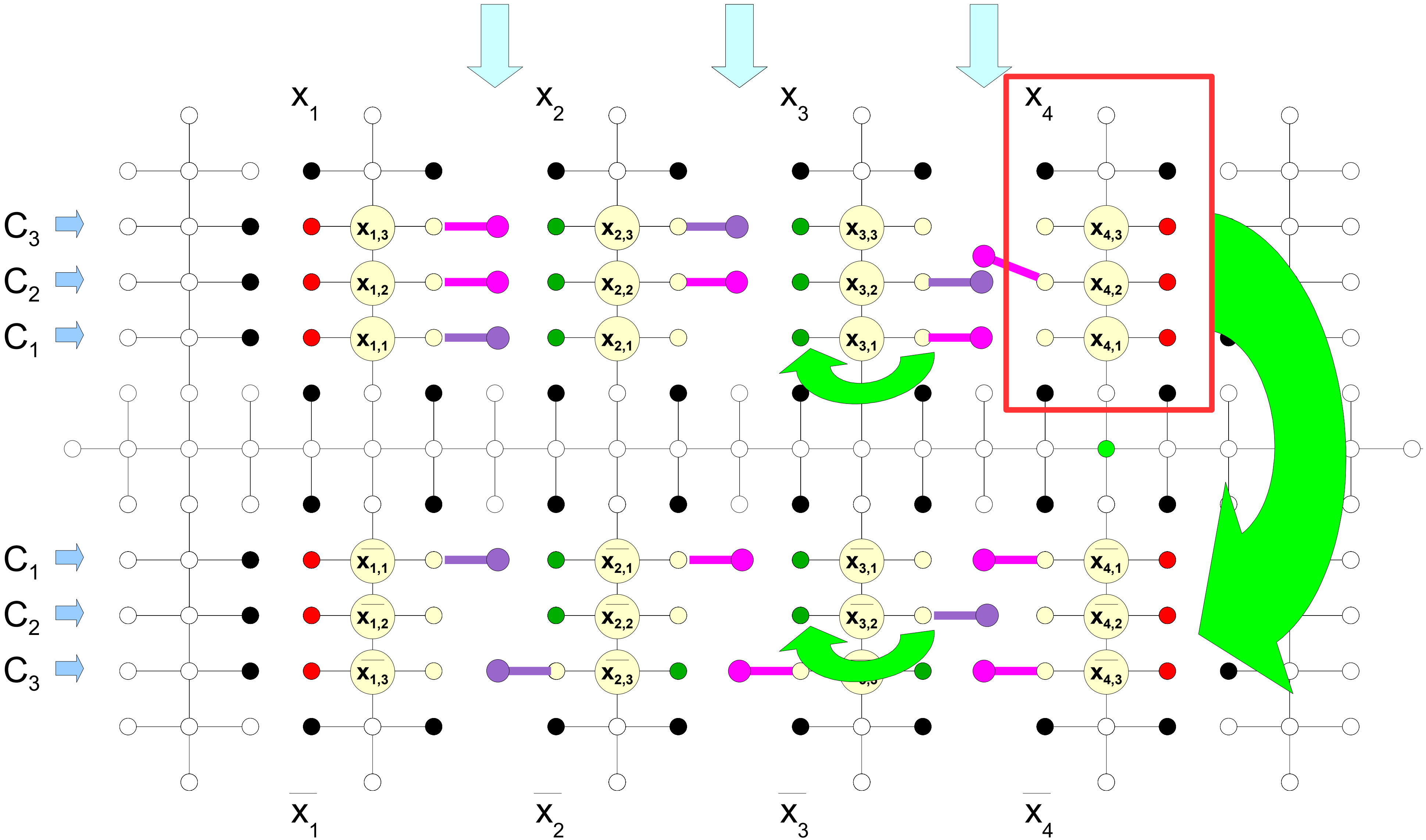
Free columns now occupied by *flags*



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

C1 C2 C3

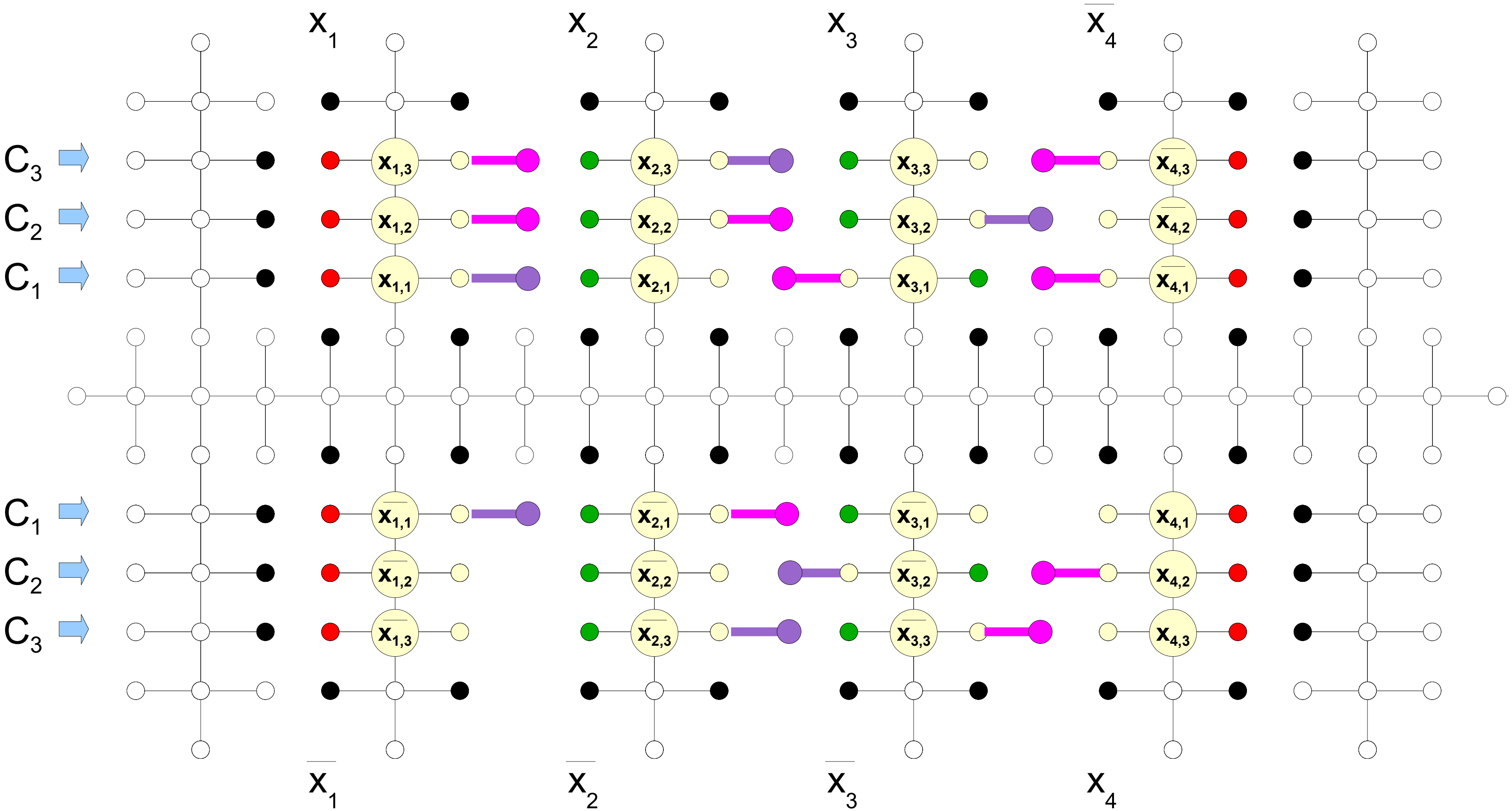
Free columns now occupied by *flags*



$$\Phi = (\bar{x}_2 \lor x_3 \lor \bar{x}_4) \land (x_1 \lor x_2 \lor x_4) \land (x_1 \lor \bar{x}_3 \lor \bar{x}_4)$$

C_1 C_2 C_3

Extended Skeleton S(Φ)

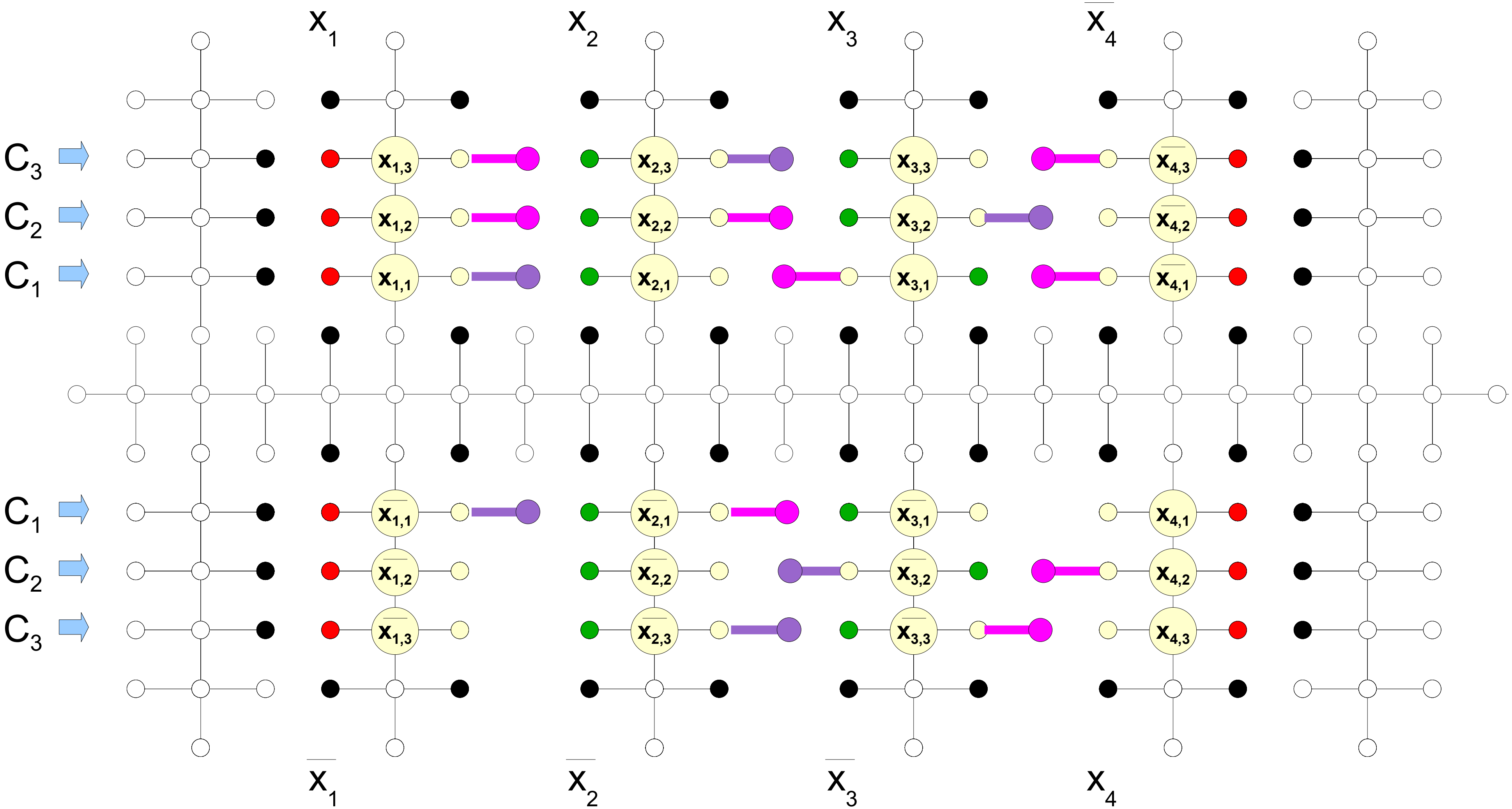


$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

C_1 C_2 C_3

Truth assignment:

$X_1 = V$ $X_2 = V$ $X_3 = V$ $X_4 = F$



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

C_1 C_2 C_3

Problem **BHATT-COSMADAKIS:**

Input: an extended skeleton $S(\Phi)$

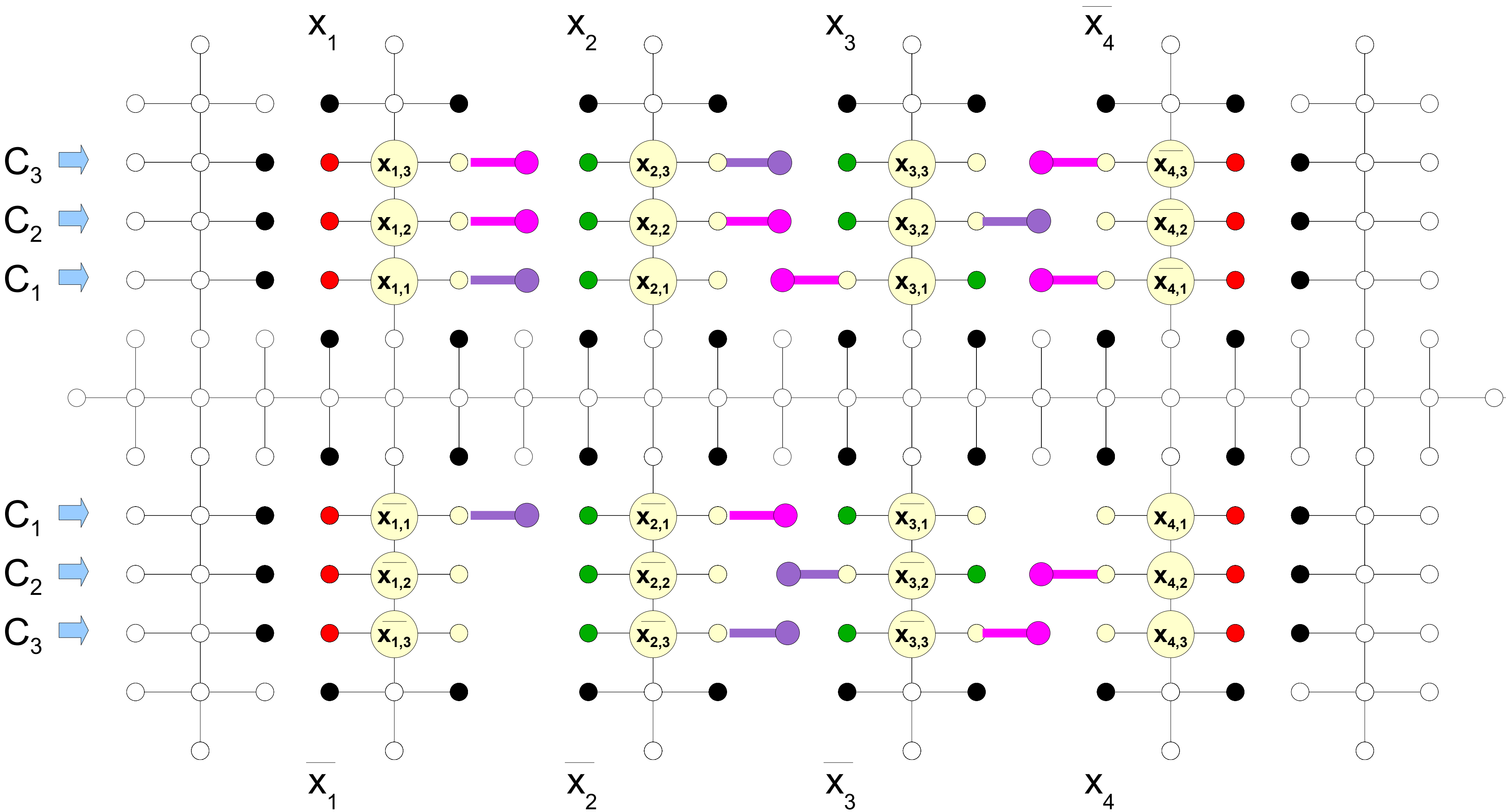
Output: YES, if $S(\Phi)$ is a partial grid;
NO, otherwise

NOT-ALL-EQUAL 3-CNF SAT $\leq_P$ BHATT-COSMADAKIS

BHATT-COSMADAKIS is NP-complete (IPL '87)

D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}				
{1,2,4}				
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**		IPL '87	[Bhatt and Cosmadakis]

Extended Skeleton S(Φ)

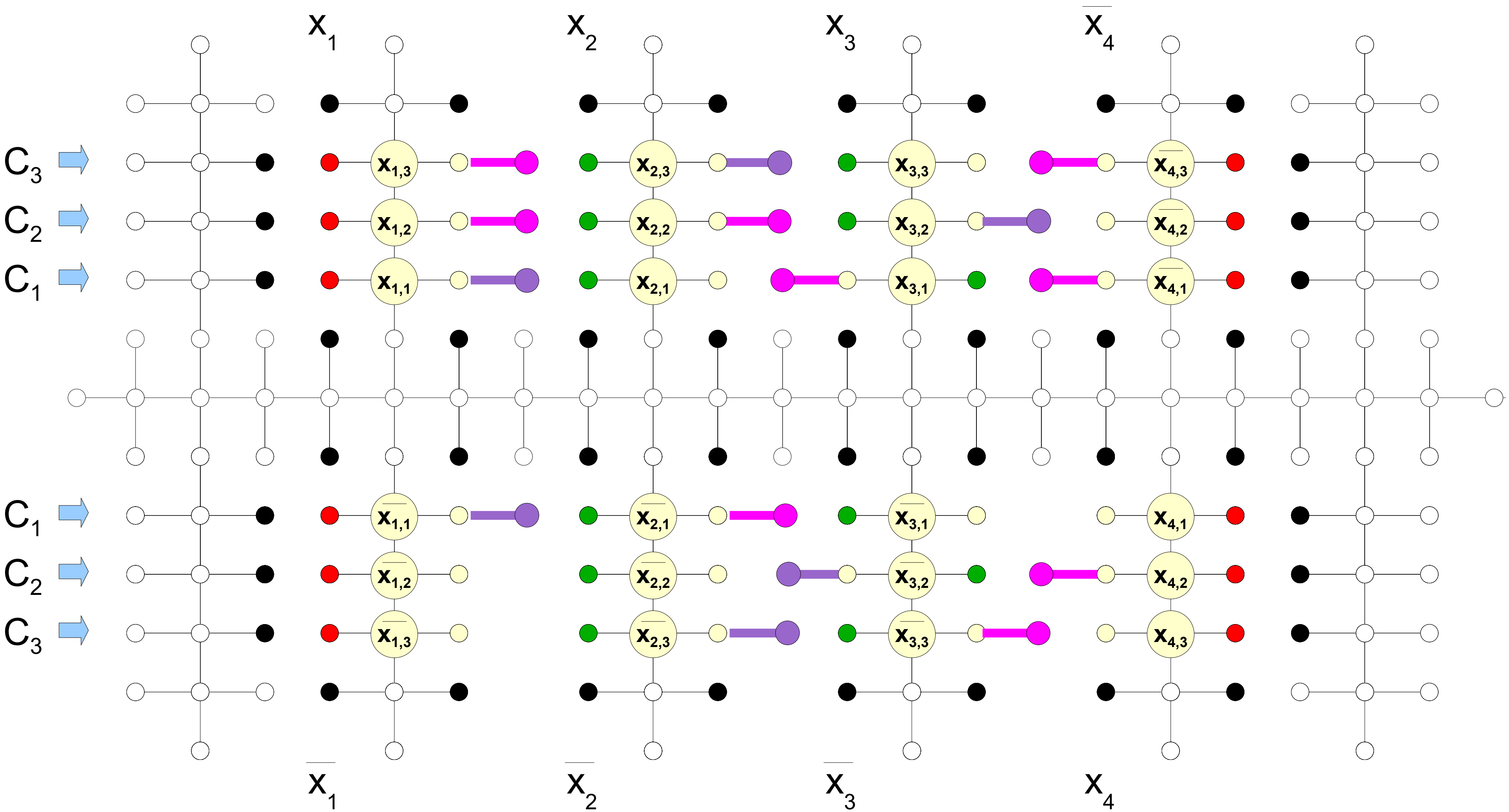


$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

C_1 C_2 C_3

D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}				
{1,2,4}				
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

