Polynomial time algorithm for the Radon number of grids in the geodetic convexity

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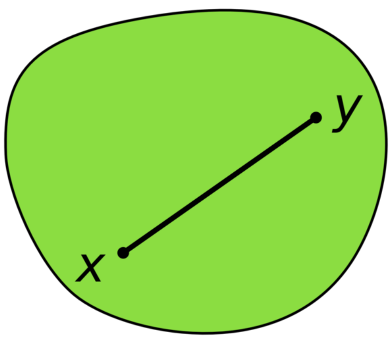
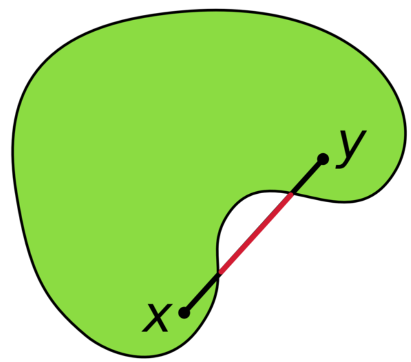
Convexity in Euclidean space

Ground set of the convexity space: the *d*-dimensional space R^d
Interval (x,y) = straight line segment between x and y

Convex set



Non-convex set



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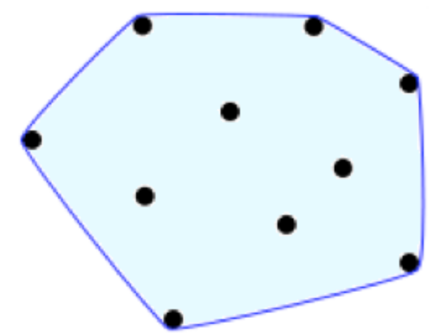

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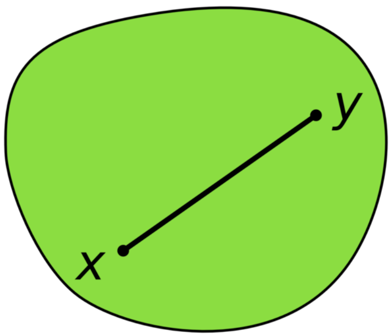

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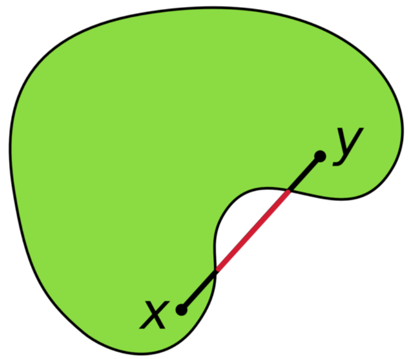

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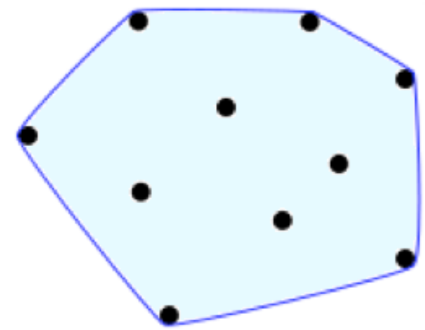

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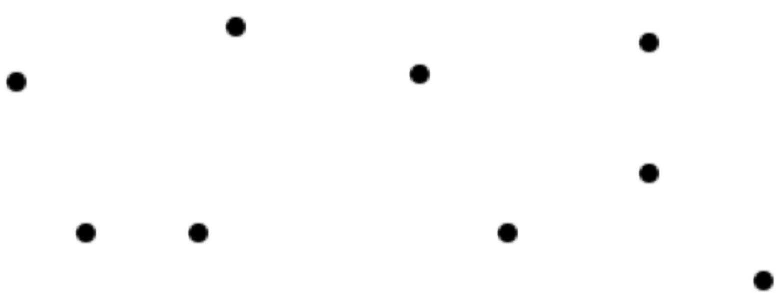

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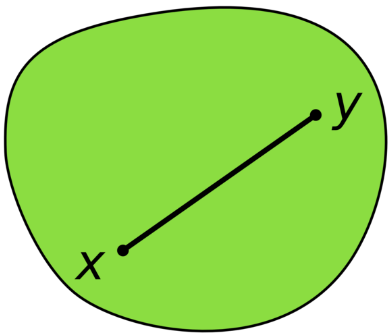

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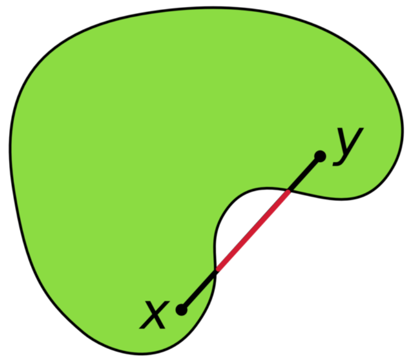

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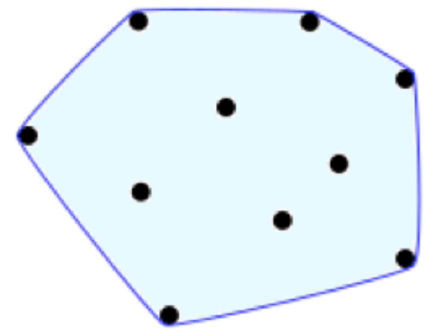

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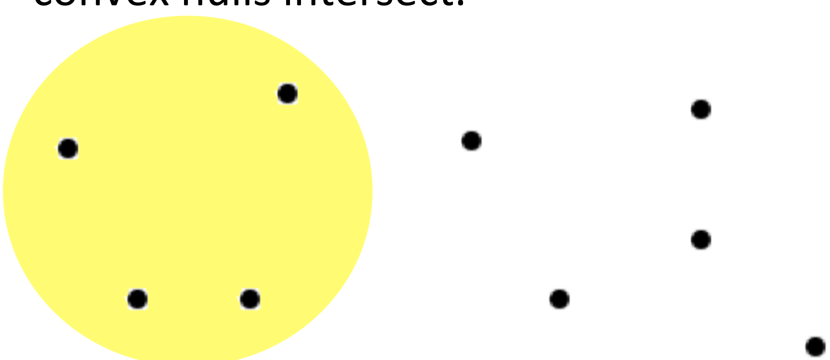

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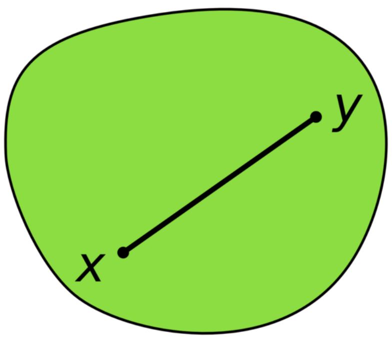

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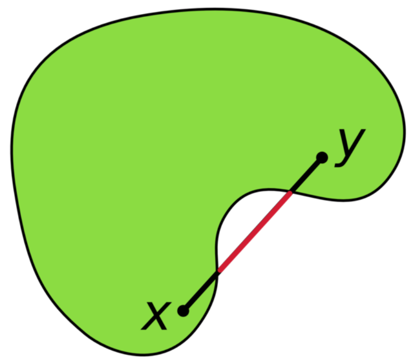

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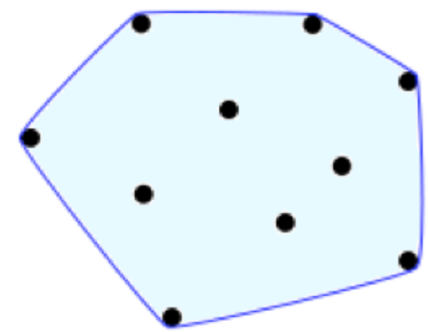

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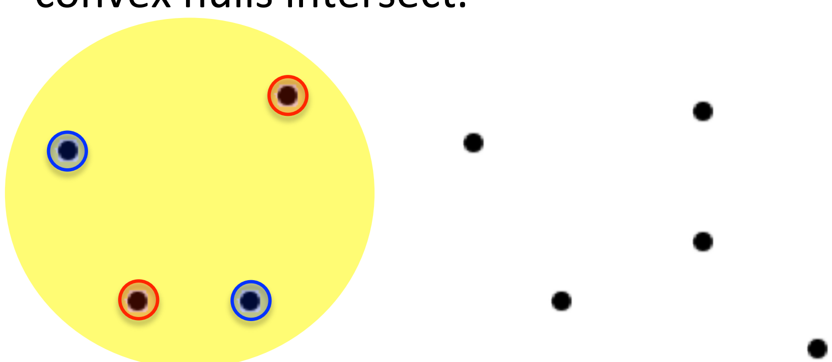

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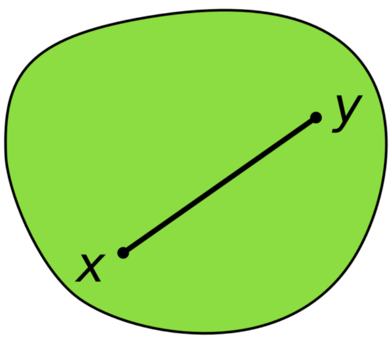

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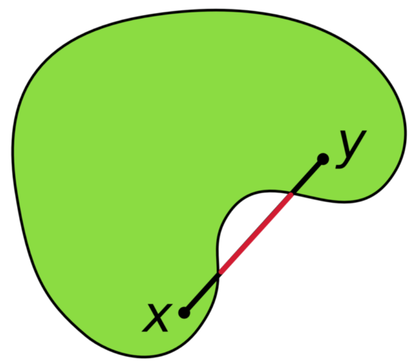