Extended Skeleton S(Φ)



$$\Phi = (\overline{x}_2 \vee x_3 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4)$$

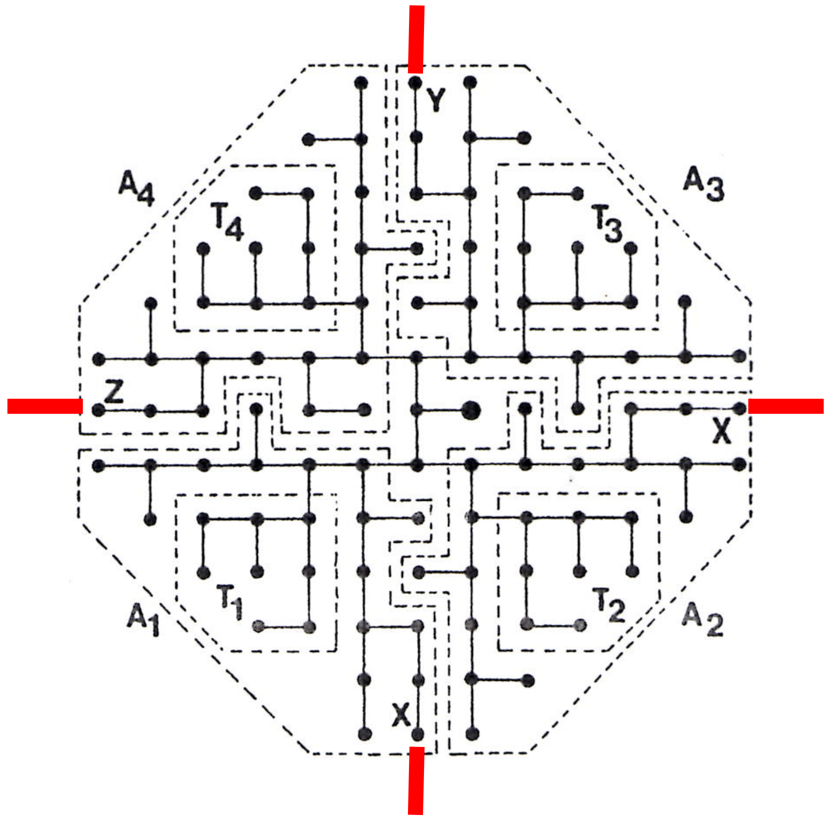
C_1 C_2 C_3

D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}				
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}		**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

Gregori's **U-tree**

a {1,2,3}-tree



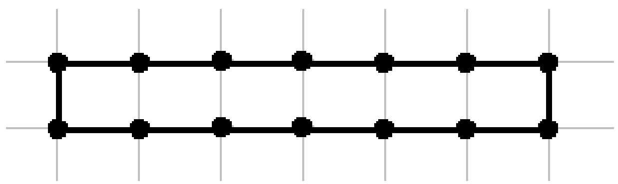
D	*D*-graphs	*D*-trees	reference	info
{1,2,3}		**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	*D*-graphs	*D*-trees	reference	info
{1}				
{2}				
{3}				
{4}				
{1,2}				
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}				
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

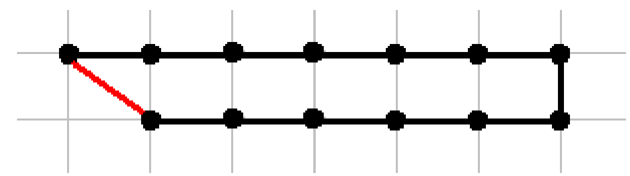
D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

P_2

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	—	trivial	YES iff |*V*| is even



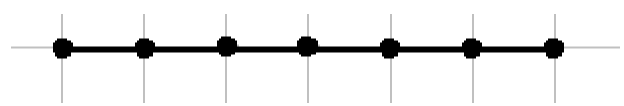
even cycle



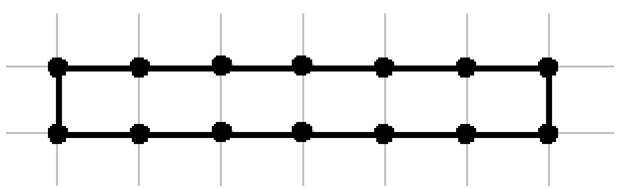
odd cycle

{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

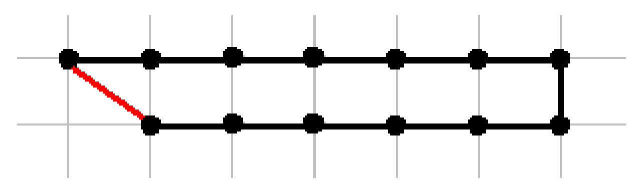
D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff |*V*| is even
{3}				
{4}				
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
[illegible]				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]



path



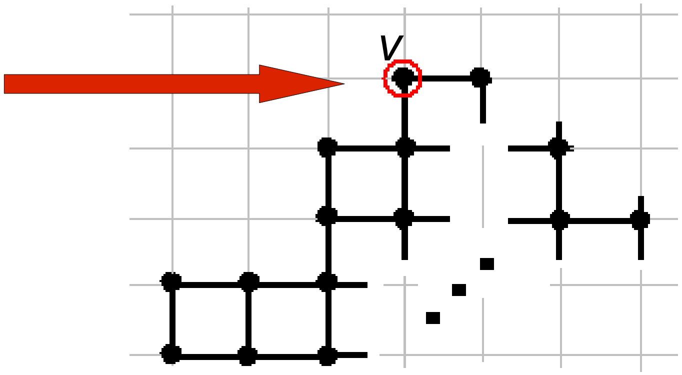
even cycle



odd cycle

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	—	trivial	YES iff $\|V\|$ is even
{3}	**P**	—	trivial	always NO
{4}	**P**	—	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}				
{1,4}				
{2,3}				
{2,4}				
{3,4}	**P**	—	trivial	always NO

deg(v) < 3



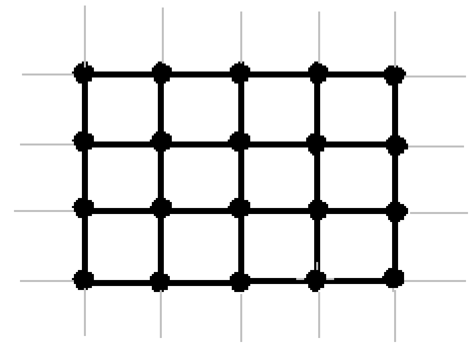
D	D-graphs	D-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff G is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				
{2,4}				
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}				
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	*D*-graphs	*D*-trees	reference	info
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				

The subgraph of *G* induced by its 4-degree vertices is a grid

⬇

G is a partial grid

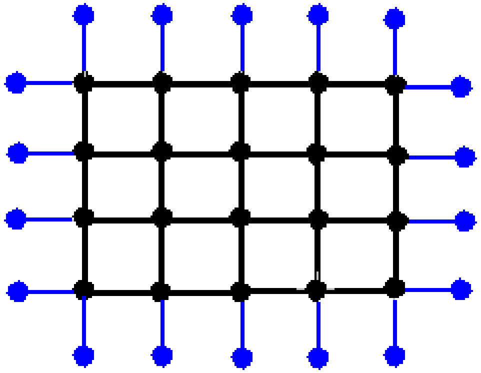


D	D-graphs	D-trees	reference	info

The subgraph of G induced by its 4-degree vertices is a grid

⬇

G is a partial grid



{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				

D	*D*-graphs	*D*-trees	reference	info
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				

The subgraph of *G* induced by its 4-degree vertices is a grid

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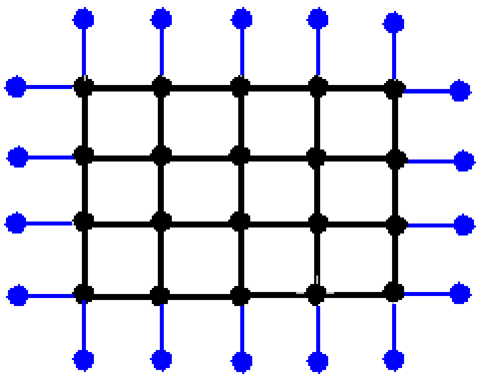
G is a partial grid



G is a partial grid

⬇

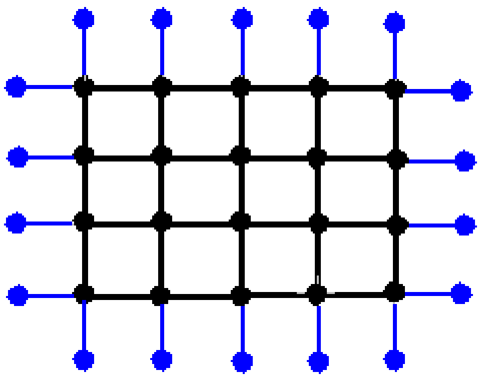
The subgraph of *G* induced by its 4-degree vertices is a grid

D	*D*-graphs	*D*-trees	reference	info

The subgraph of *G* induced by its 4-degree vertices is a grid

⬇

G is a partial grid



{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				

Hypothesis: **A** ∧ $\overline{\textbf{B}}$

G is a partial grid **(A)**

⬇

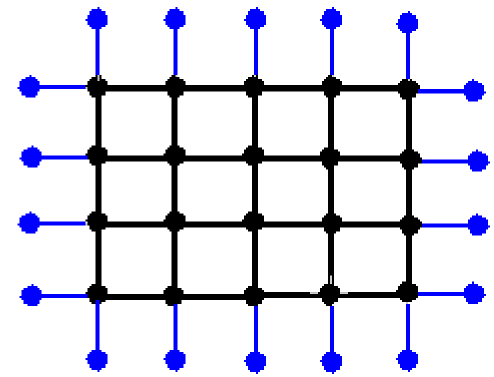
The subgraph of *G* induced by its 4-degree vertices is a grid **(B)**

D	*D*-graphs	*D*-trees	reference	info
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				

The subgraph of *G* induced by its 4-degree vertices is a grid

⬇

G is a partial grid

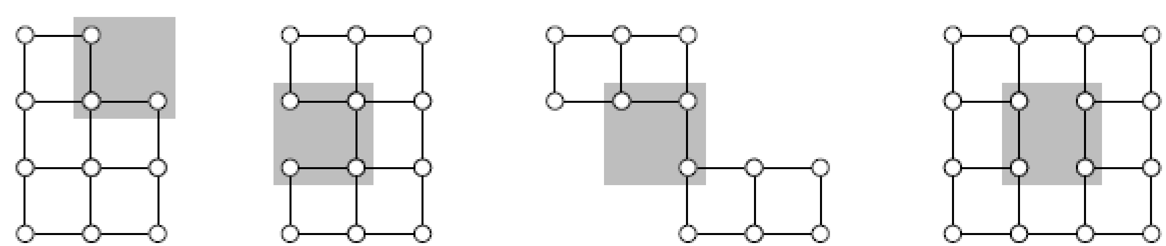


Hypothesis: **A** ∧ **$\overline{\text{B}}$** ➡ **CONTRADICTION!**

G is a partial grid **(A)**

⬇

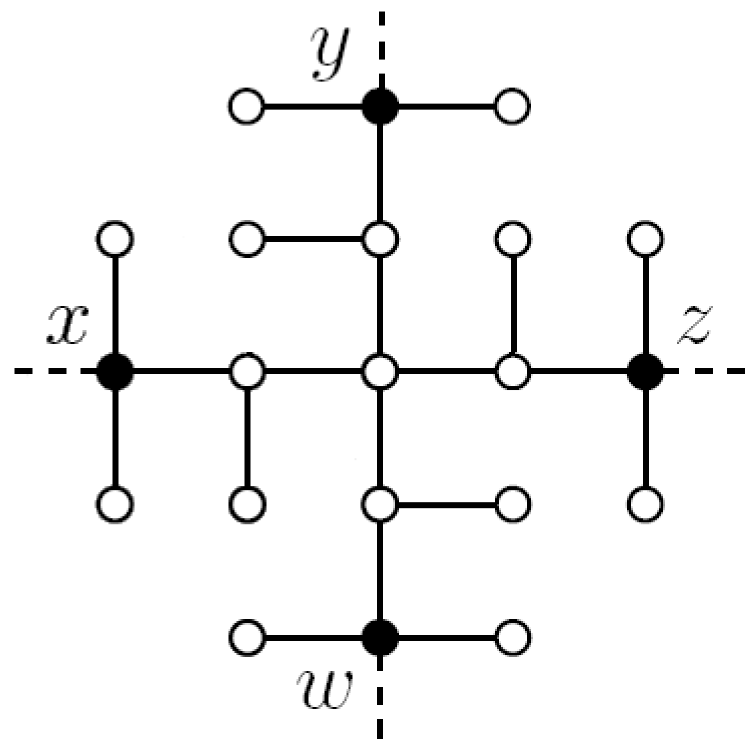
The subgraph of *G* induced by its 4-degree vertices is a grid **(B)**



D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				
{2,4}				
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

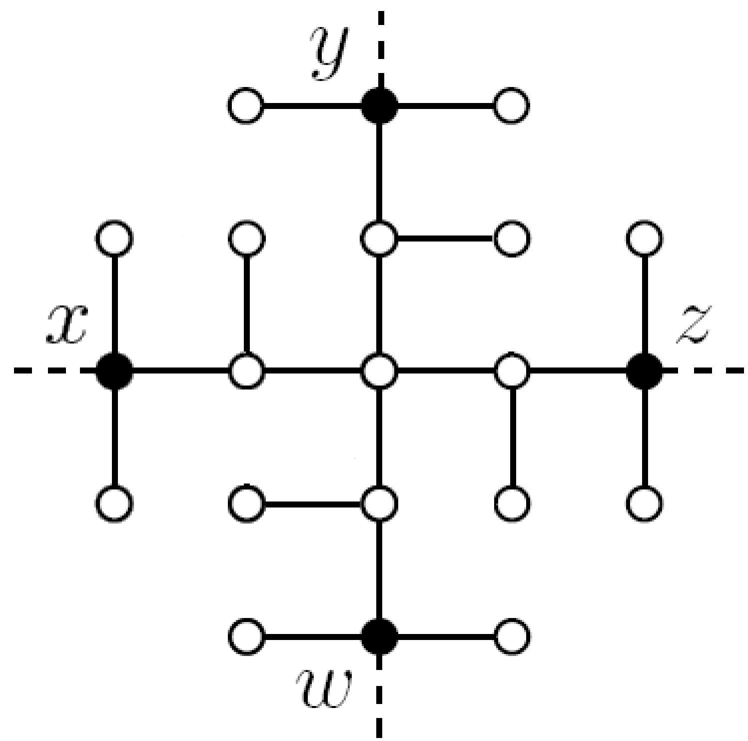
The windmill tree

(*gadget* for {1,3,4}-trees)



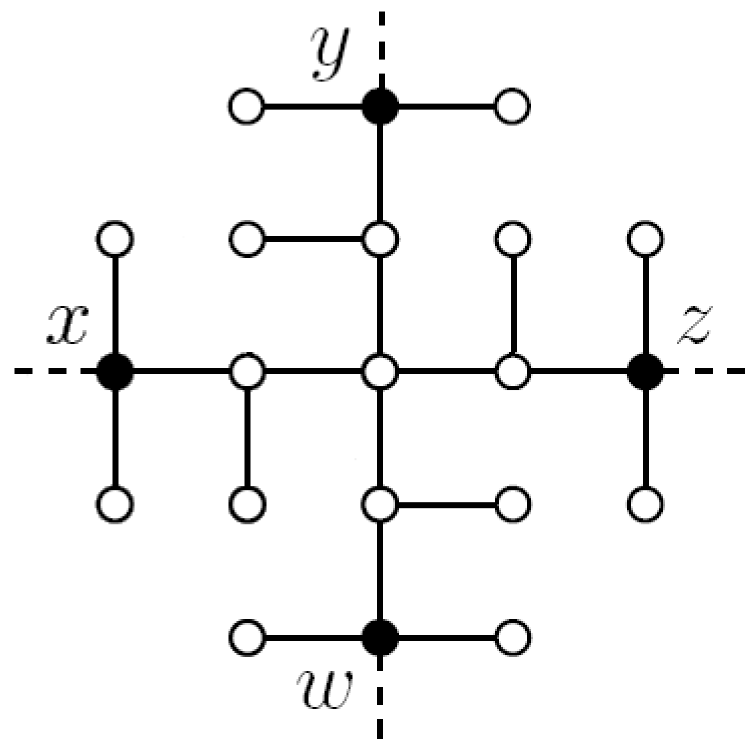
The windmill tree

(*gadget* for {1,3,4}-trees)



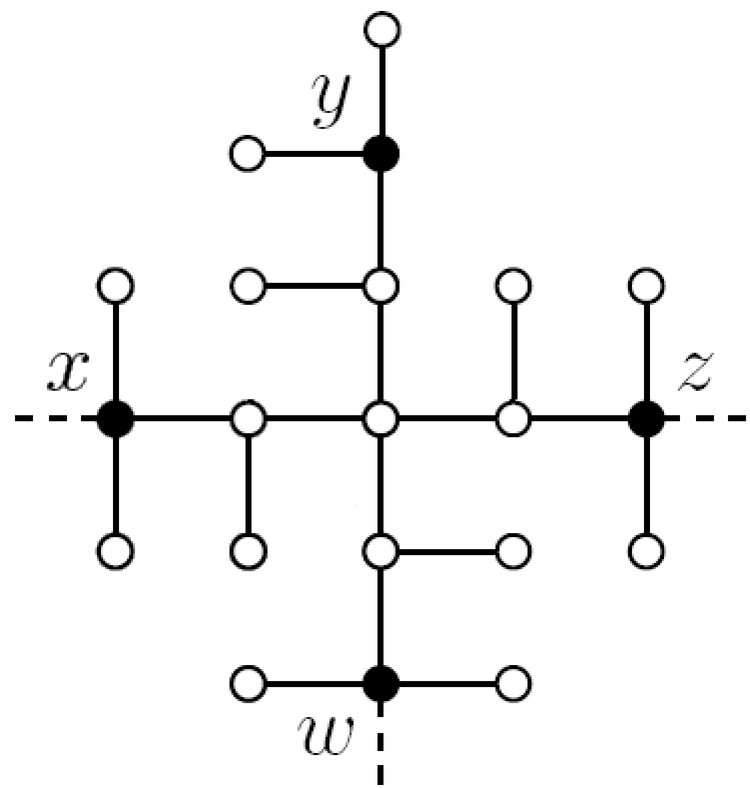
The windmill tree

(*gadget* for {1,3,4}-trees)



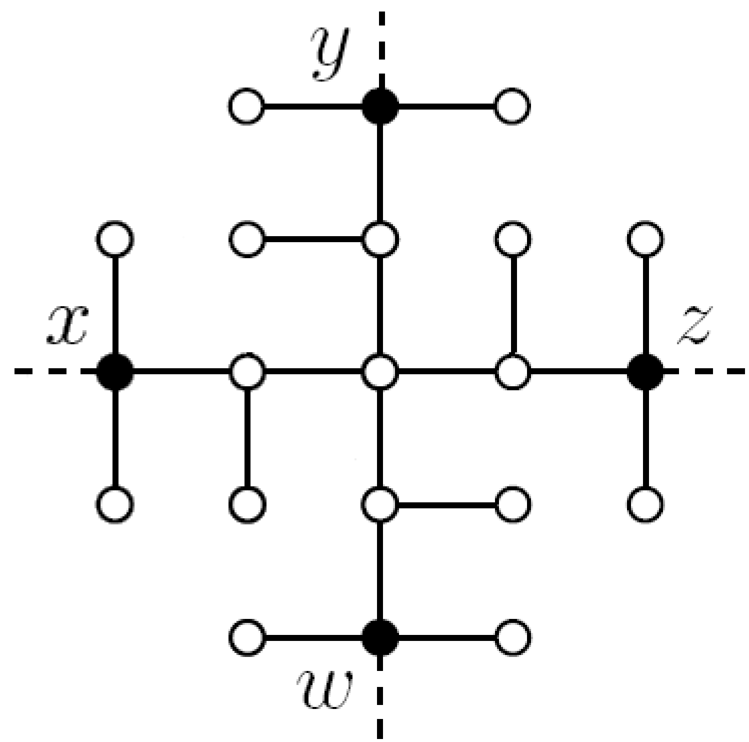
The windmill tree

(*gadget* for {1,3,4}-trees)



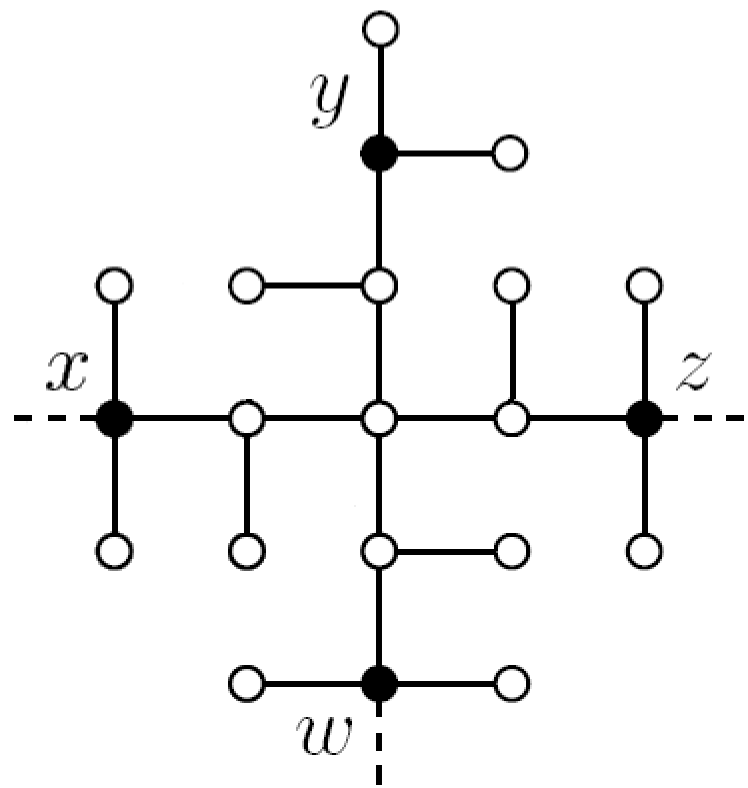
The windmill tree

(*gadget* for {1,3,4}-trees)



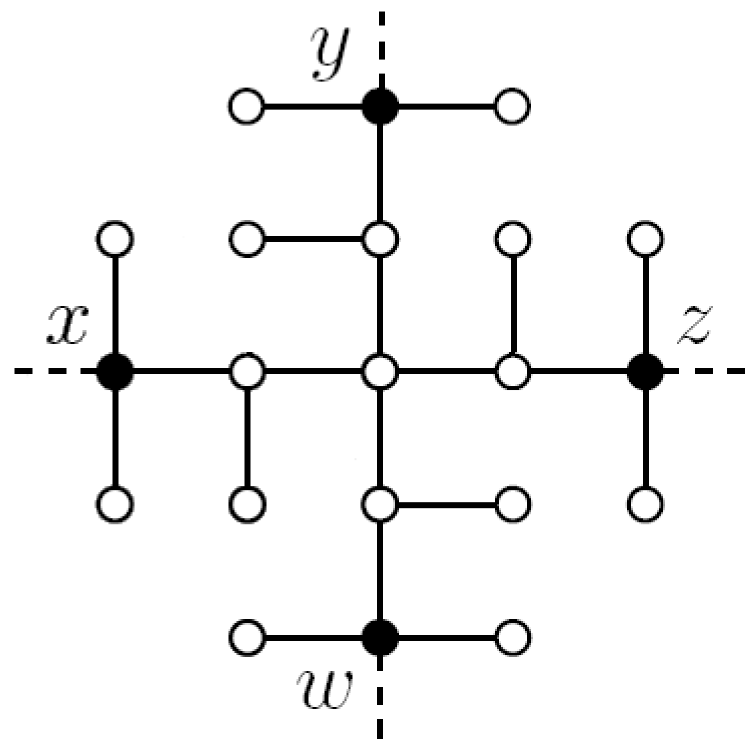
The windmill tree

(*gadget* for {1,3,4}-trees)



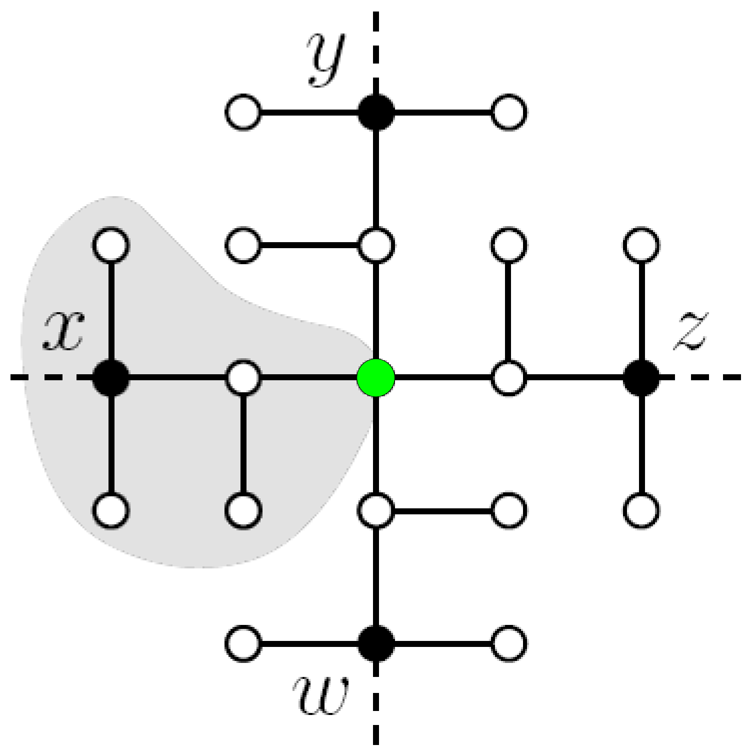
The windmill tree

(*gadget* for {1,3,4}-trees)



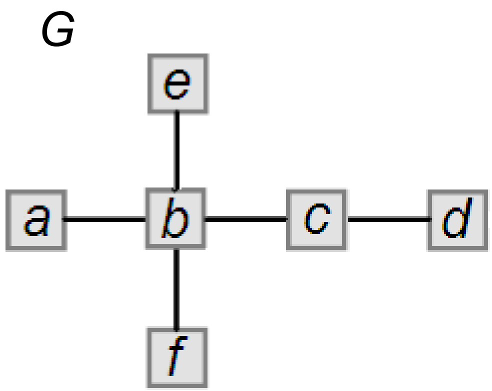
The windmill tree

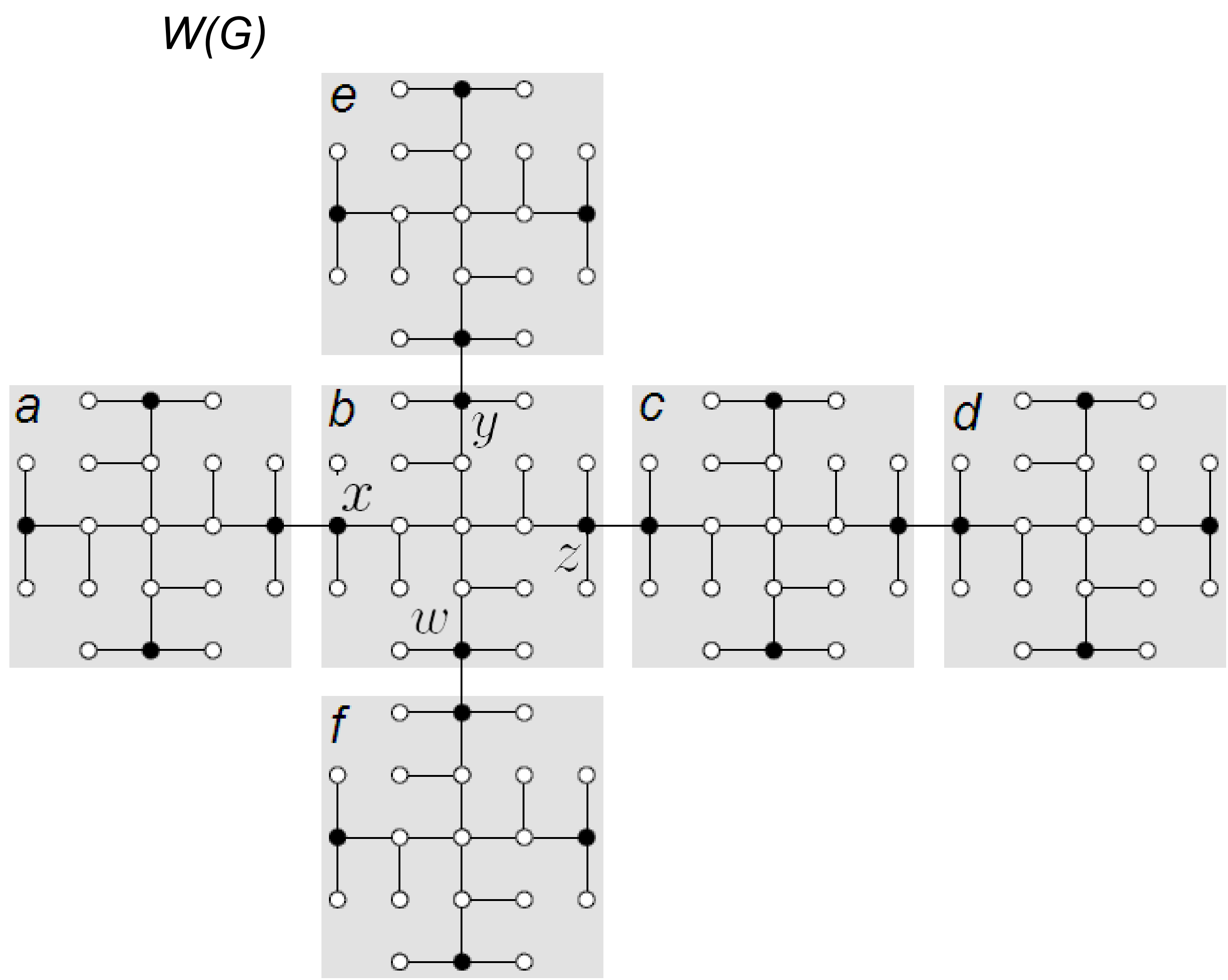
(*gadget* for {1,3,4}-trees)



The windmill tree

(*gadget* for {1,3,4}-trees)

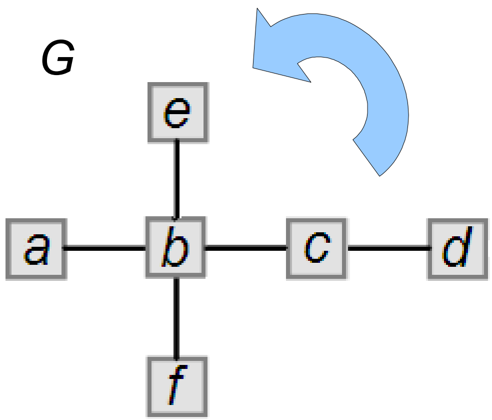




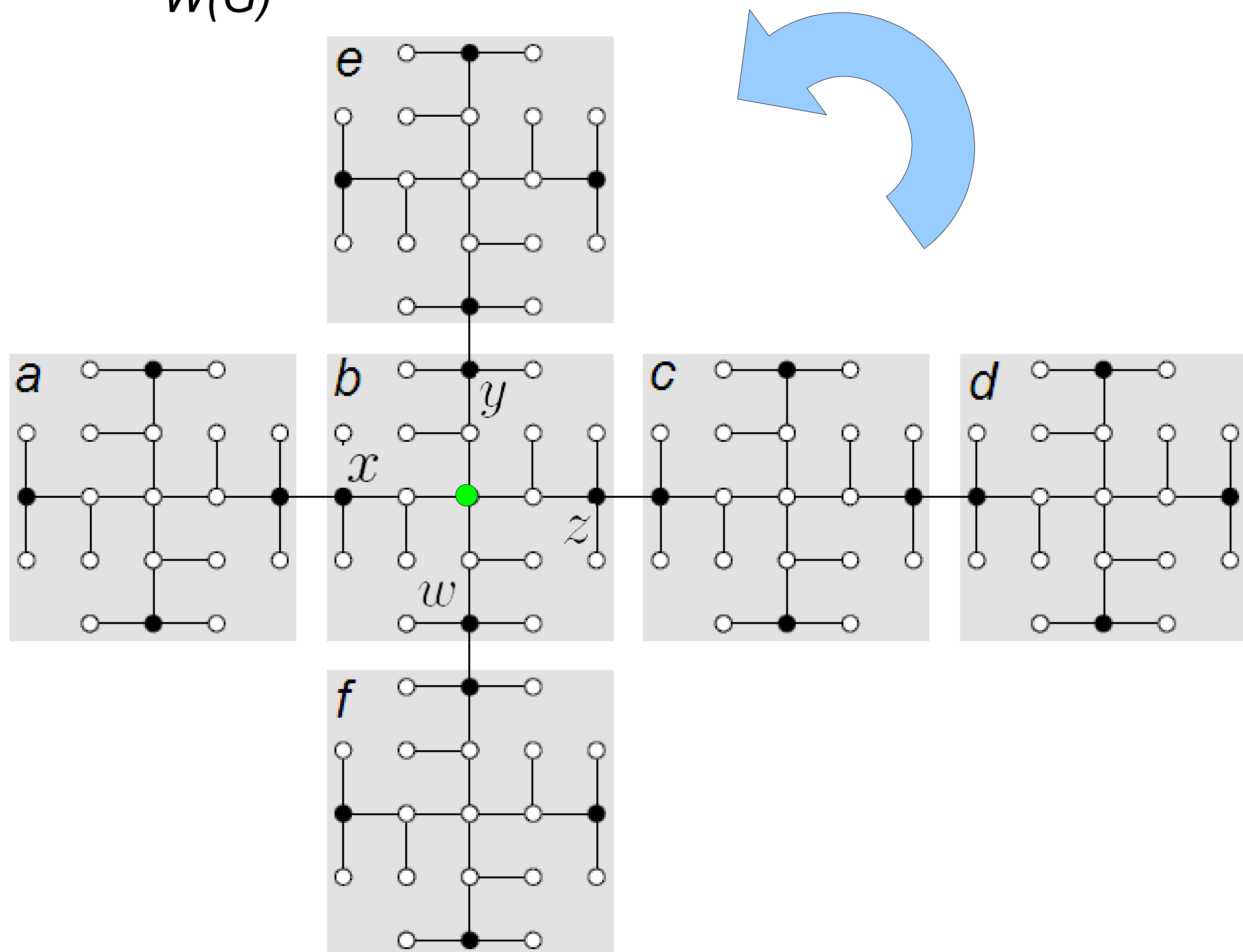
The windmill tree

(*gadget* for {1,3,4}-trees)