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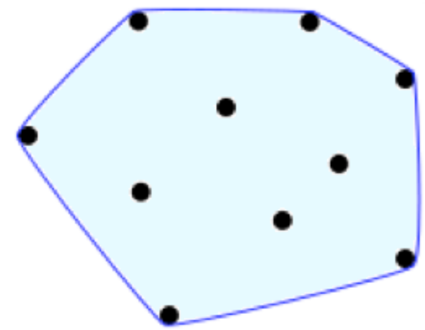

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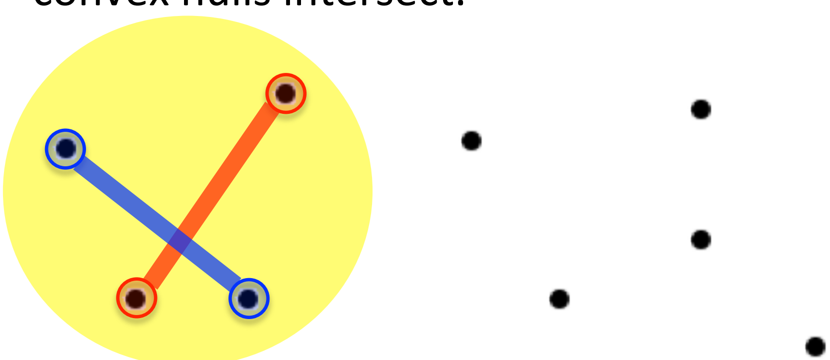

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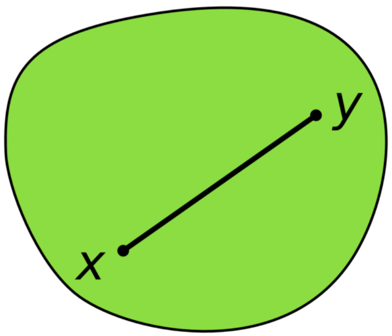

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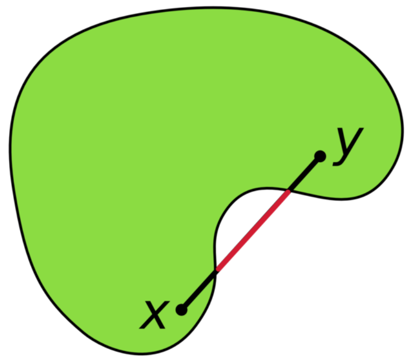

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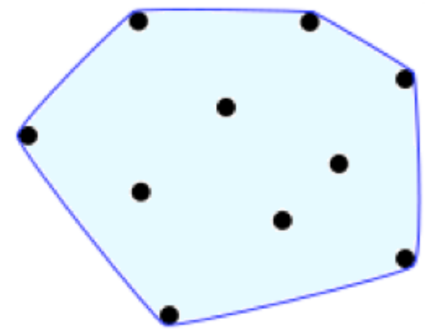

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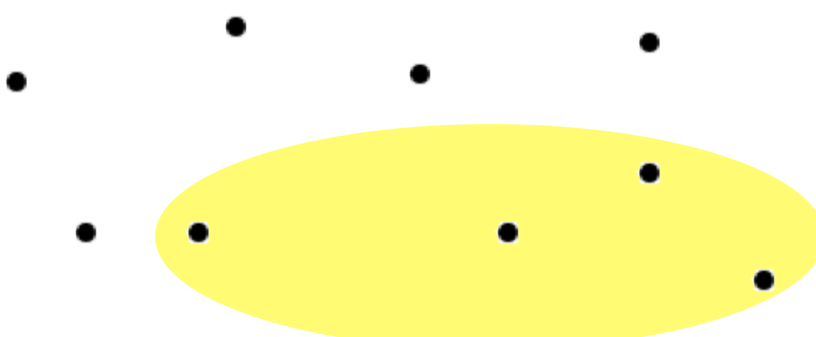

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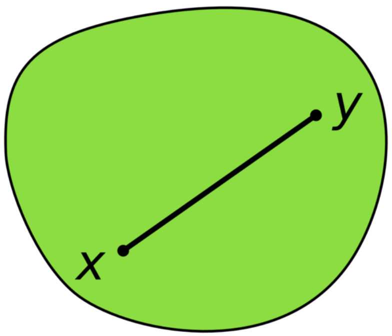

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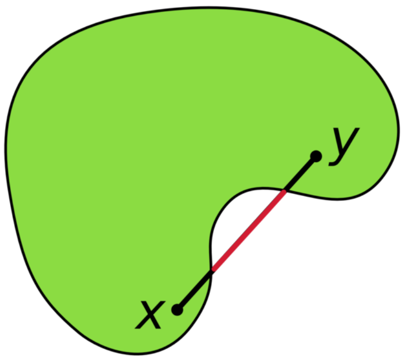

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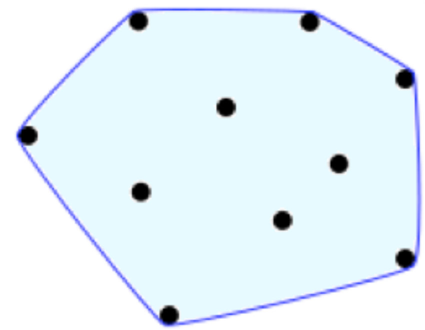

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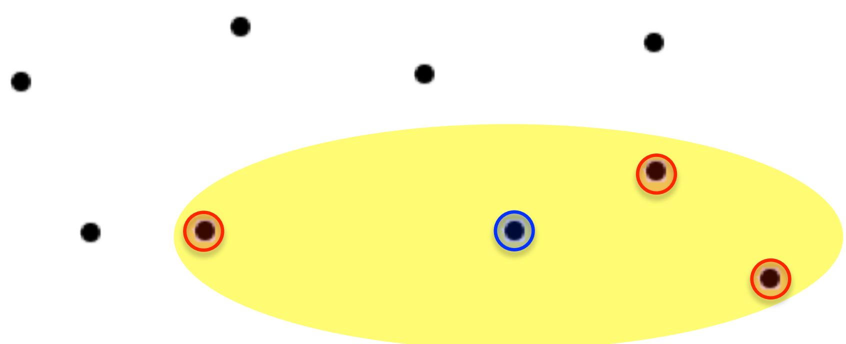

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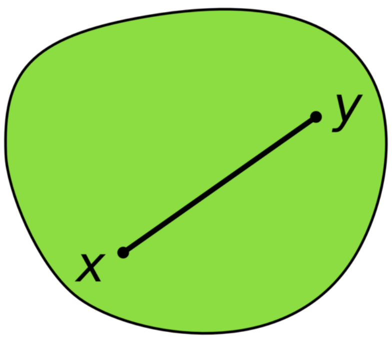

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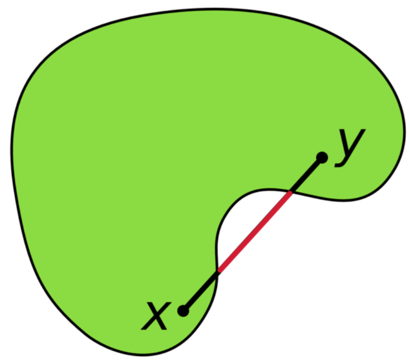

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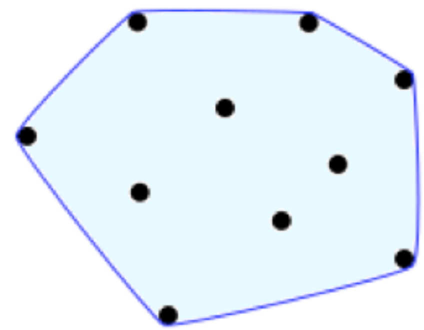

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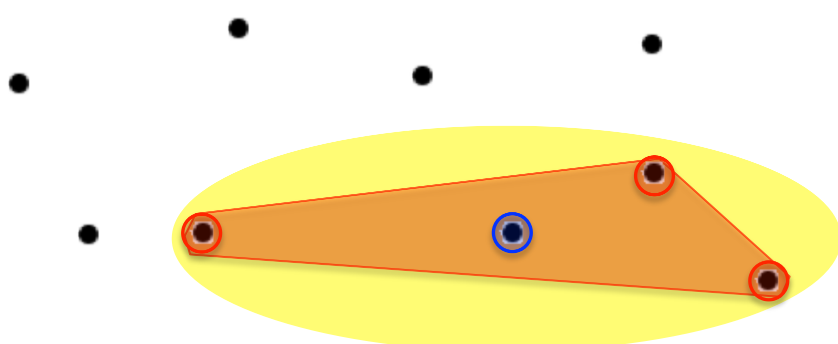

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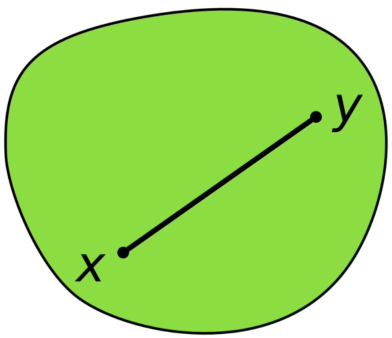