G

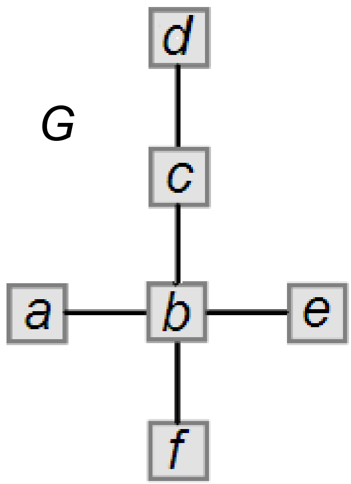


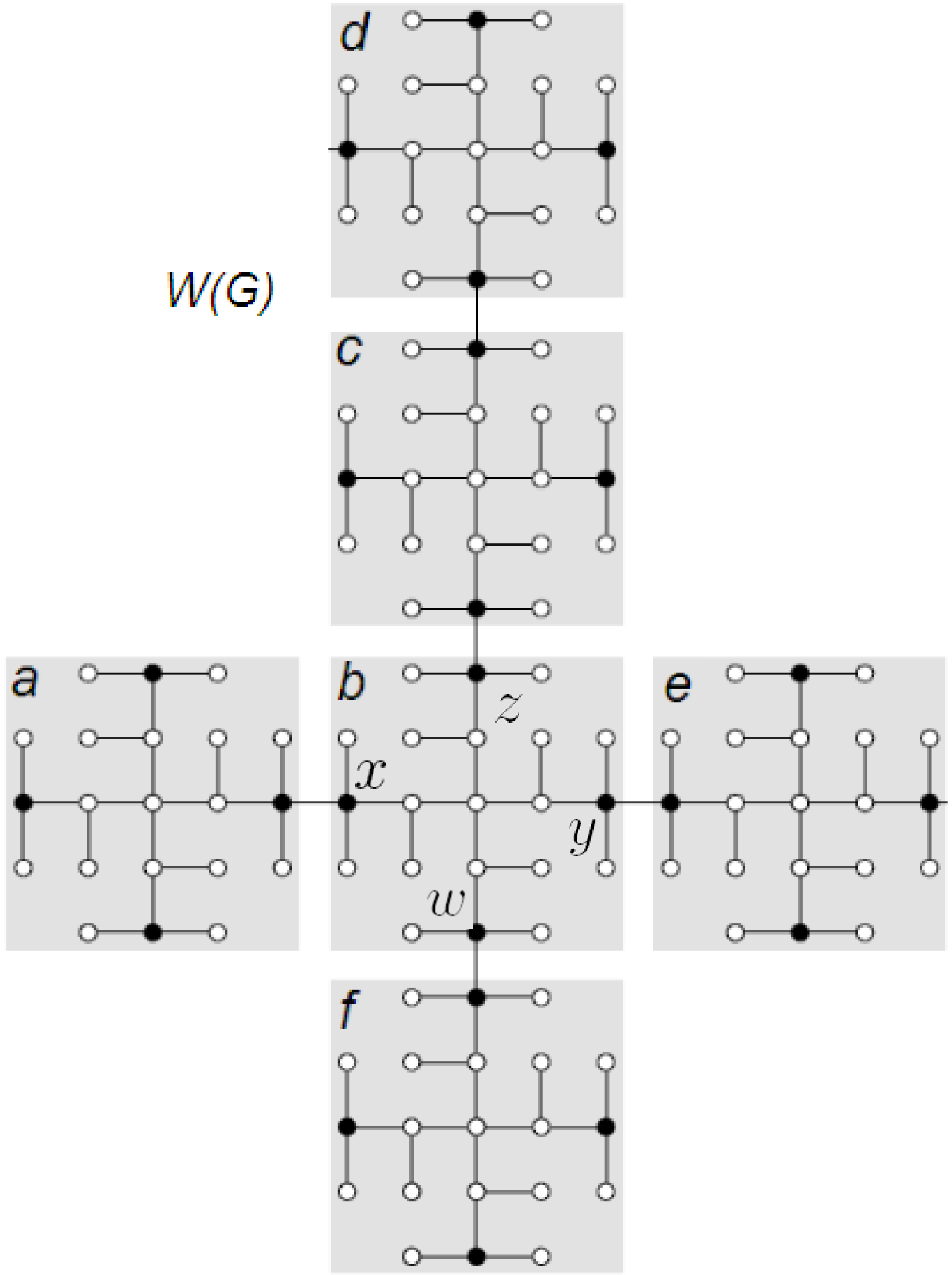
W(G)



The windmill tree

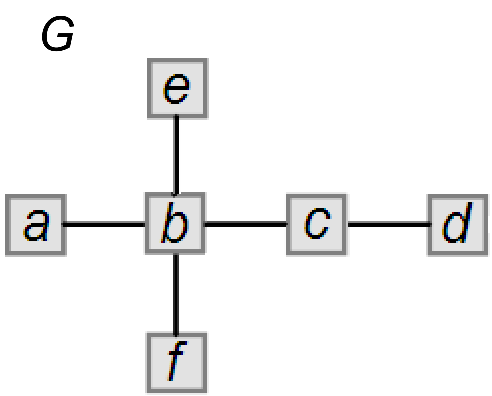
(*gadget* for {1,3,4}-trees)

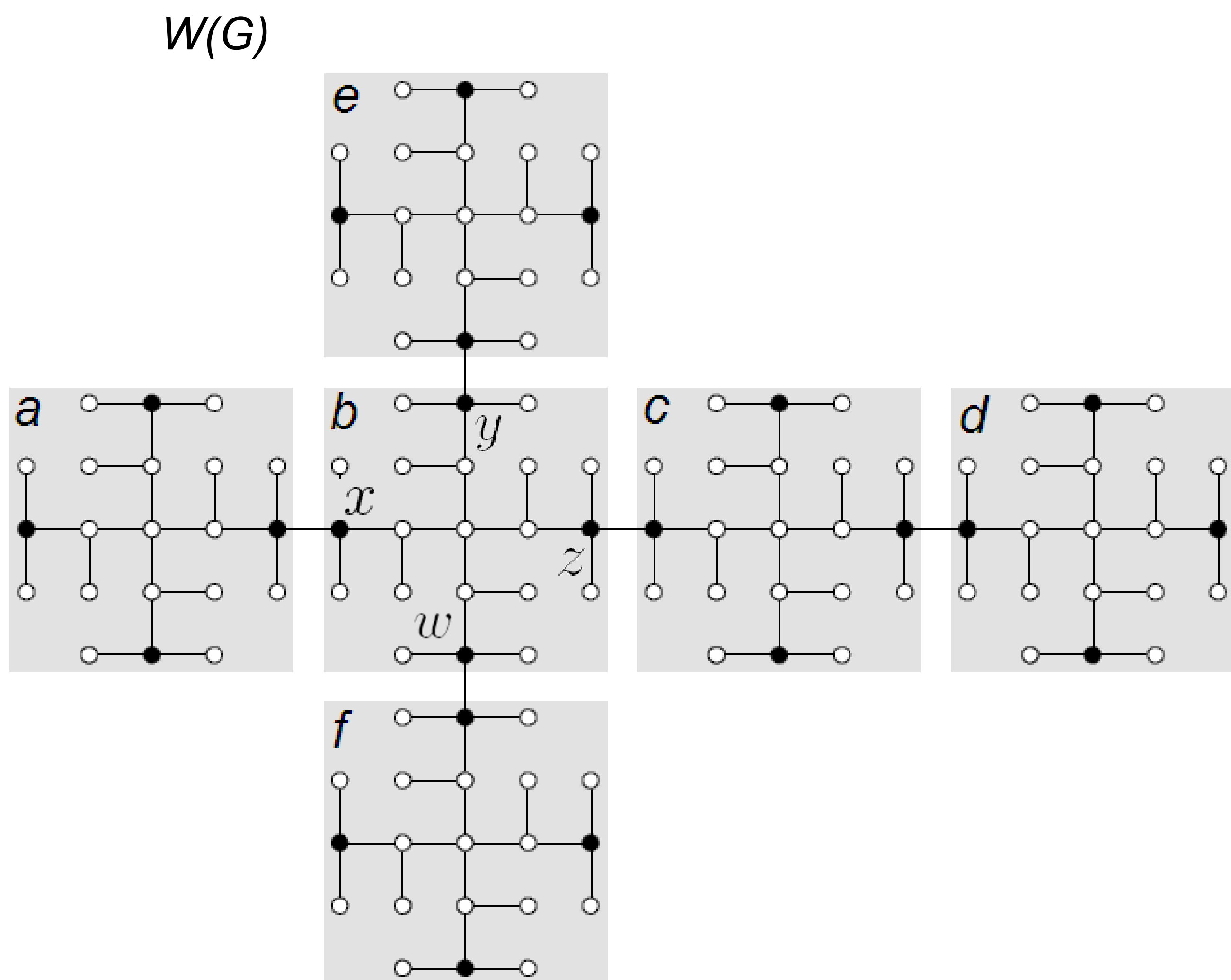




The windmill tree

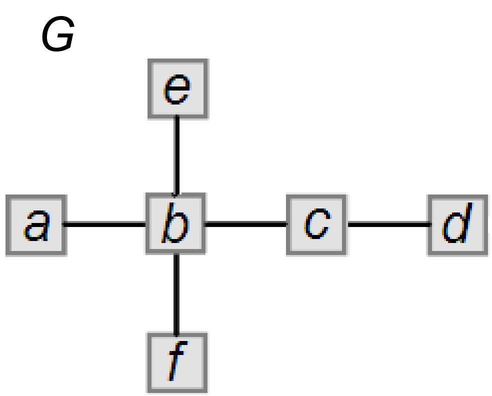
(*gadget* for {1,3,4}-trees)

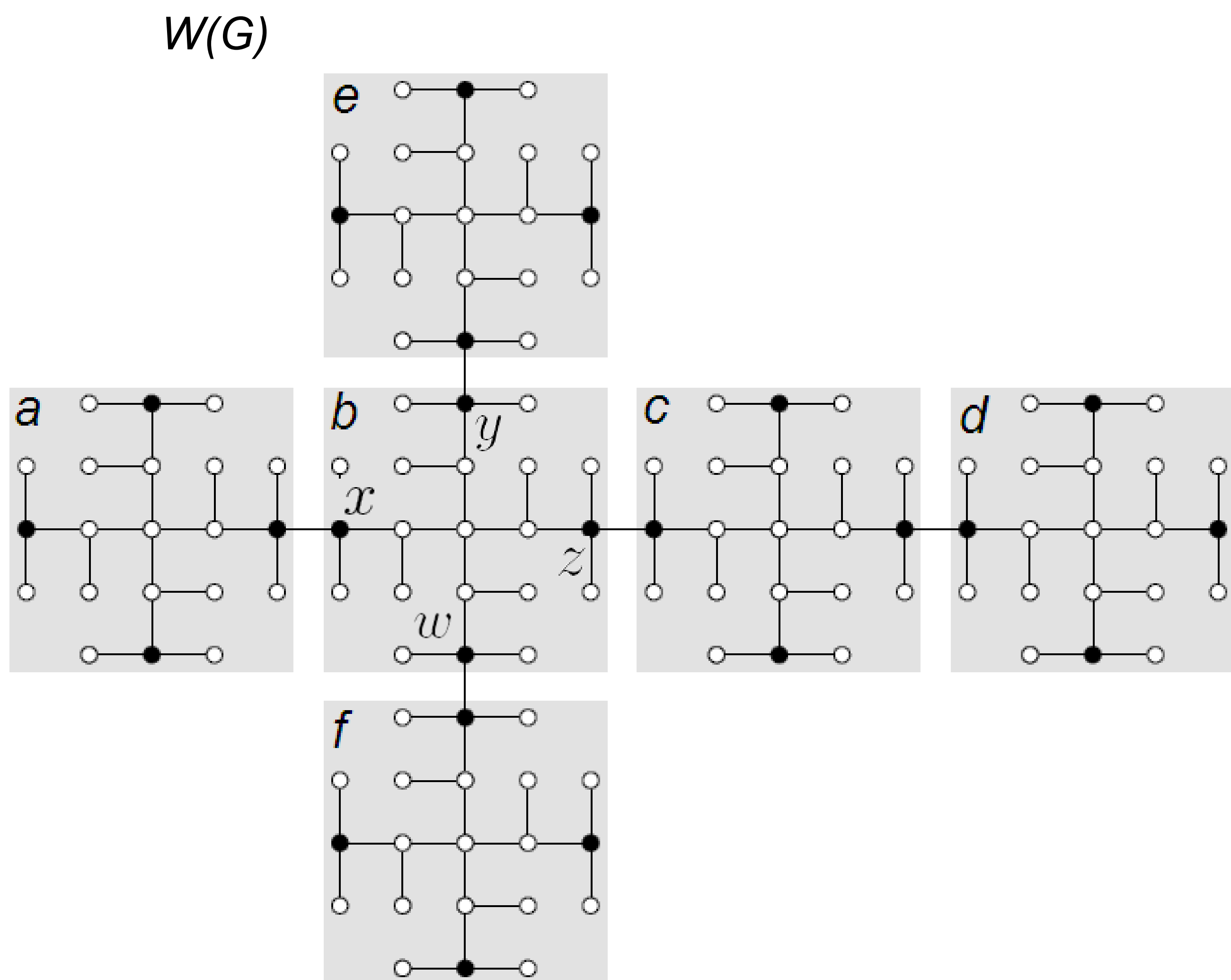




The windmill tree

(*gadget* for {1,3,4}-trees)





G is a partial grid

⬍

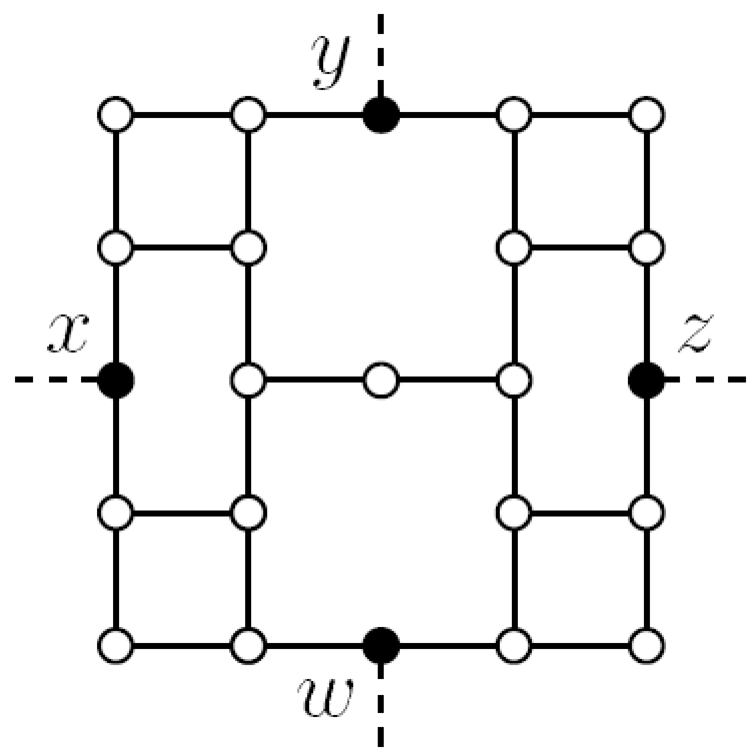
W(G) is a partial grid

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid deg(v)=1\}$ is a grid
{2,3}				
{2,4}				
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	D-graphs	D-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff G is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	
{2,4}				
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

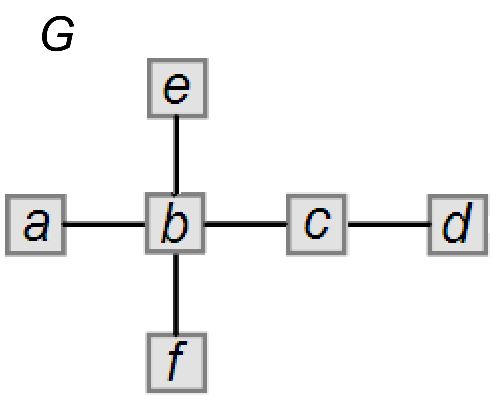
The double ladder

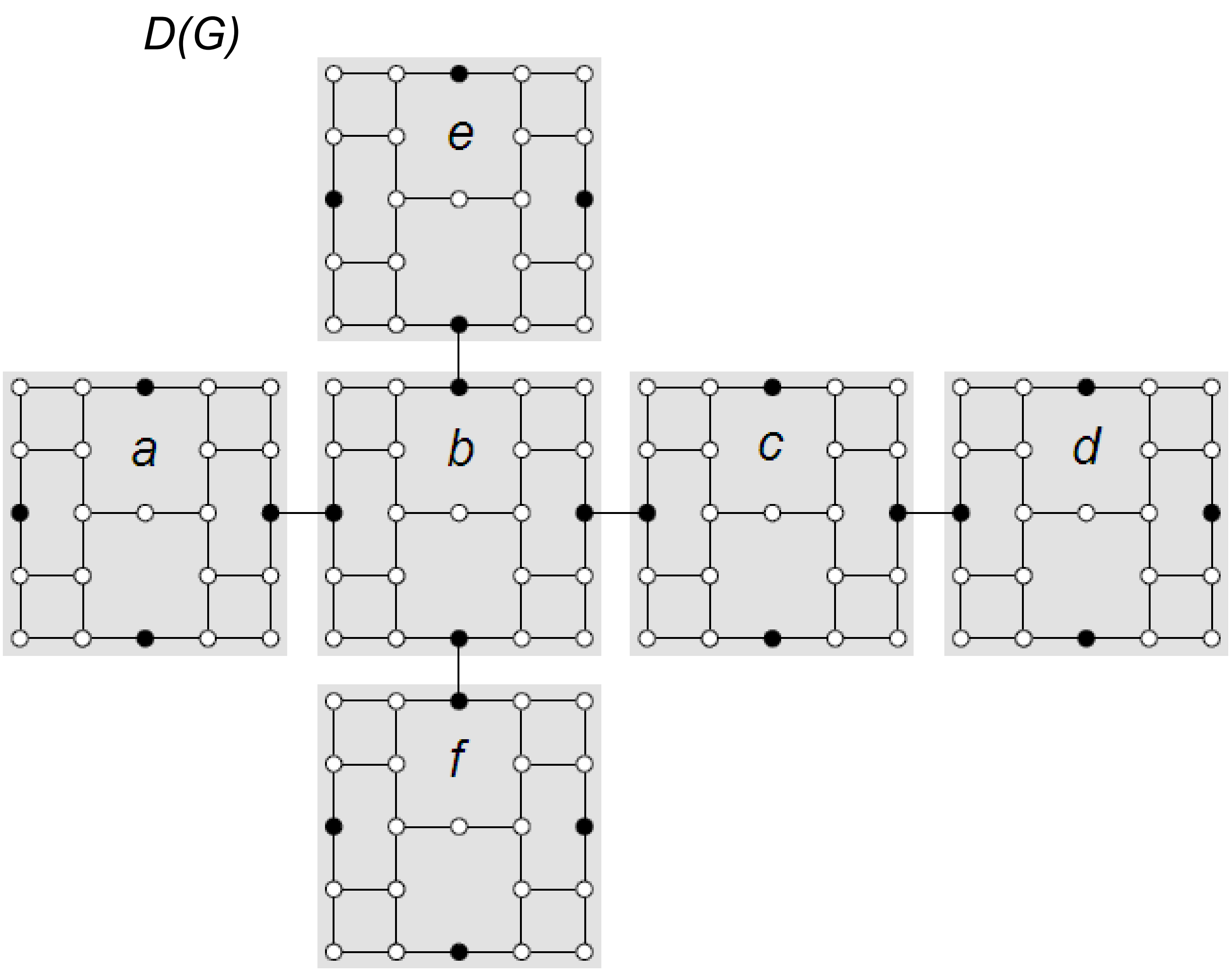
(*gadget* for {2,3}-graphs)