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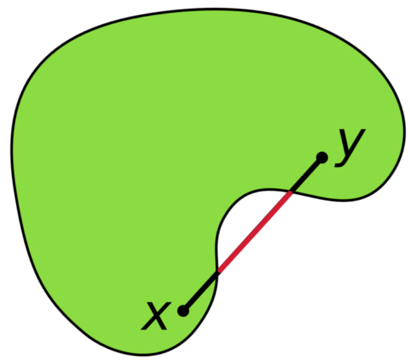

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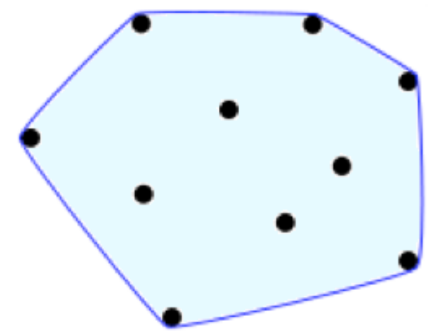

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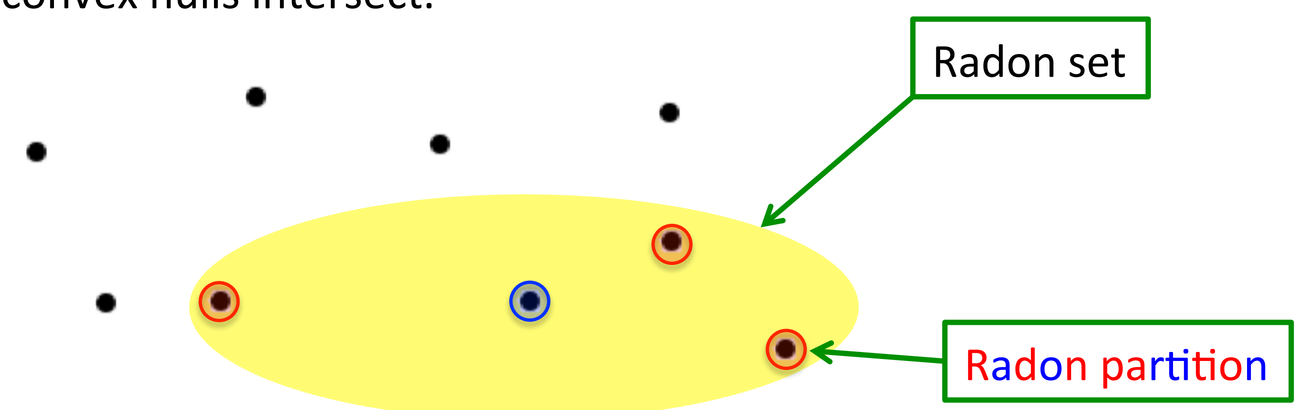

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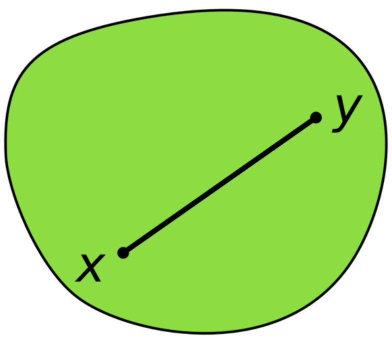

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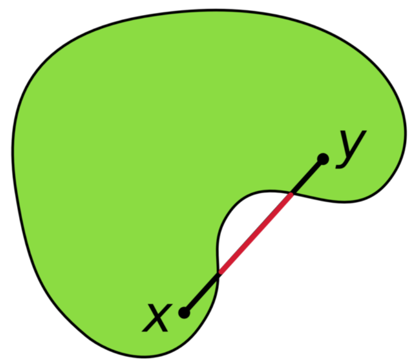

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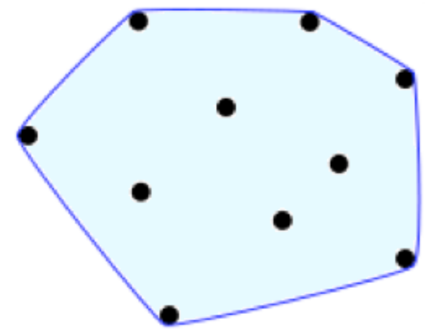

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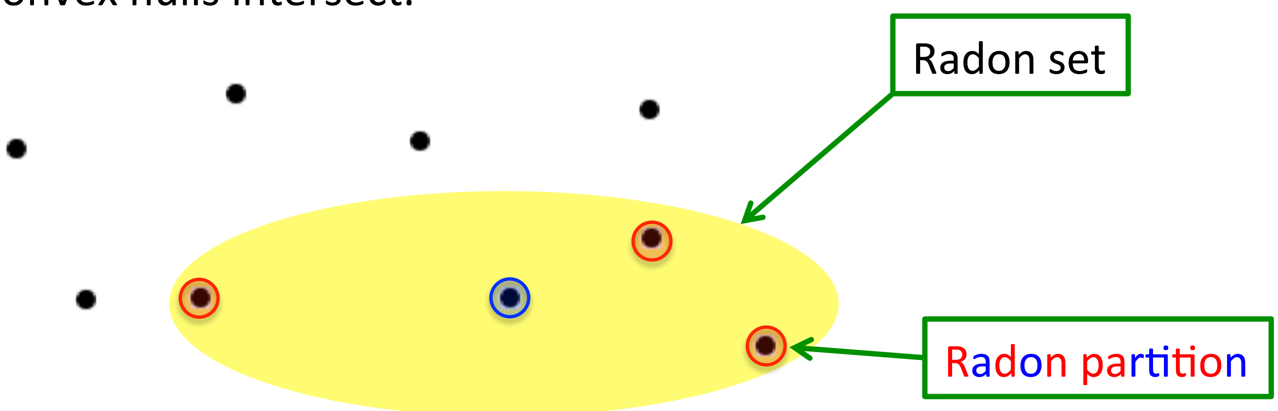
<u>Radon number</u>: the smallest value r such that every set with at least r vertices is a Radon set



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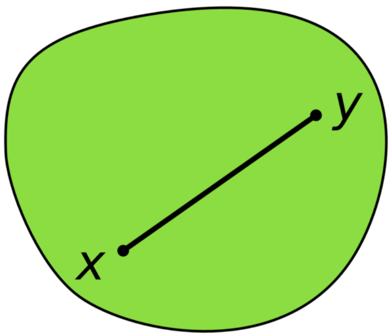

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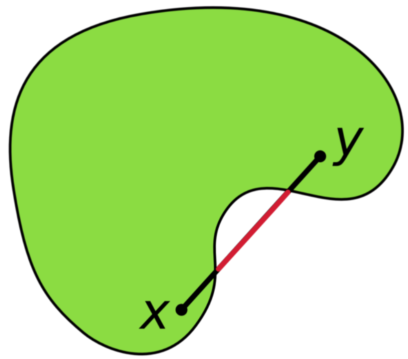

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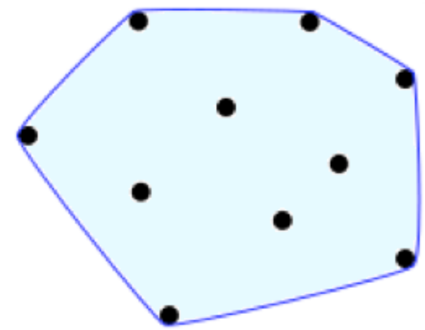

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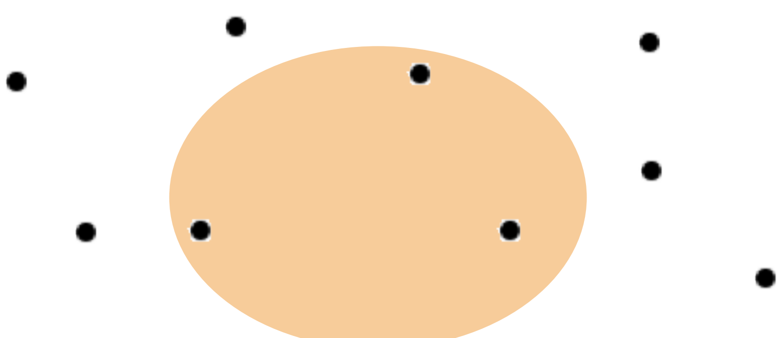

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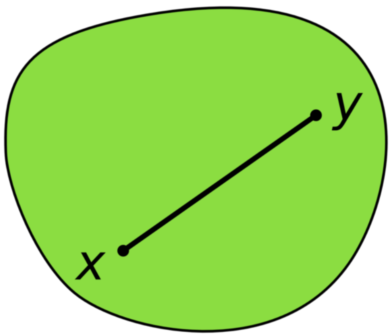

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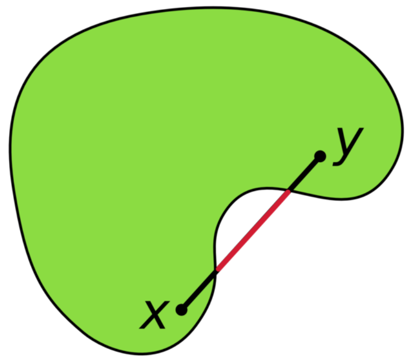

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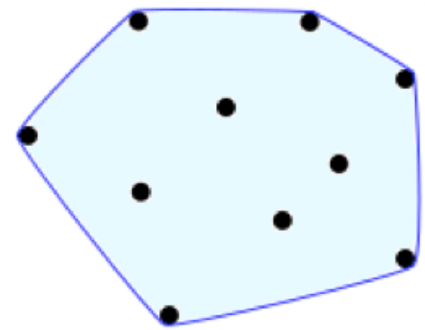

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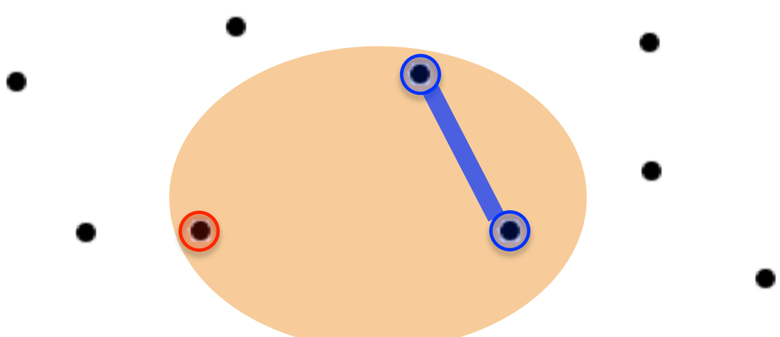

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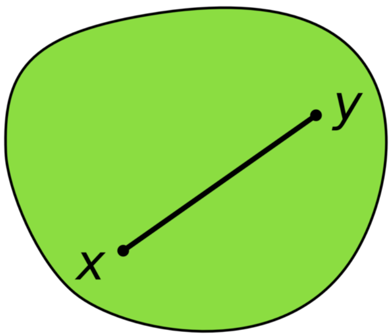

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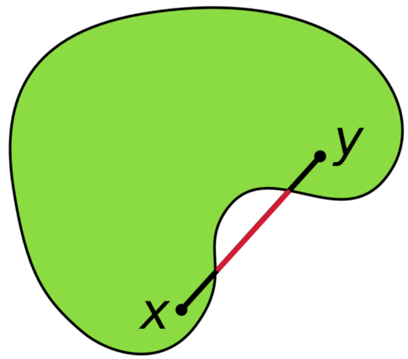

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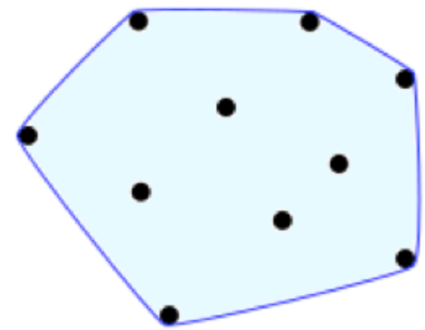

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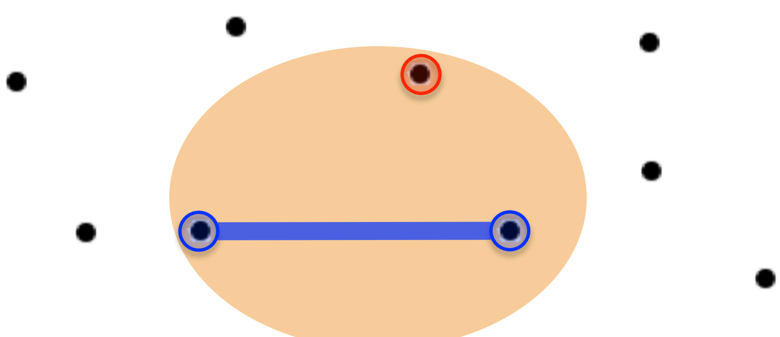

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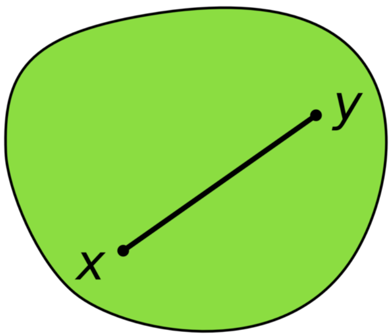

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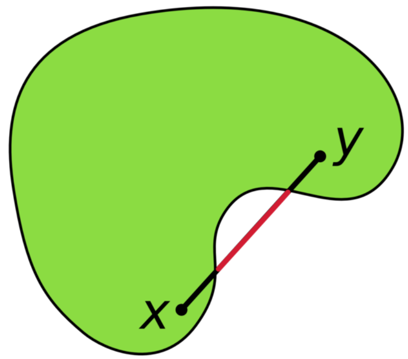

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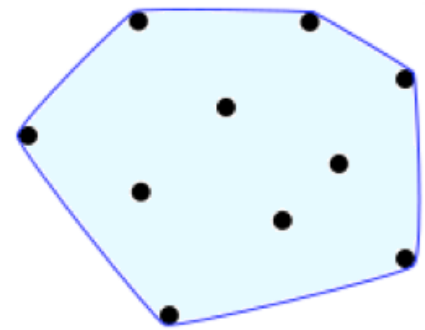

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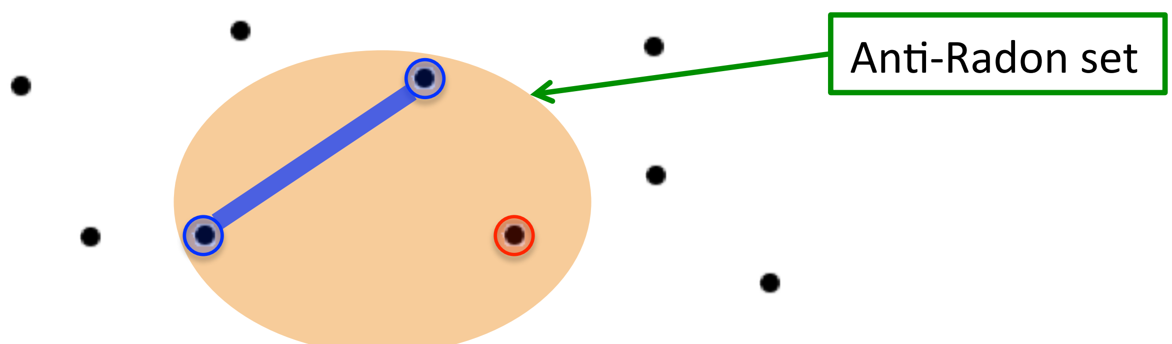

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Geodetic convexity in graphs

Ground set of the convexity space: vertices *V* of some connected graph *G* (*V*, *E*)
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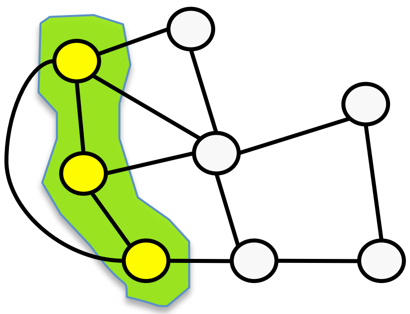
Convex subset of *V*



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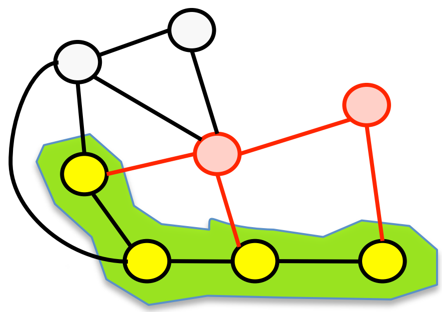

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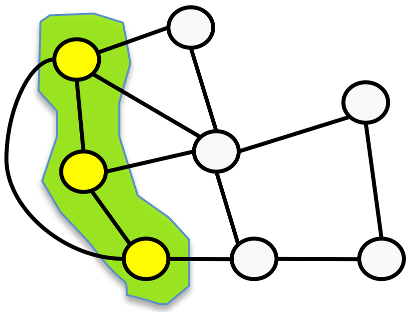

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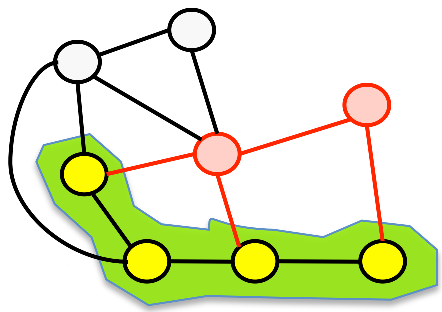

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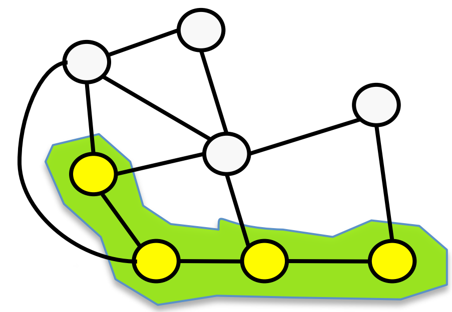