The double ladder

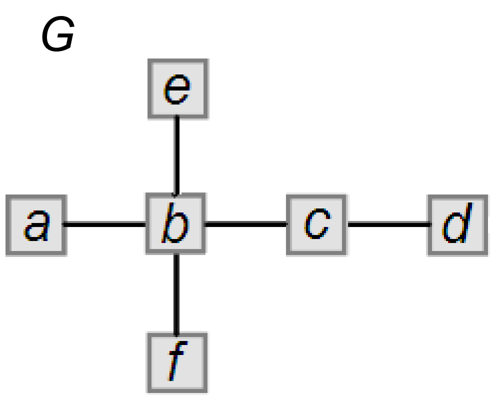
(*gadget* for {2,3}-graphs)





The double ladder

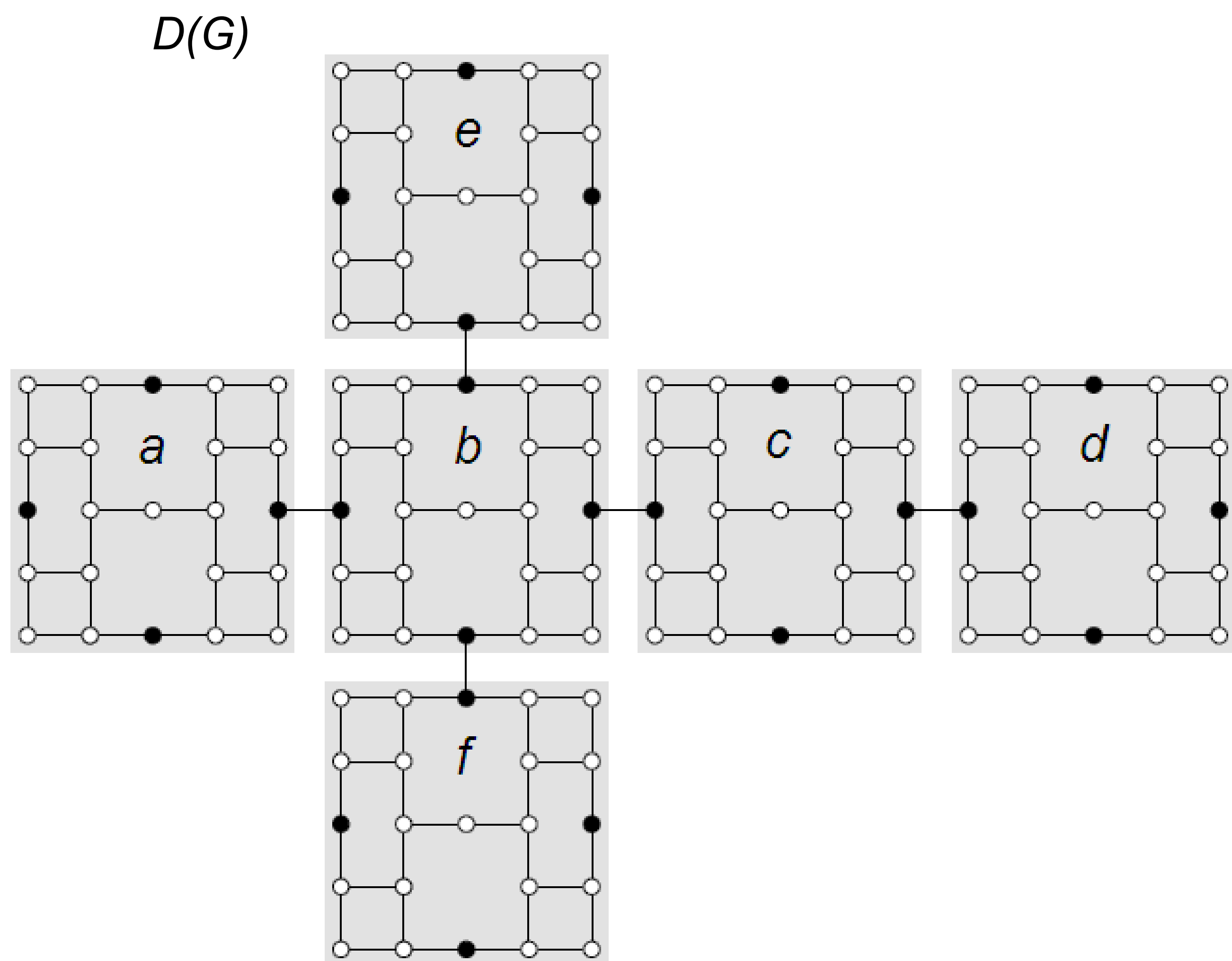
(*gadget* for {2,3}-graphs)



G is a partial grid

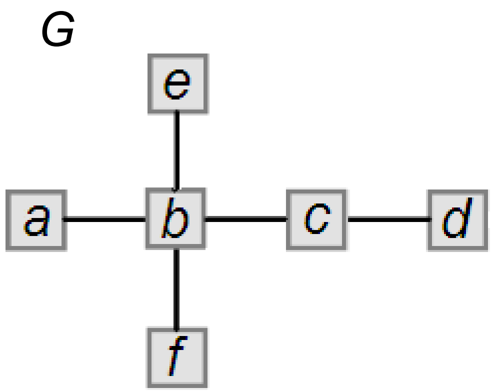
⬍

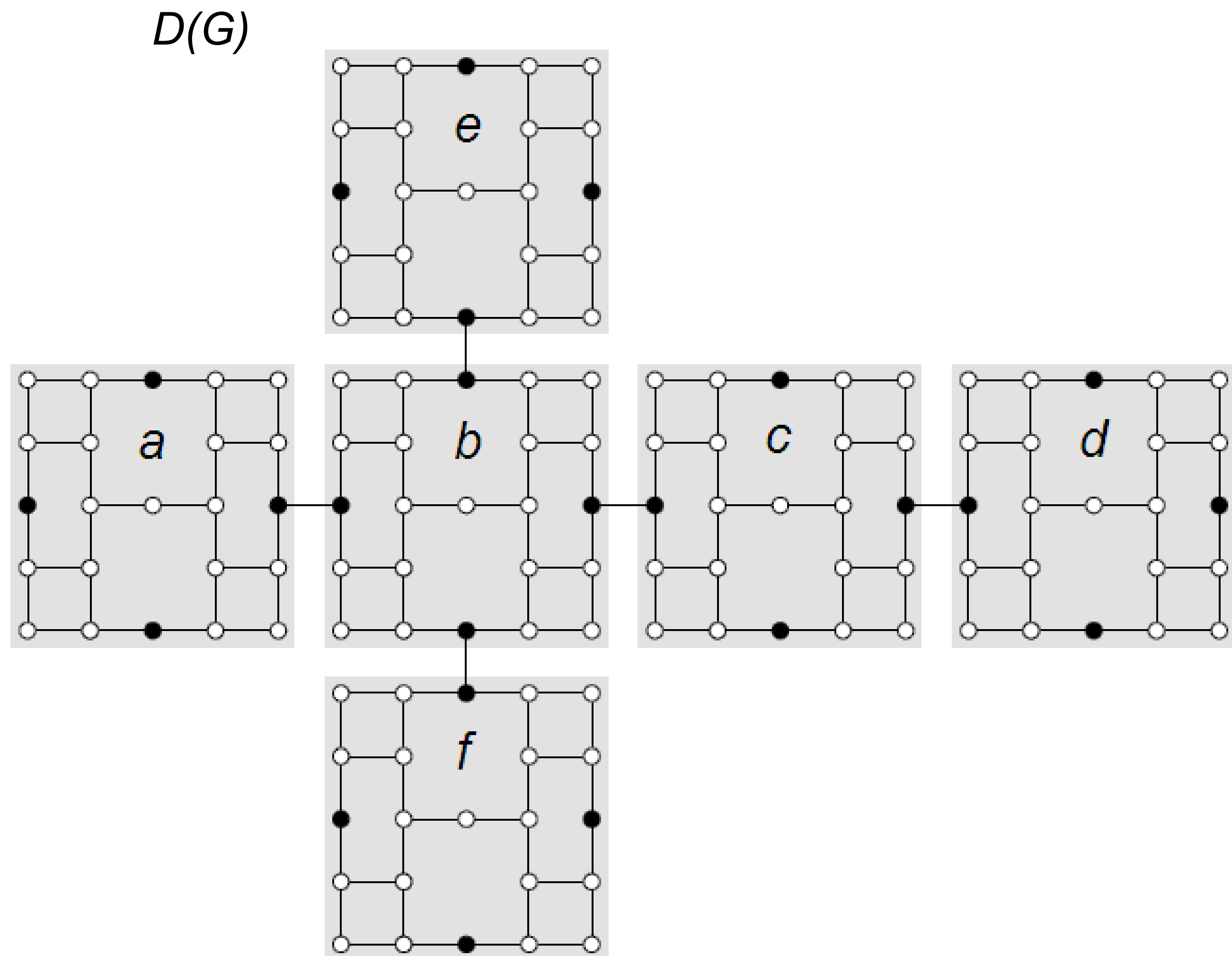
D(G) is a partial grid



The double ladder

(*gadget* for {2,3}-graphs)





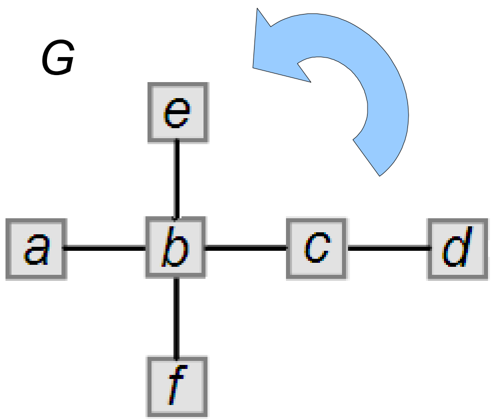
G is a partial grid

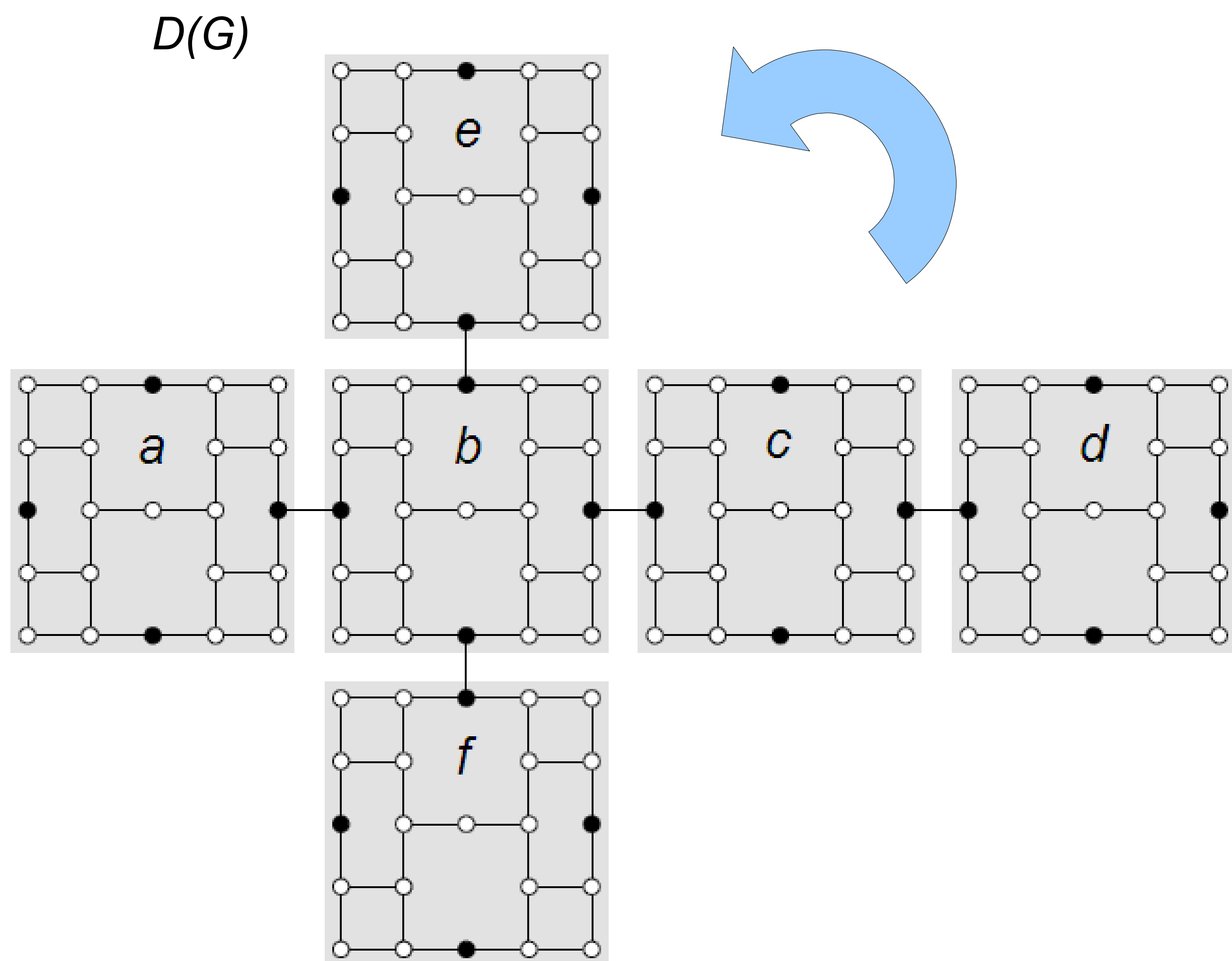
⇕

D(G) is a partial grid

The double ladder

(*gadget* for {2,3}-graphs)





G is a partial grid

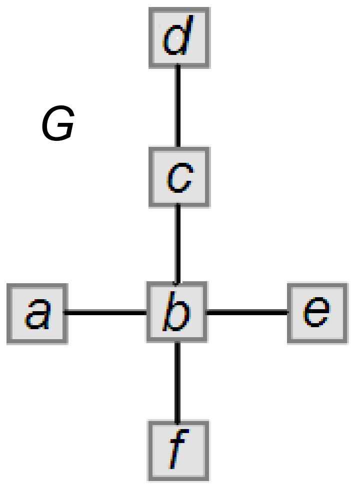
⇕

D(G) is a partial grid

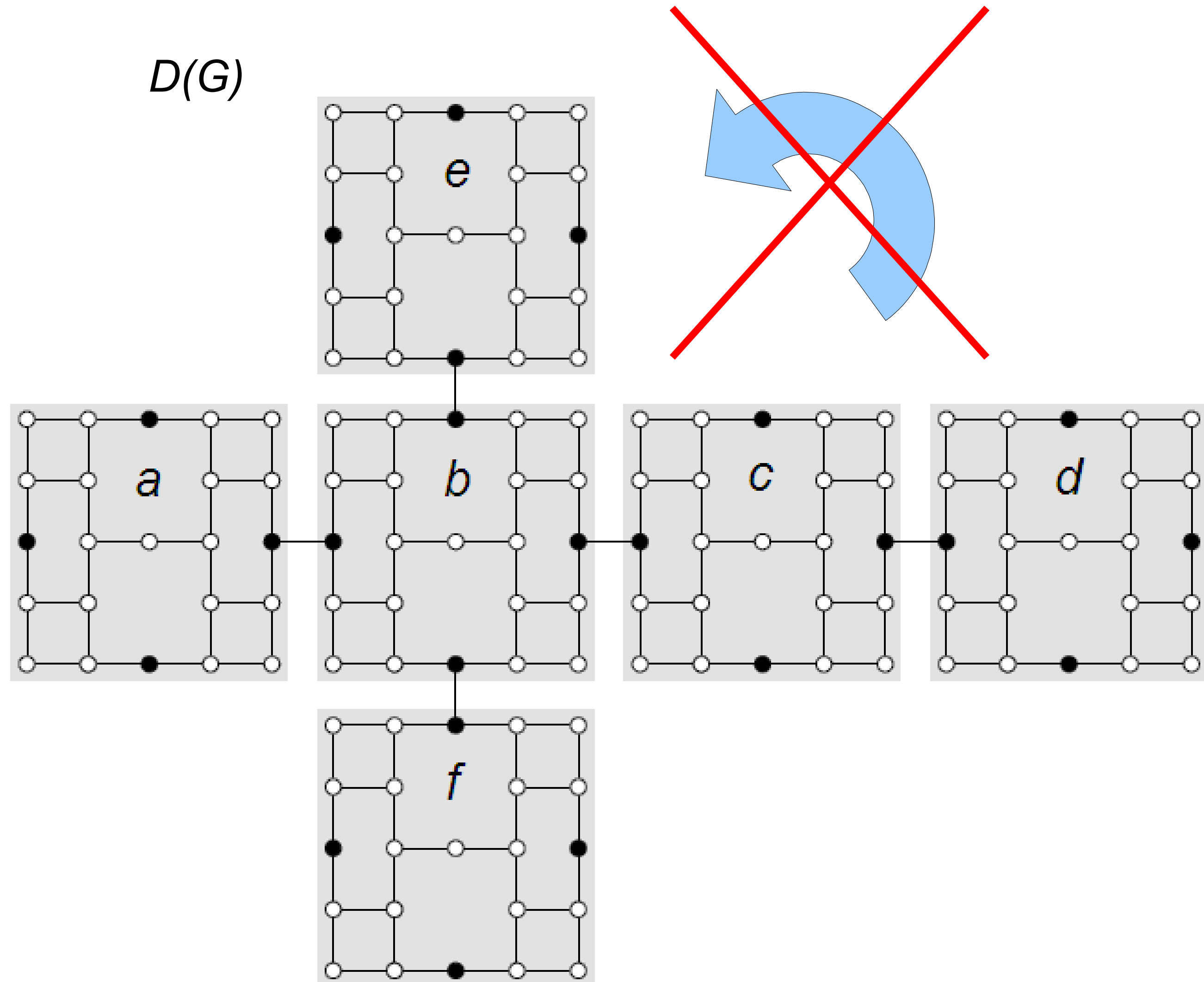
The double ladder

(*gadget* for {2,3}-graphs)