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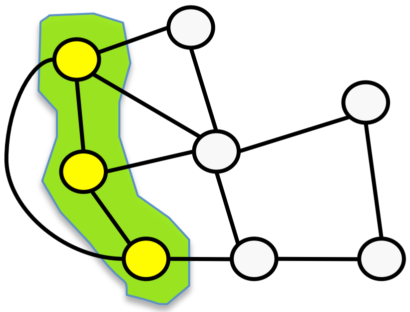
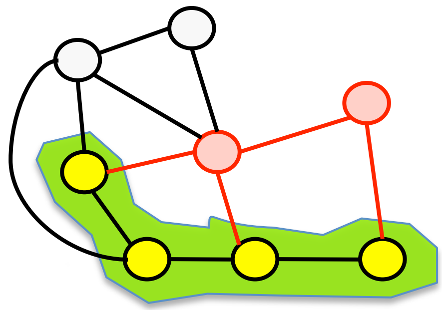
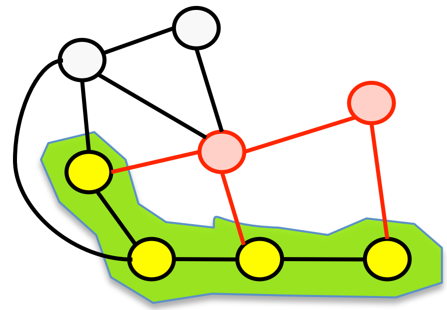


Non-convex subset of *V*



Convex hull



Geodetic convexity in graphs

Ground set of the convexity space: vertices *V* of some connected graph *G* (*V*, *E*)
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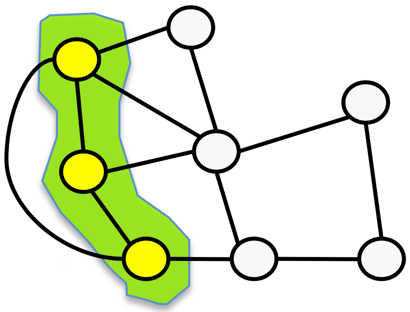

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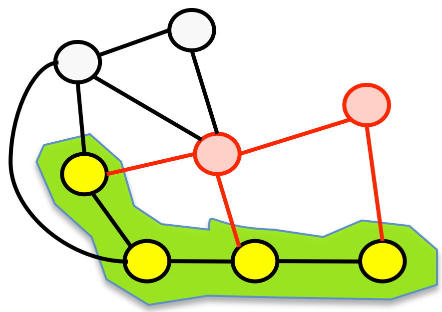

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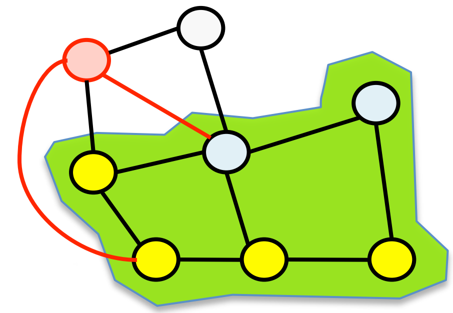

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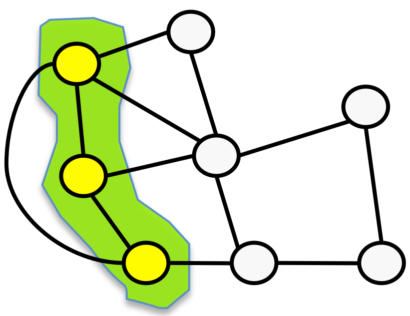

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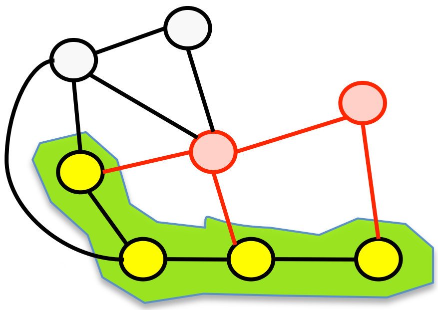

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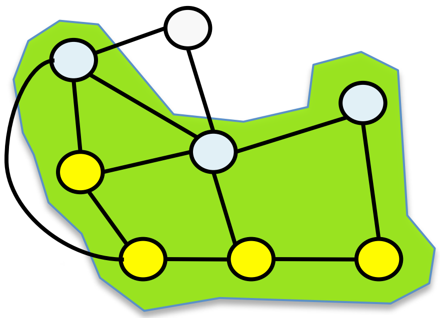

Geodetic Radon number

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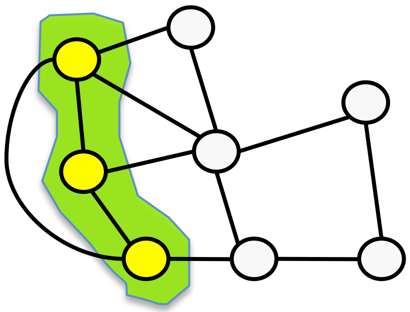

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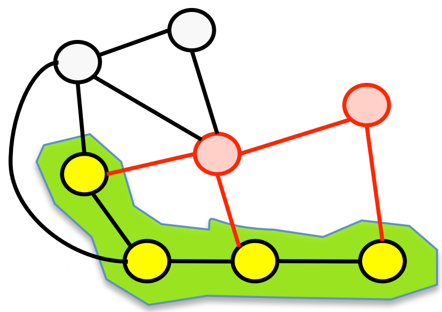

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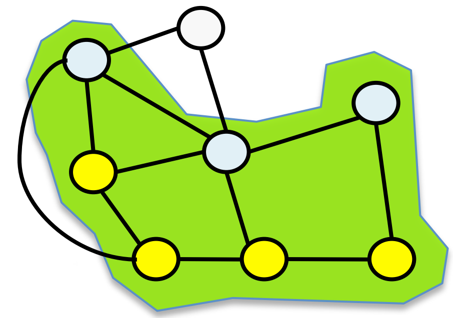

Every subset of *V* with at least __??__ vertices can be partitioned into two sets whose convex hulls have a non-empty intersection.

Radon number: the smallest value *r* that satisfies the statement

Geodetic Radon number

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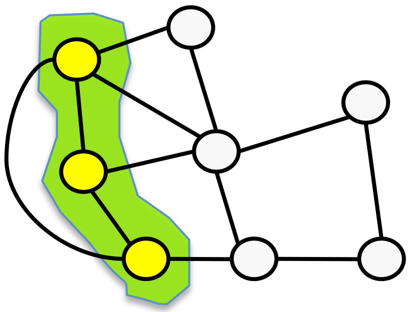
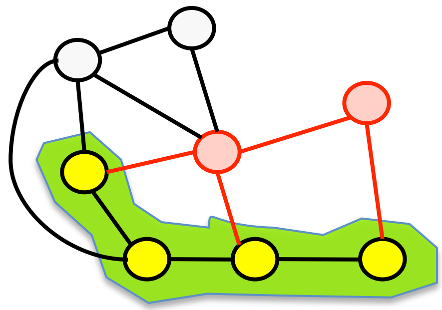
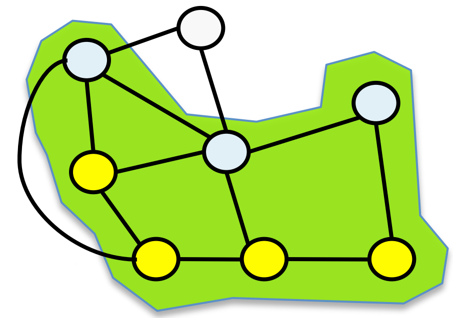


Non-convex subset of *V*



Convex hull



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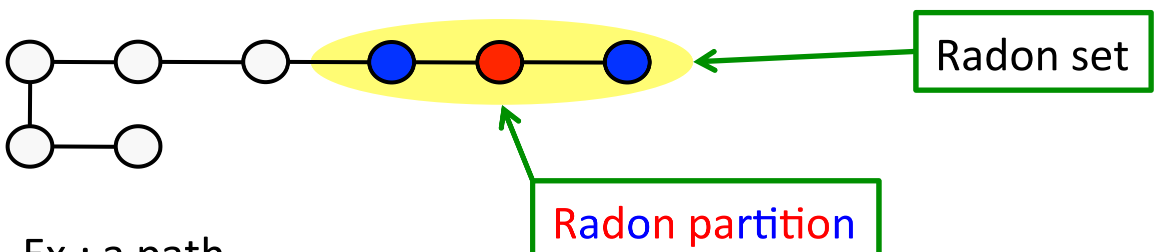

Ex.: a path

Geodetic Radon number

<u>RADON NUMBER</u>:

Input: Graph G (V,E)
Output: the smallest r such that every subset S of V, $|S| \geq r$, is a Radon set.

or equivalently…

<u>ANTI-RADON SET</u>:

Input: Graph G (V,E)
Output: an anti-Radon set of G
with maximum size

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Radon number

size of maximum anti-Radon set
+ 1

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NP-hard, even for bipartite graphs.

Radon number

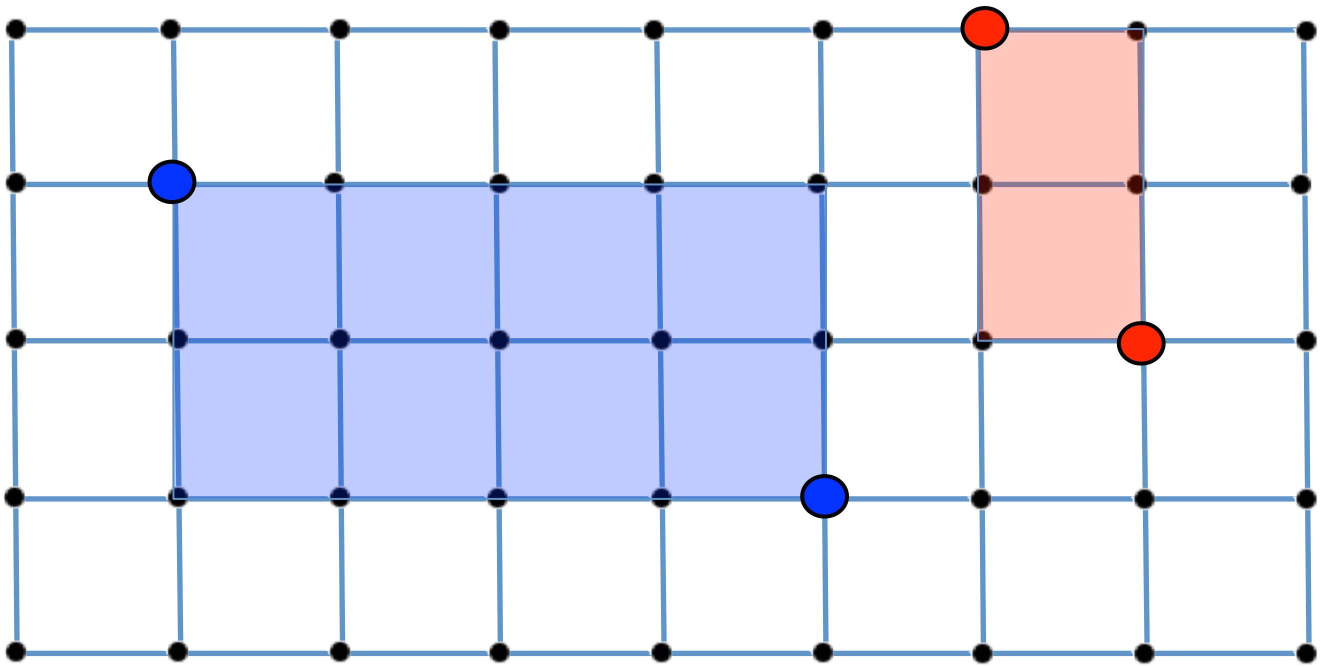
↕

size of maximum anti-Radon set + 1

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Ex.: Grid (9, 5)



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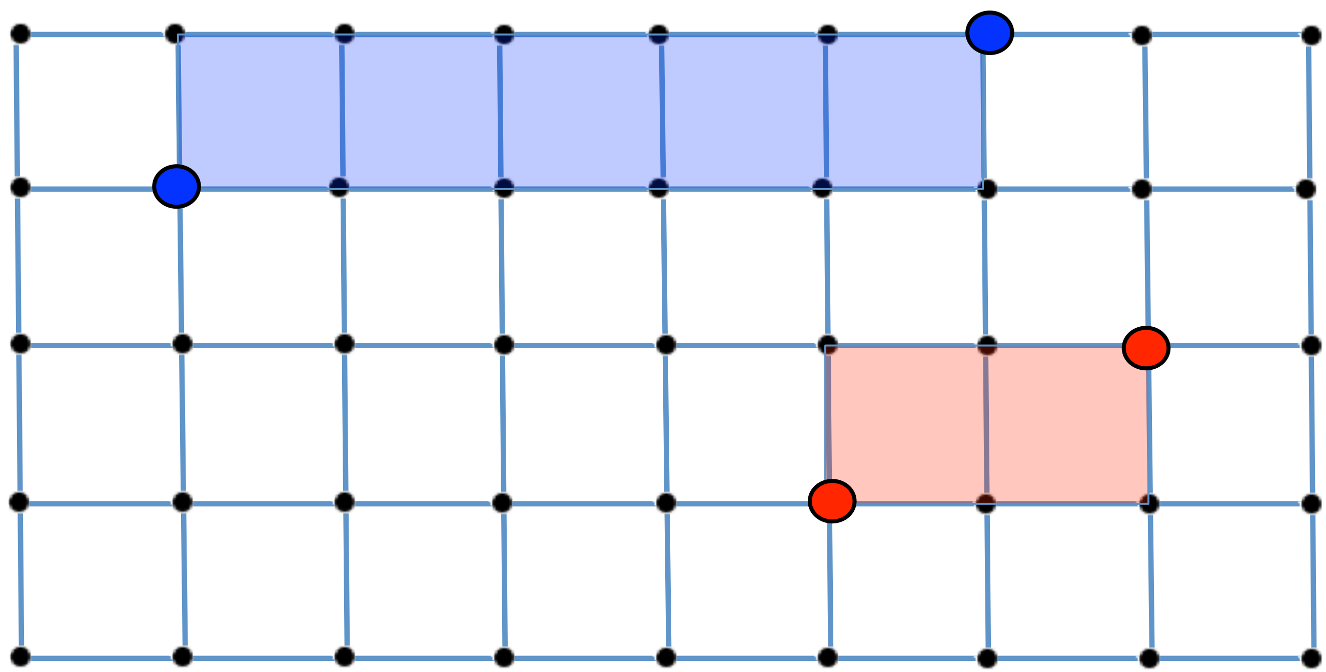

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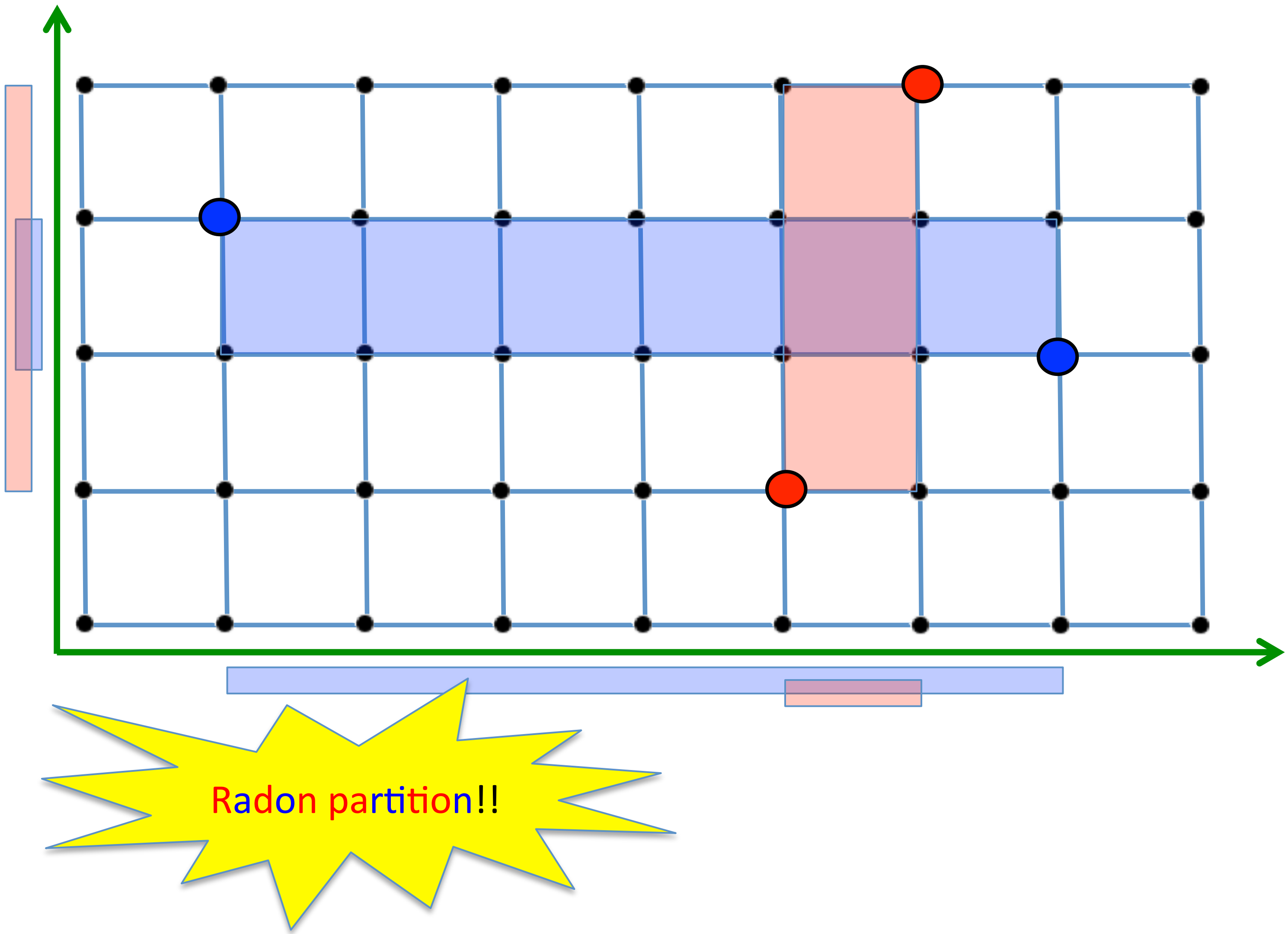