G



D(G)

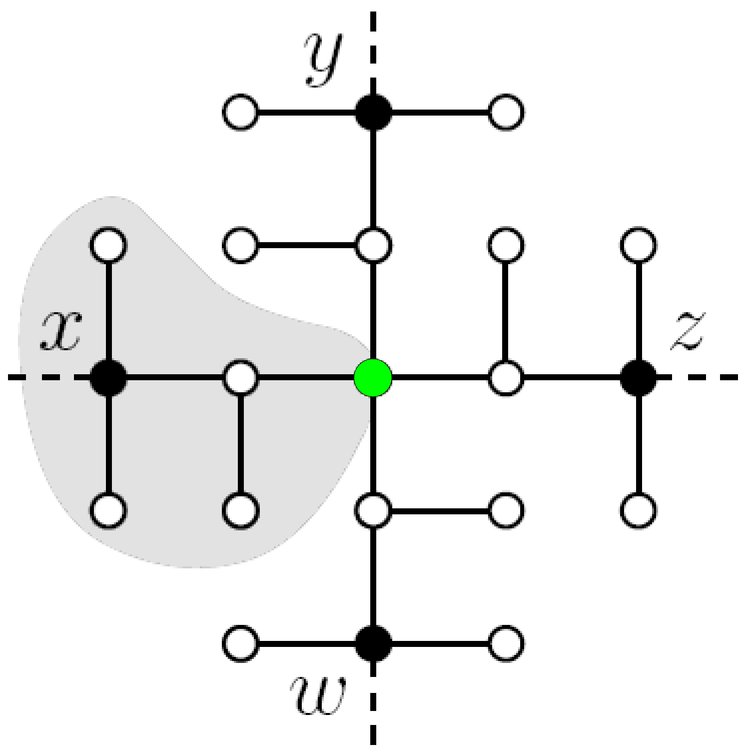


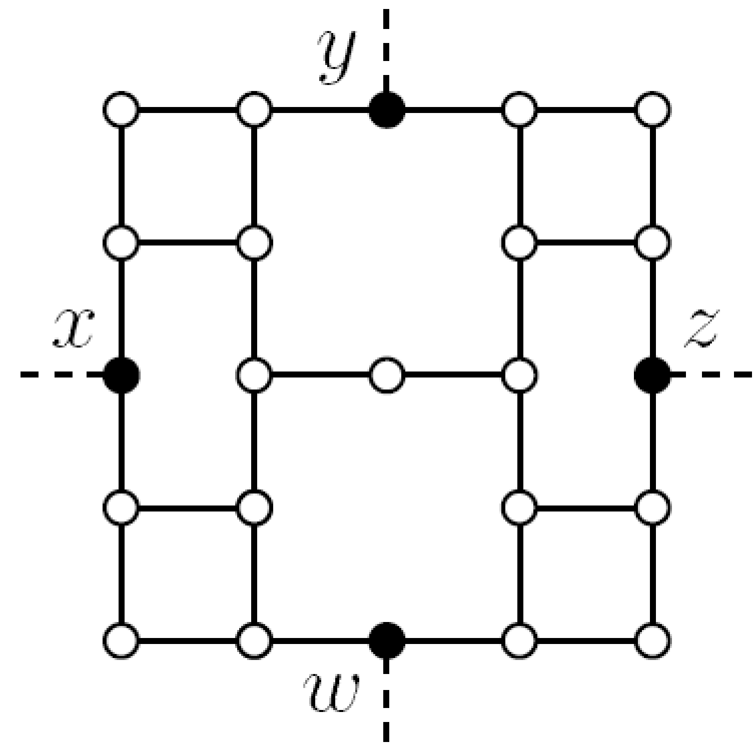
G is a partial grid

⇕

D(G) is a partial grid

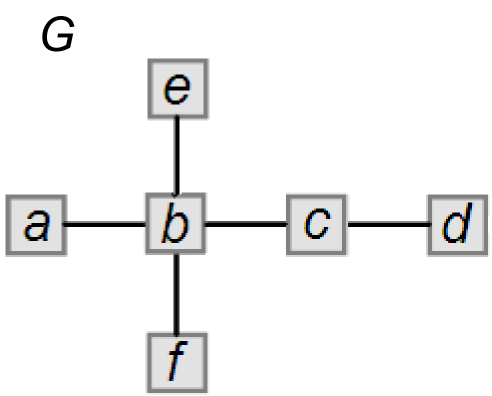
Windmill tree *vs.* Double ladder

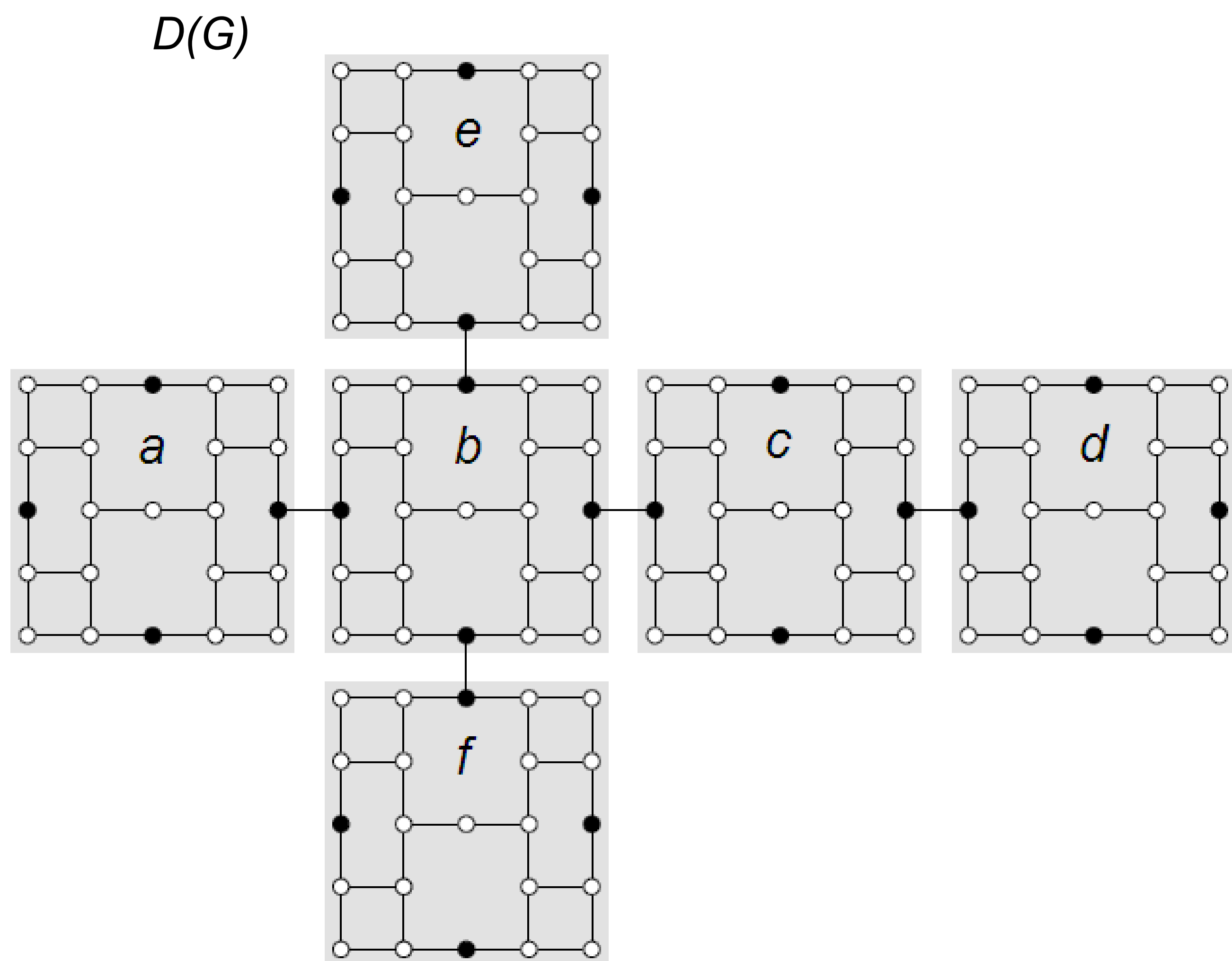




The double ladder

(*gadget* for {2,3}-graphs)





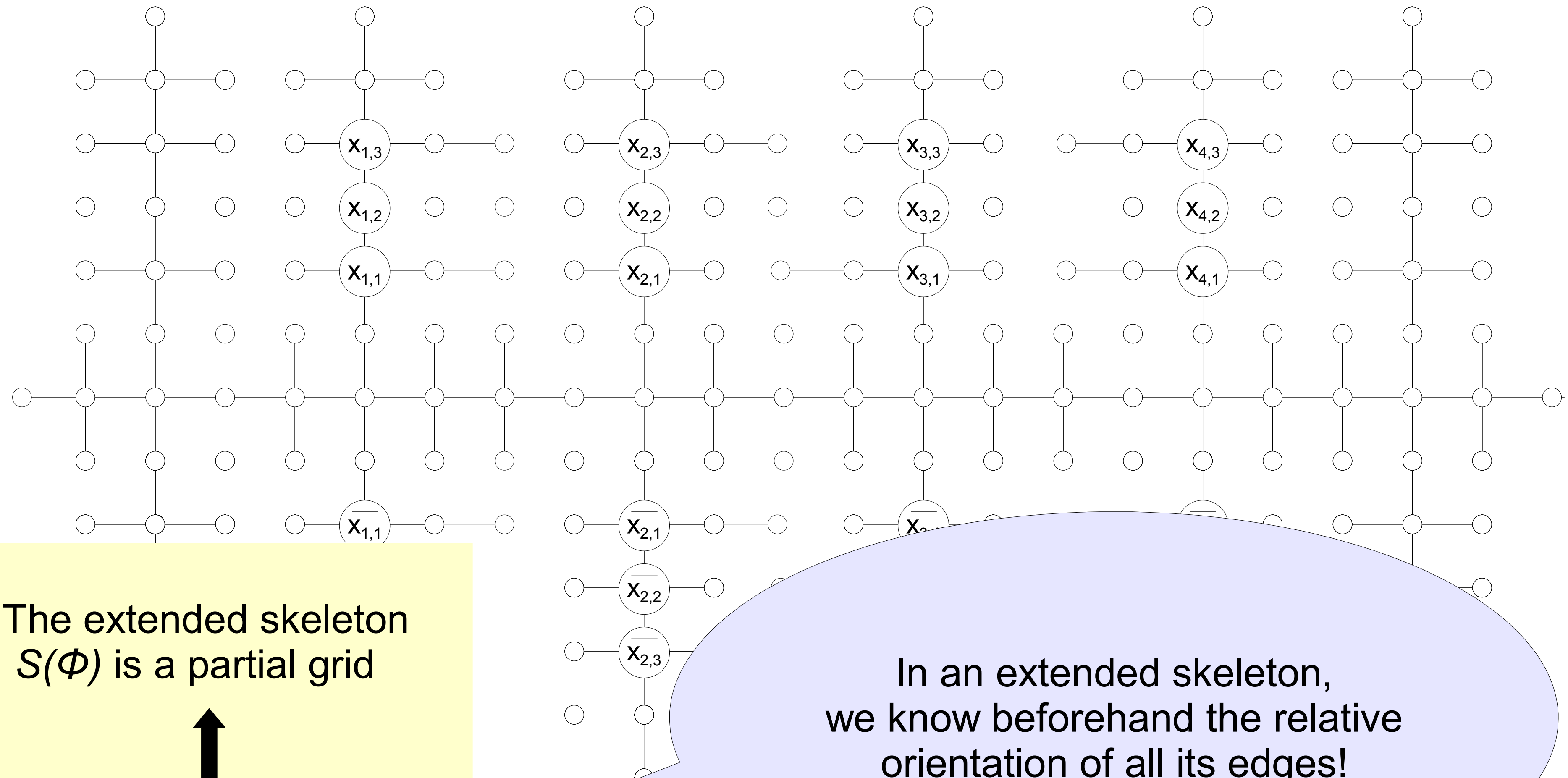
G is a partial grid

⇕

D(G) is a partial grid

The double ladder

(*gadget* for {2,3}-graphs)



The extended skeleton *S(Φ)* is a partial grid

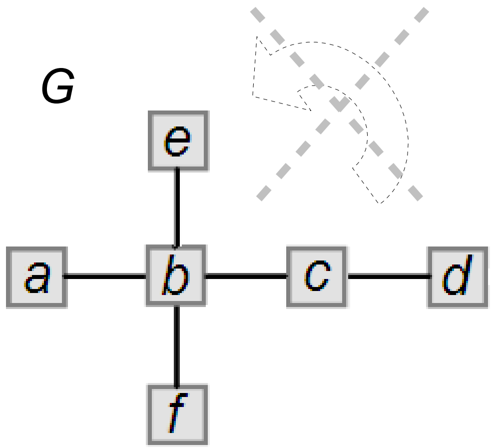
⬍

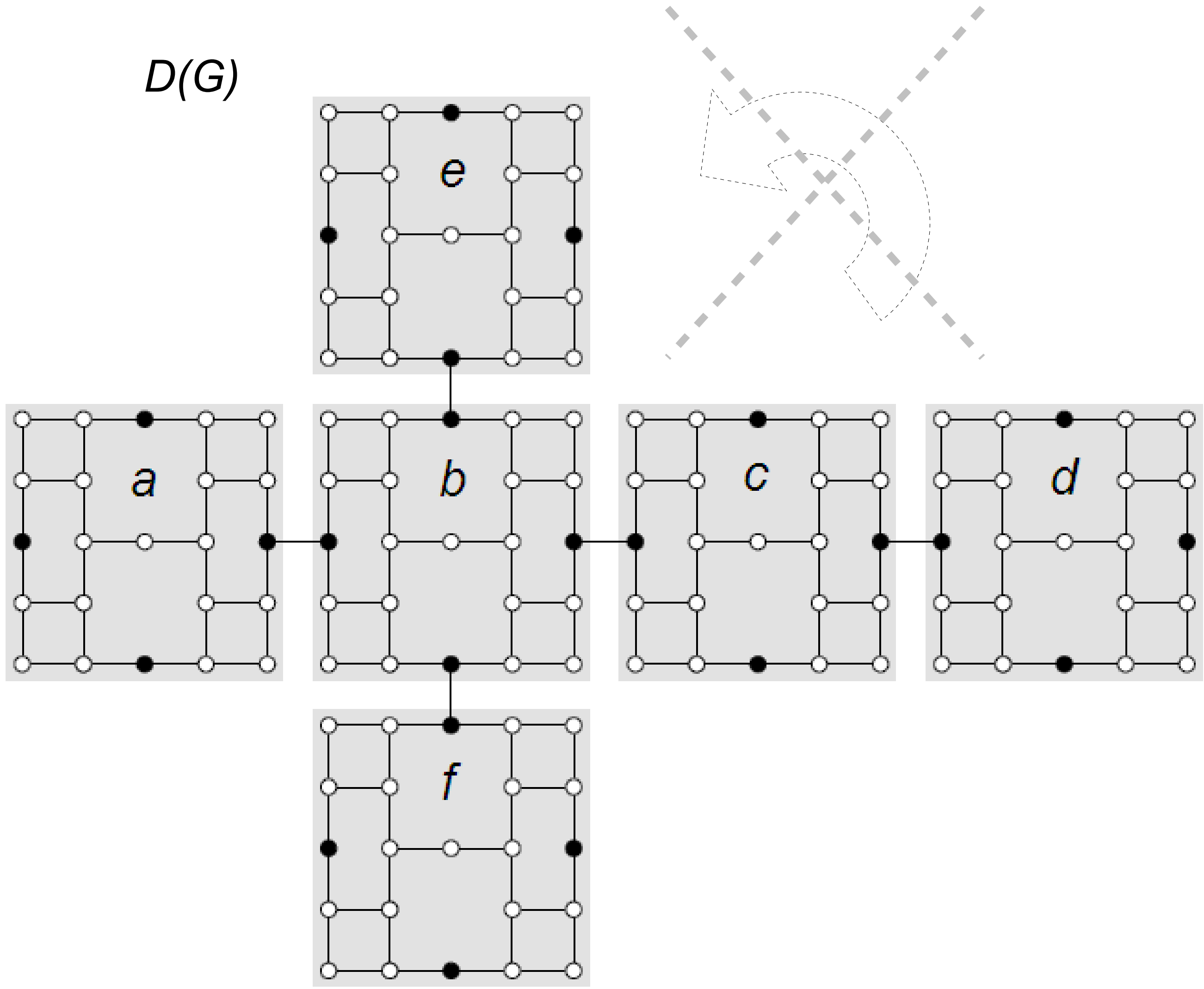
D(S(Φ)) is a partial grid

In an extended skeleton, we know beforehand the relative orientation of all its edges!