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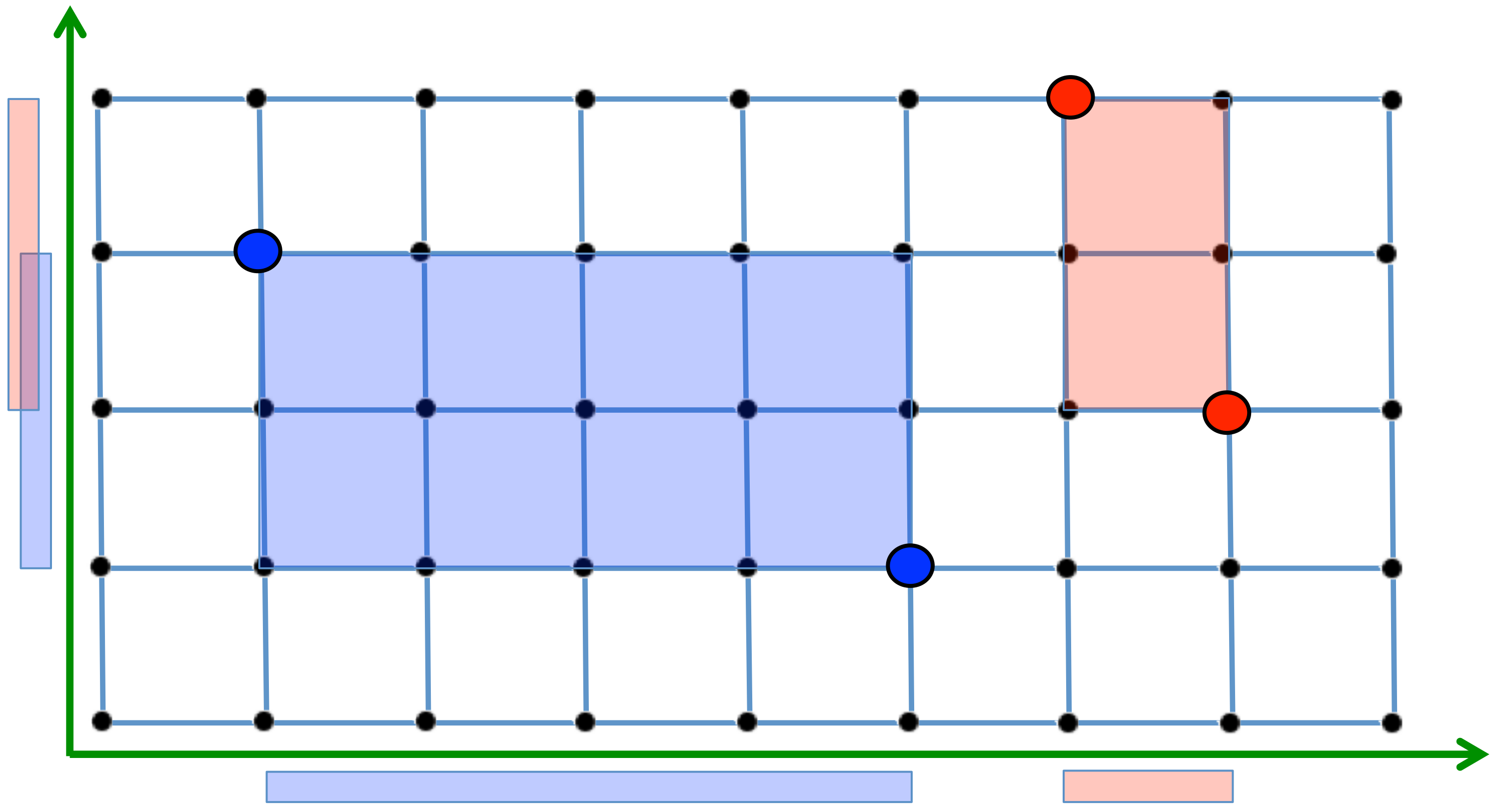

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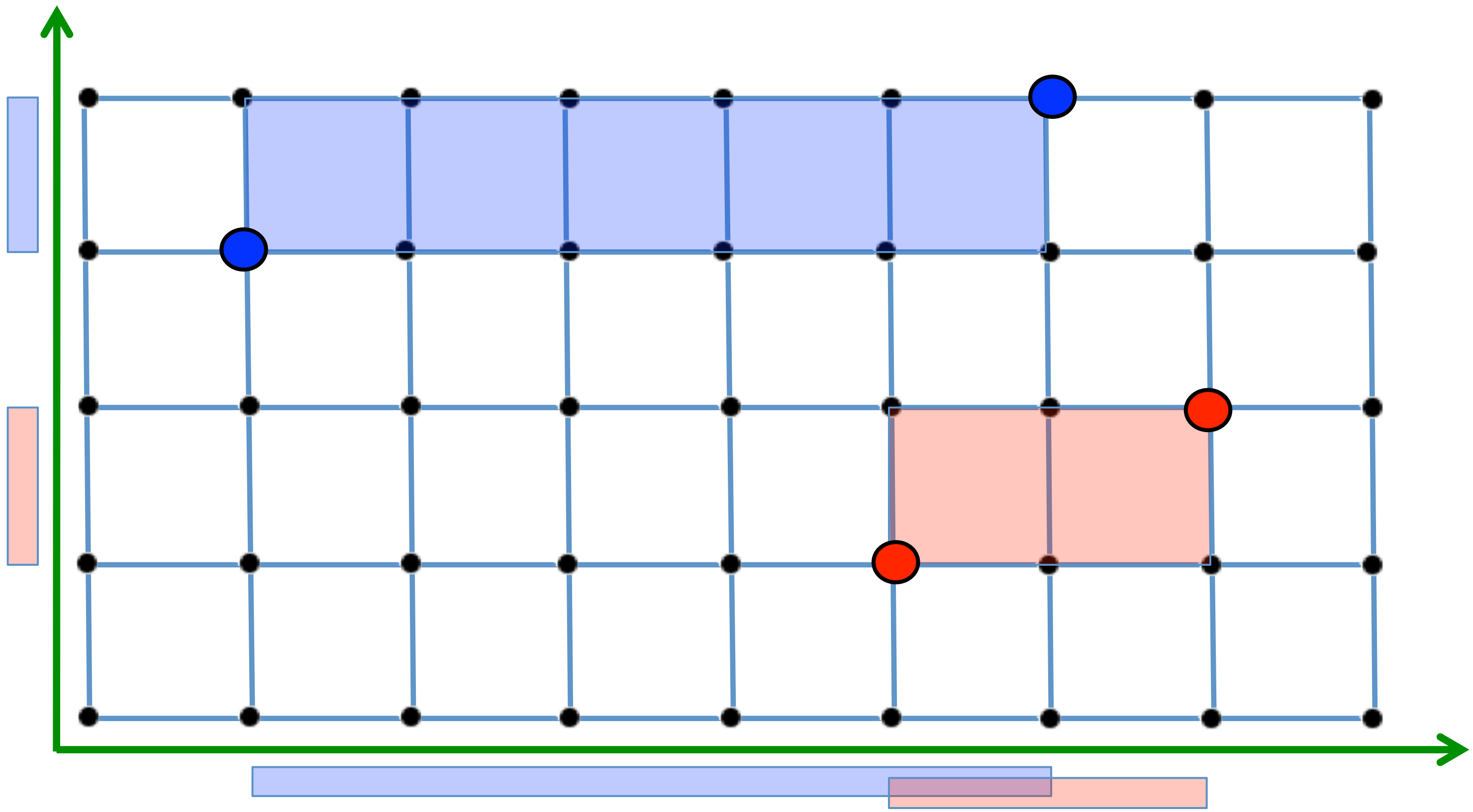

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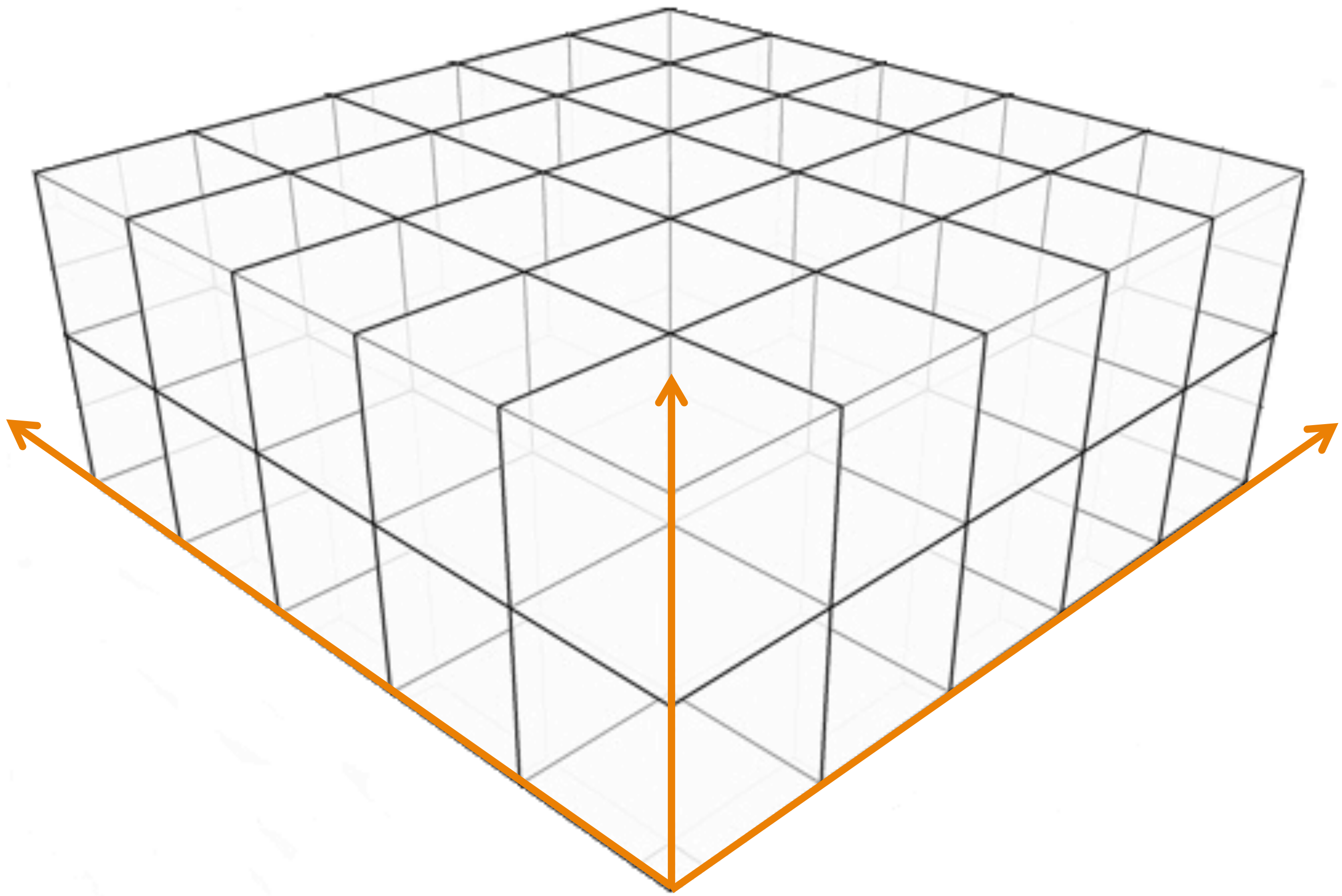