The double ladder

(*gadget* for {2,3}-graphs)





The extended skeleton *S(Φ)* is a partial grid

⬍

D(S(Φ)) is a partial grid

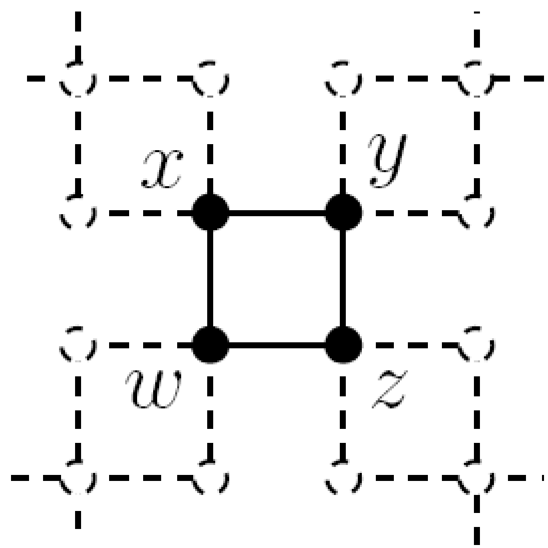
D	D-graphs	D-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff G is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}				
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}				
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff |*V*| is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}				
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

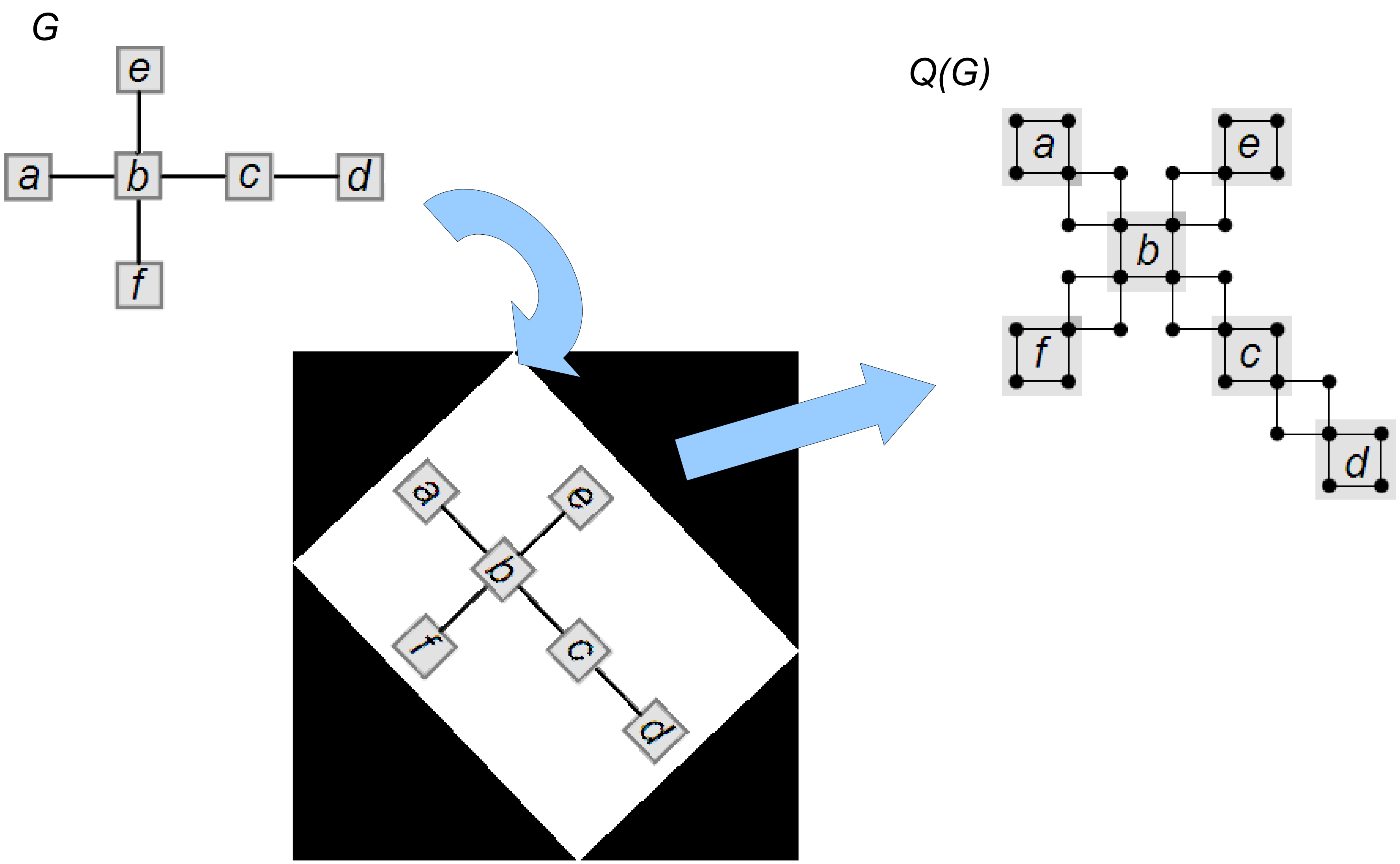
The square

(*gadget* for {2,4}-graphs)



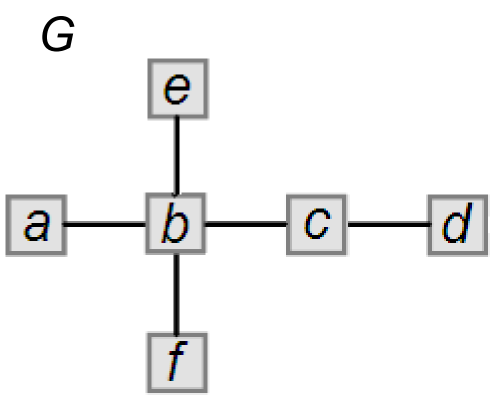
The square

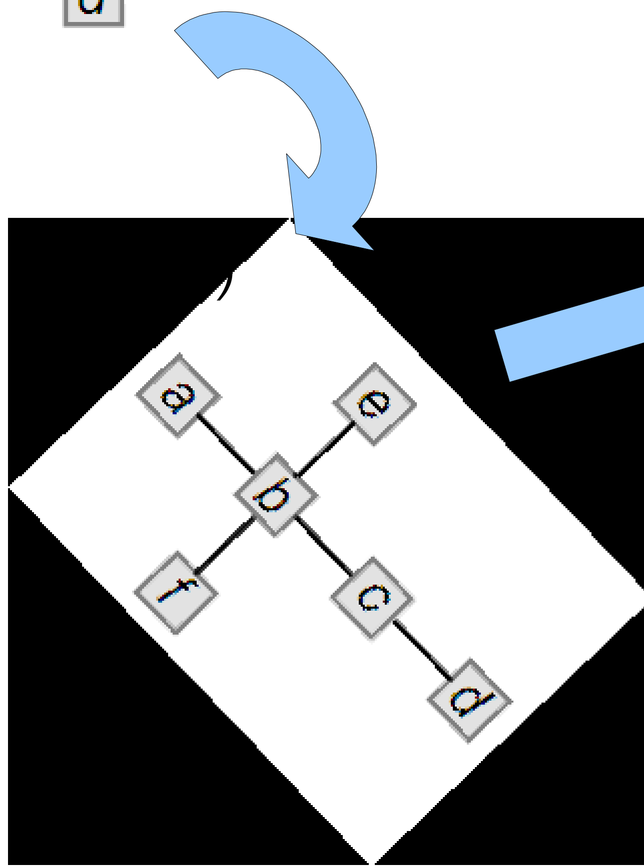
(*gadget* for {2,4}-graphs)

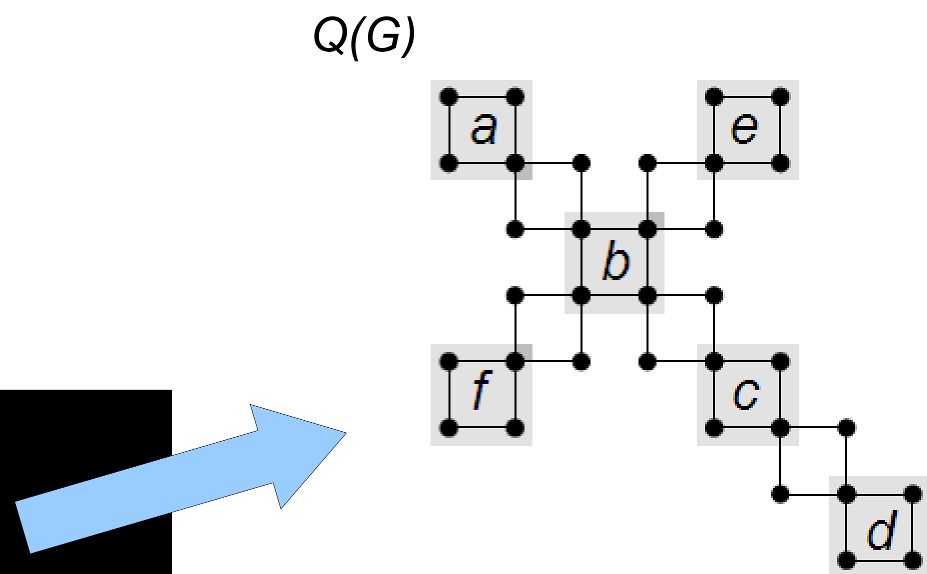


The square

(*gadget* for {2,4}-graphs)







The extended skeleton *S(Φ)* is a partial grid

⬍

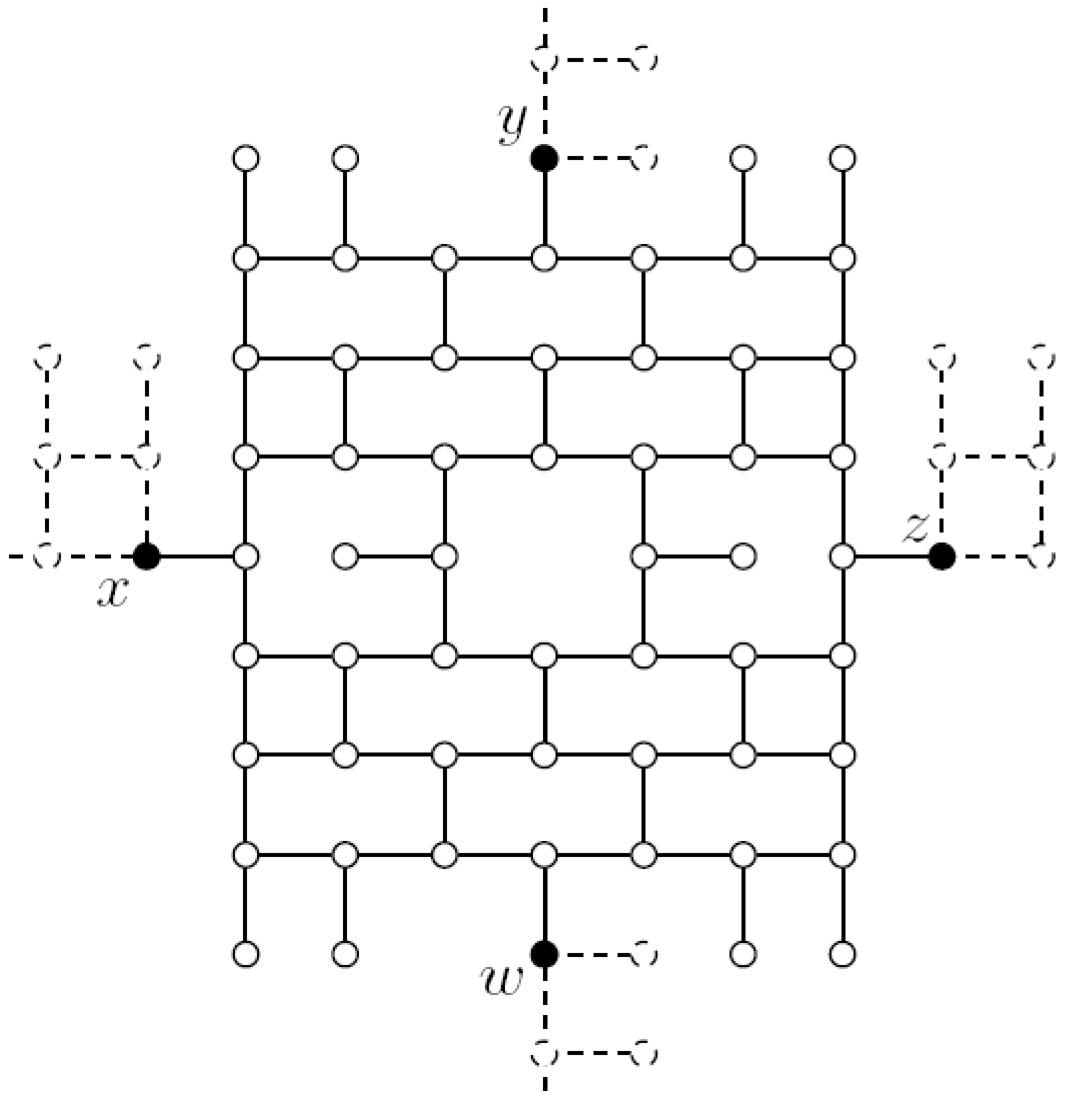
Q(S(Φ)) is a partial grid

D	D-graphs	D-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff G is path or even cycle
{1,3}				
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff |*V*| is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}			new result	
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

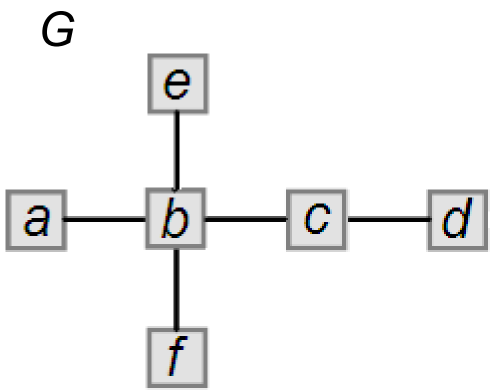
The brick wall

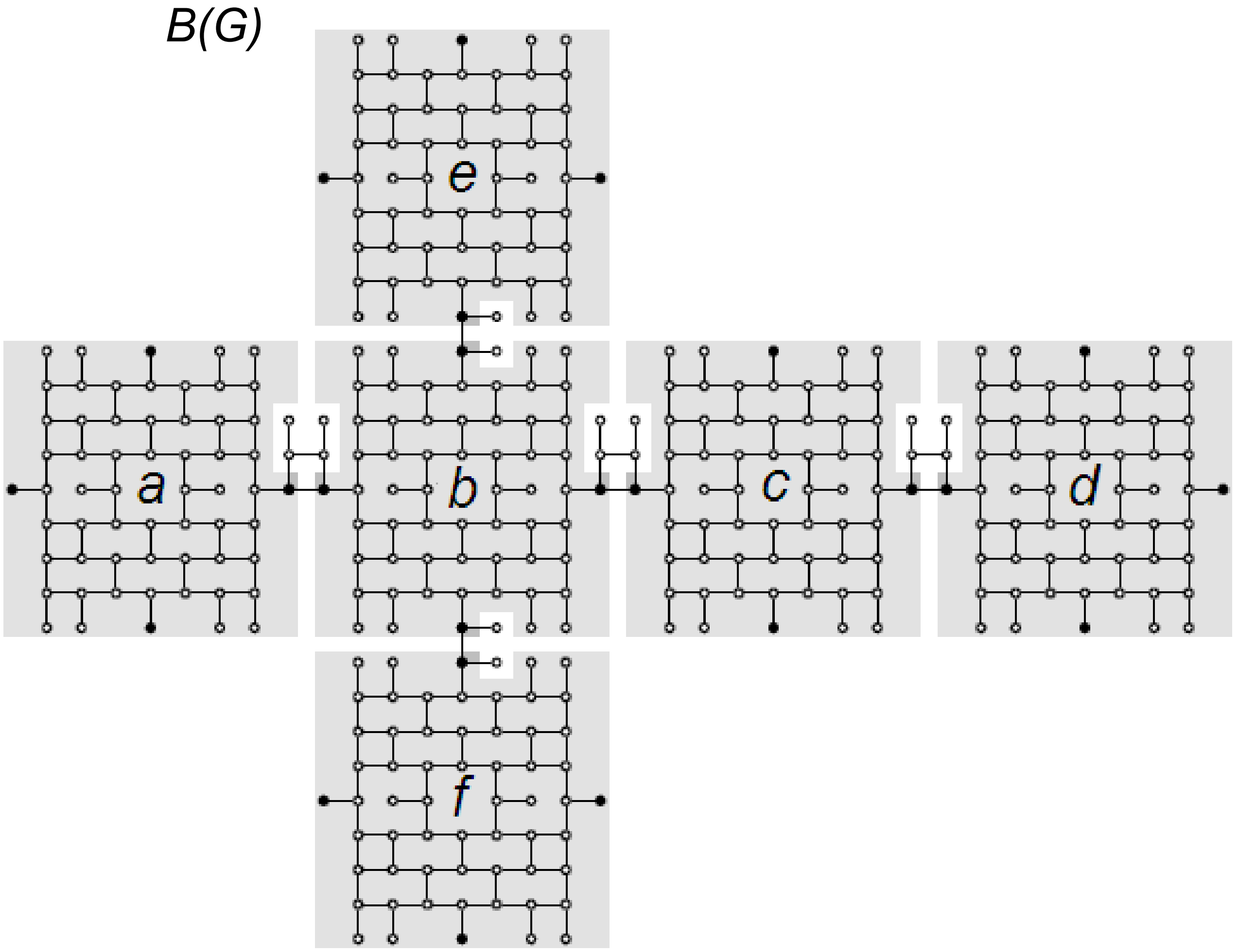
(*gadget* for {1,3}-graphs)



The brick wall

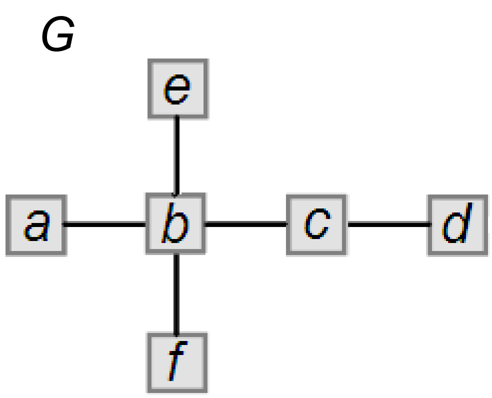
(*gadget* for {1,3}-graphs)

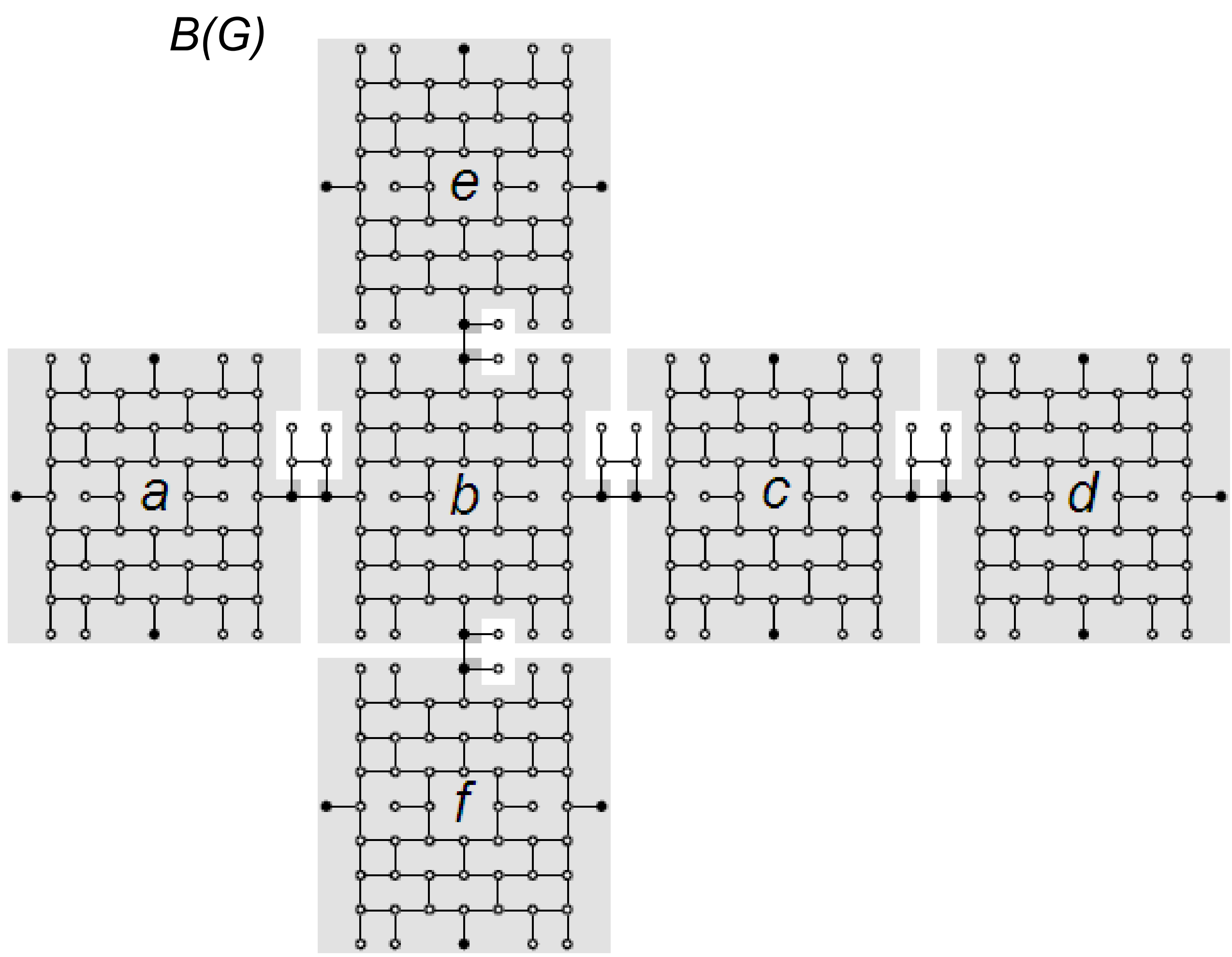




The brick wall

(*gadget* for {1,3}-graphs)





The extended skeleton *S(Φ)* is a partial grid

⬍

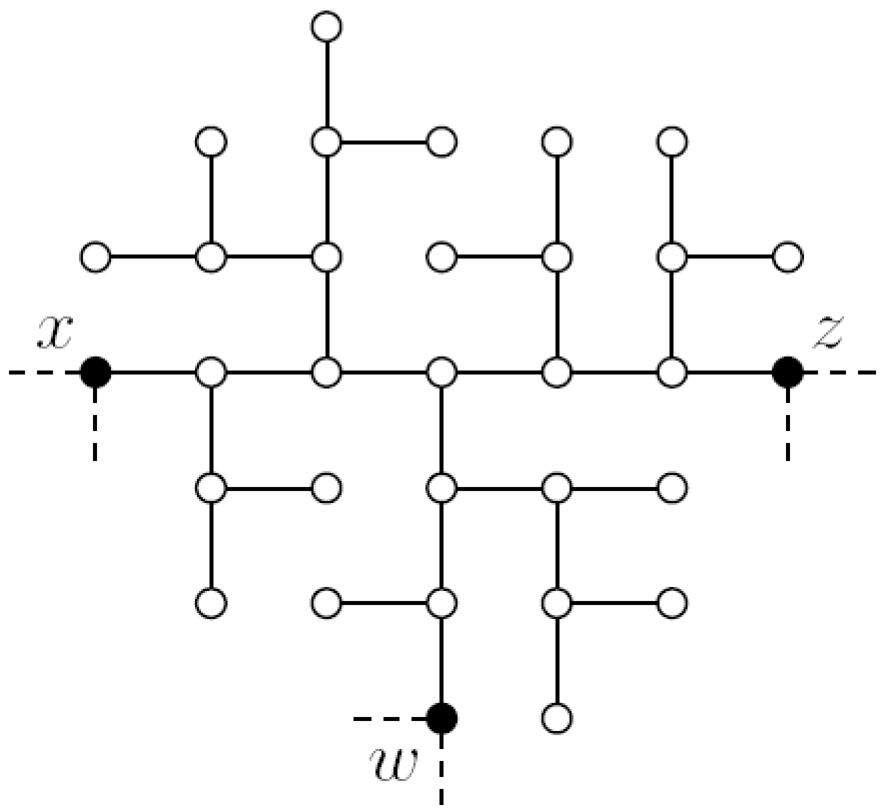
B(S(Φ)) is a partial grid

D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}	**NP-C**		new result	BHATT-COSMADAKIS reduction
{1,4}	**P**	**P**	new result	YES iff $G\setminus\{v \in G\,\|\,\deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

D	D-graphs	D-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff G is path or even cycle
{1,3}	**NP-C**	**????**	new result	BHATT-COSMADAKIS reduction
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

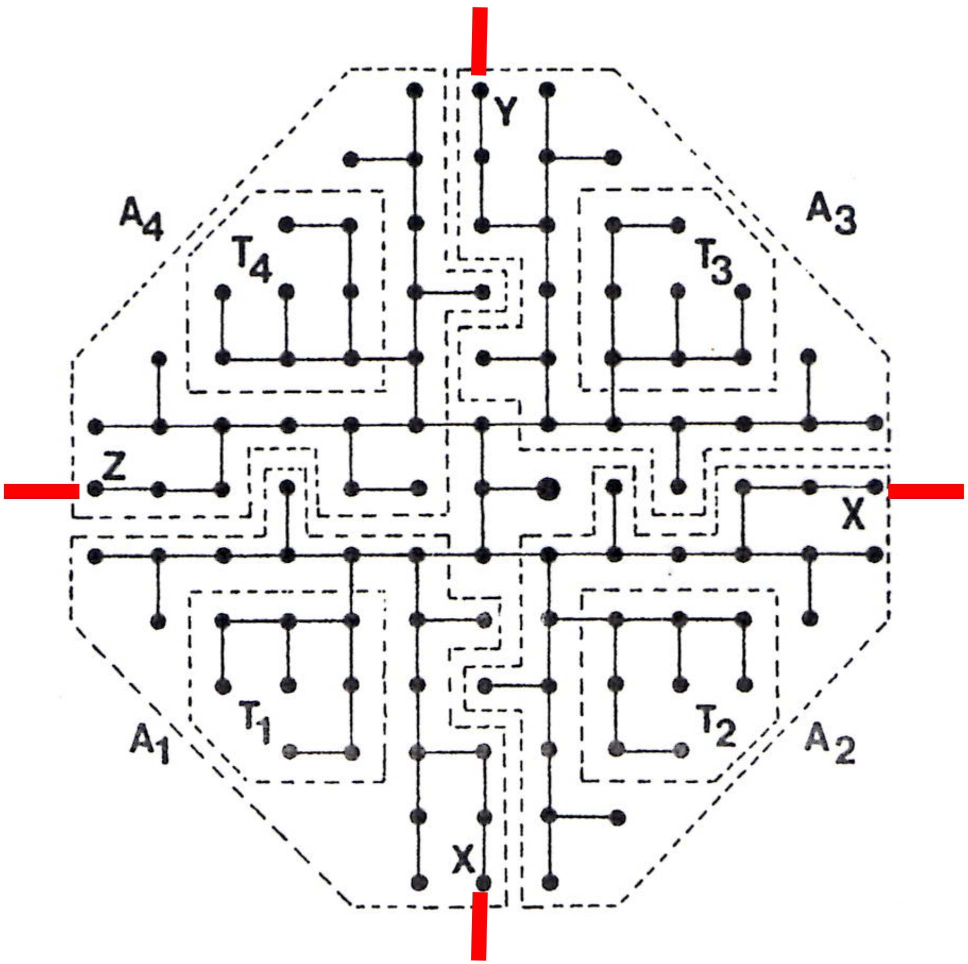
The unnamed gadget

(*gadget* for {1,3}-trees)

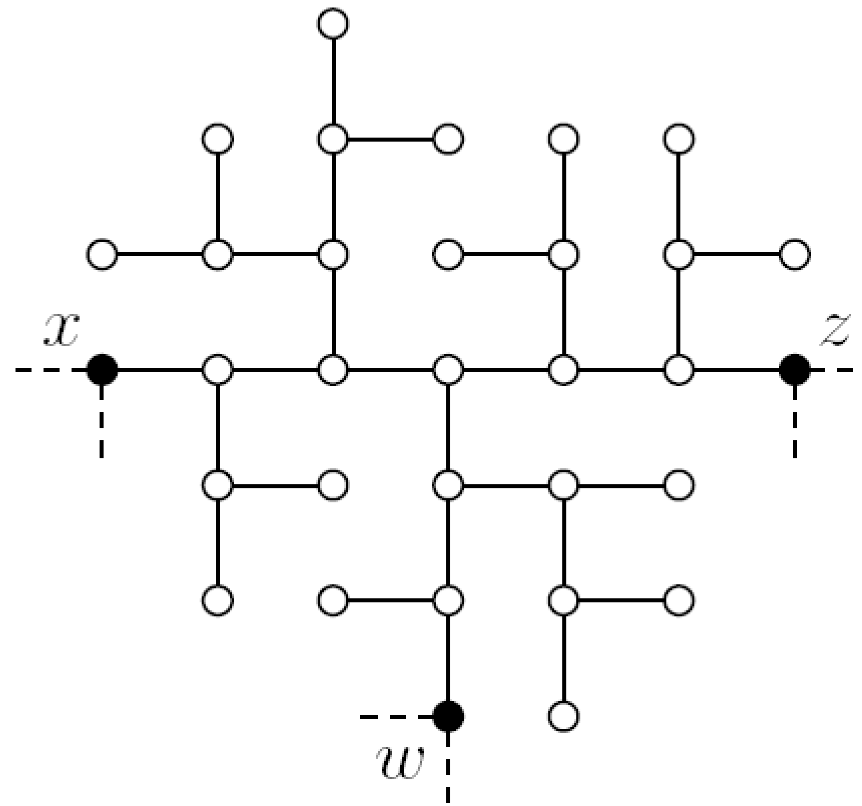


The unnamed gadget

(*gadget* for {1,3}-trees)



+

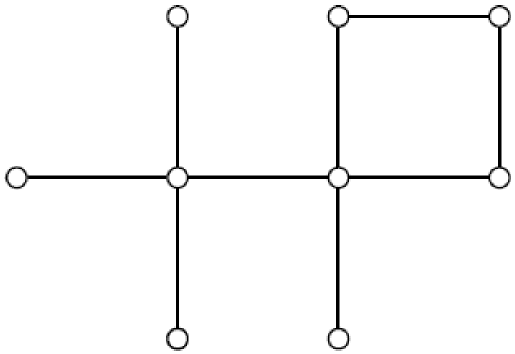


D	D-graphs	D-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff G is path or even cycle
{1,3}	**NP-C**	**NP-C**	new result	BHATT-COSMADAKIS reduction
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

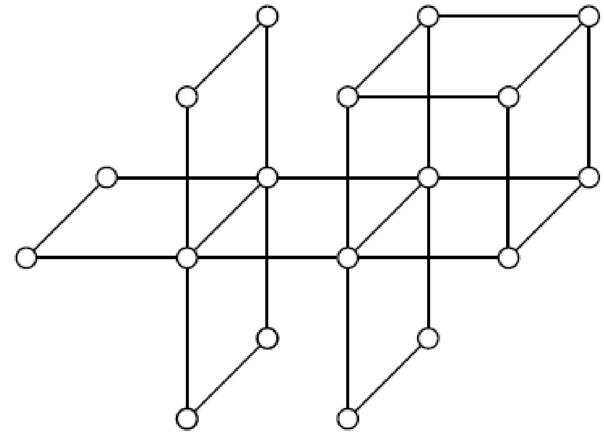
D	*D*-graphs	*D*-trees	reference	info
{1}	**P**	**P**	trivial	always YES
{2}	**P**	**—**	trivial	YES iff $\|V\|$ is even
{3}	**P**	**—**	trivial	always NO
{4}	**P**	**—**	trivial	always NO
{1,2}	**P**	**P**	trivial	YES iff *G* is path or even cycle
{1,3}	**NP-C**	**NP-C**	new result	BHATT-COSMADAKIS reduction
{1,4}	**P**	**P**	new result	YES iff $G \setminus \{v \in G \mid \deg(v)=1\}$ is a grid
{2,3}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{2,4}	**NP-C**	**—**	new result	BHATT-COSMADAKIS reduction
{3,4}	**P**	**—**	trivial	always NO
{1,2,3}	**NP-C**	**NP-C**	IPL '89	[Gregori]
{1,2,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]
{1,3,4}	**NP-C**	**NP-C**	new result	{1,2,3,4} reduction
{2,3,4}	**NP-C**	**—**	new result	generalizes {2,3}
{1,2,3,4}	**NP-C**	**NP-C**	IPL '87	[Bhatt and Cosmadakis]

3-dimensional grids

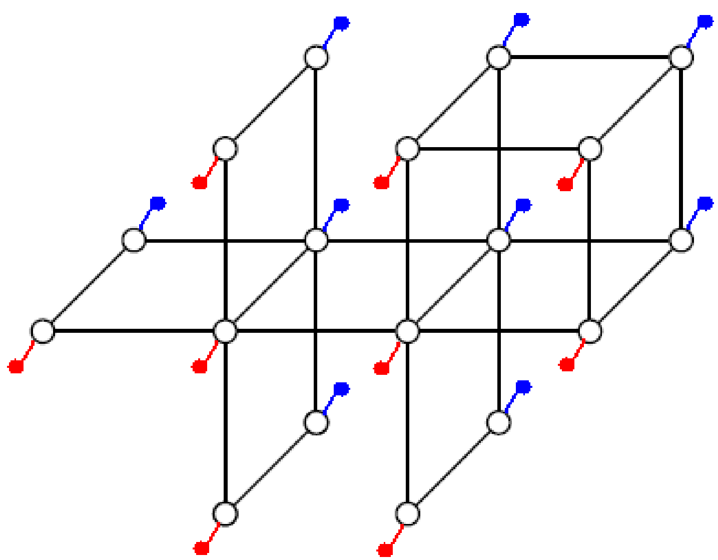
G



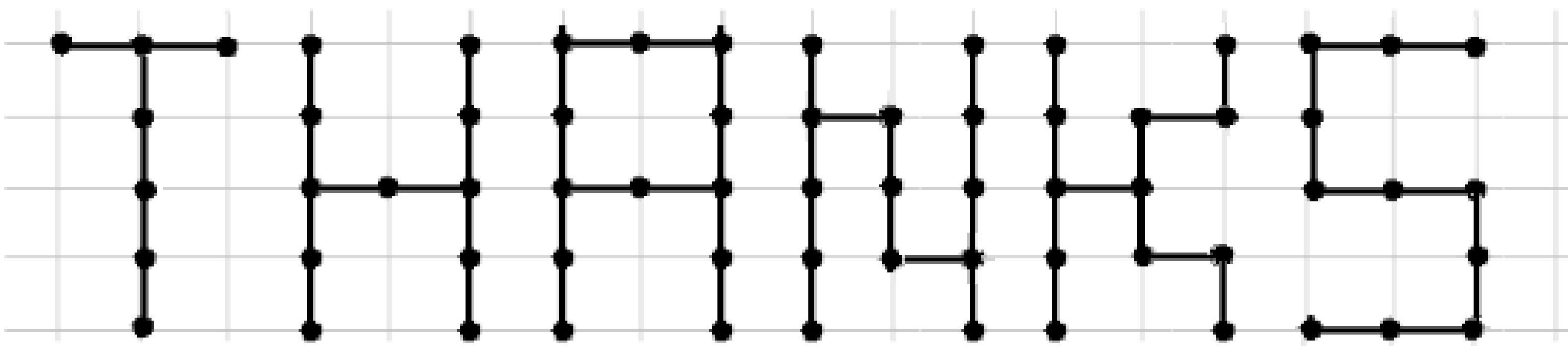
Prism(*G*)



Prism with thorns



Settled the complexity of
50 out of 63
possible input degree sets



Thank you!