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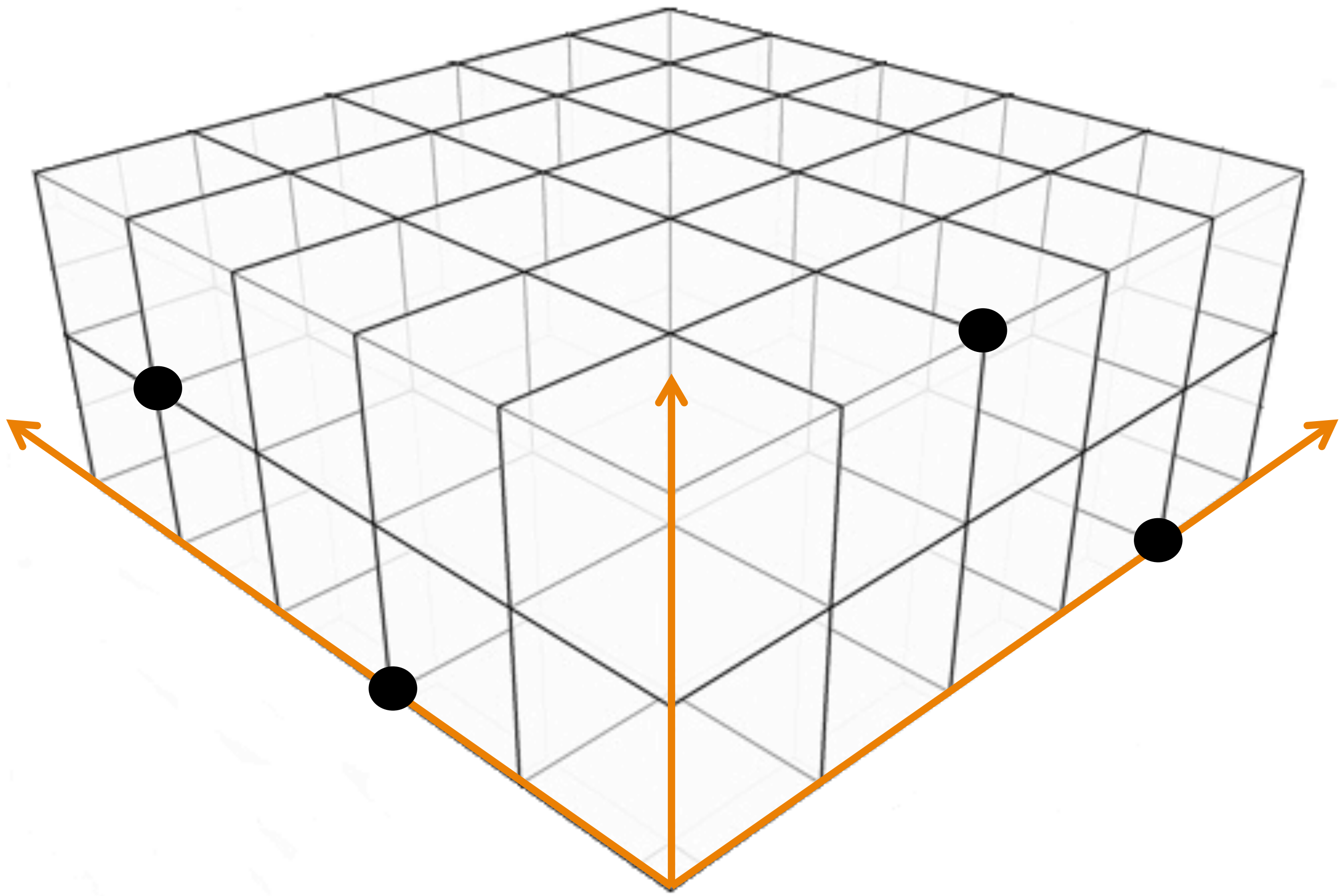

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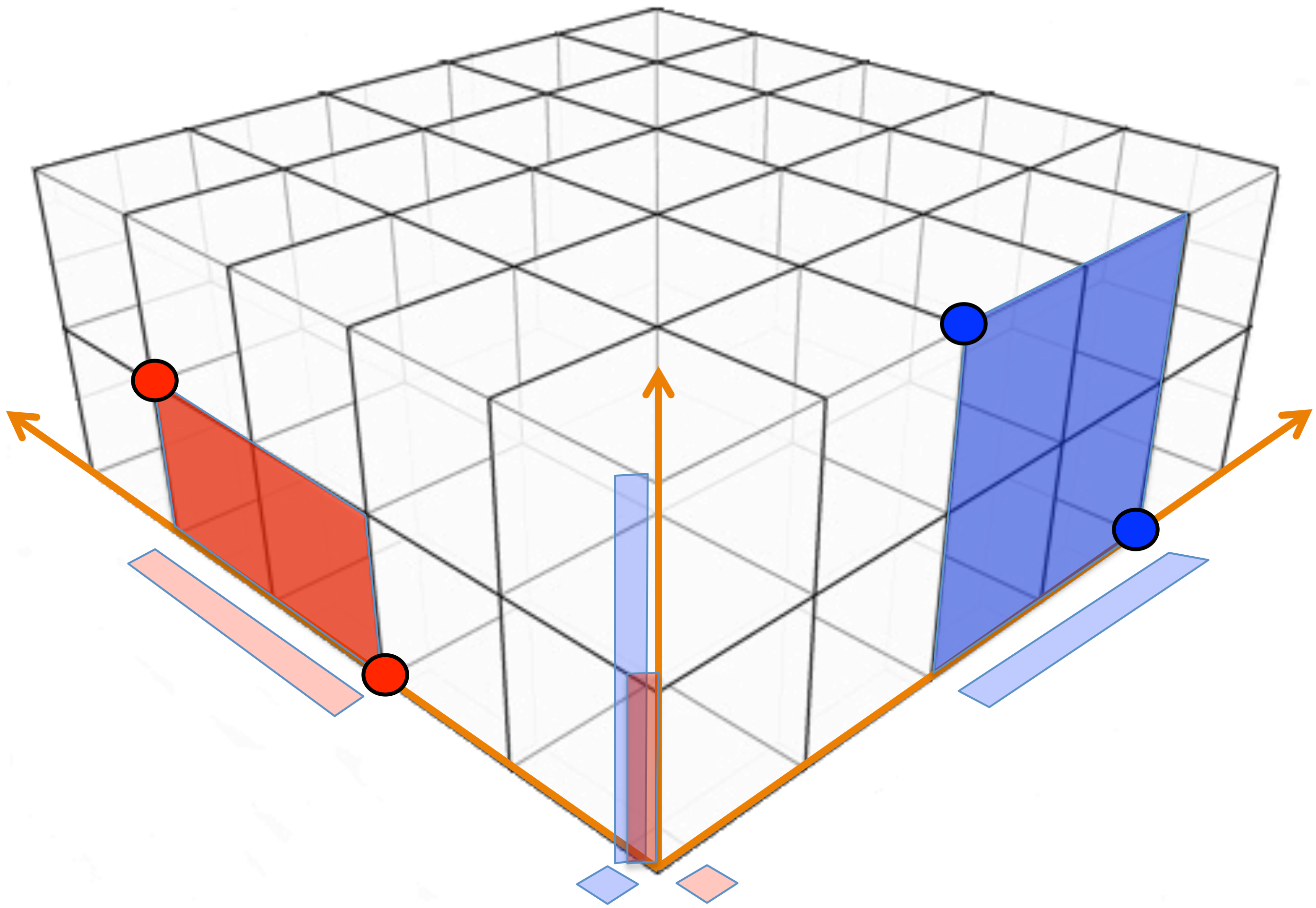

Geodetic Radon number of grids

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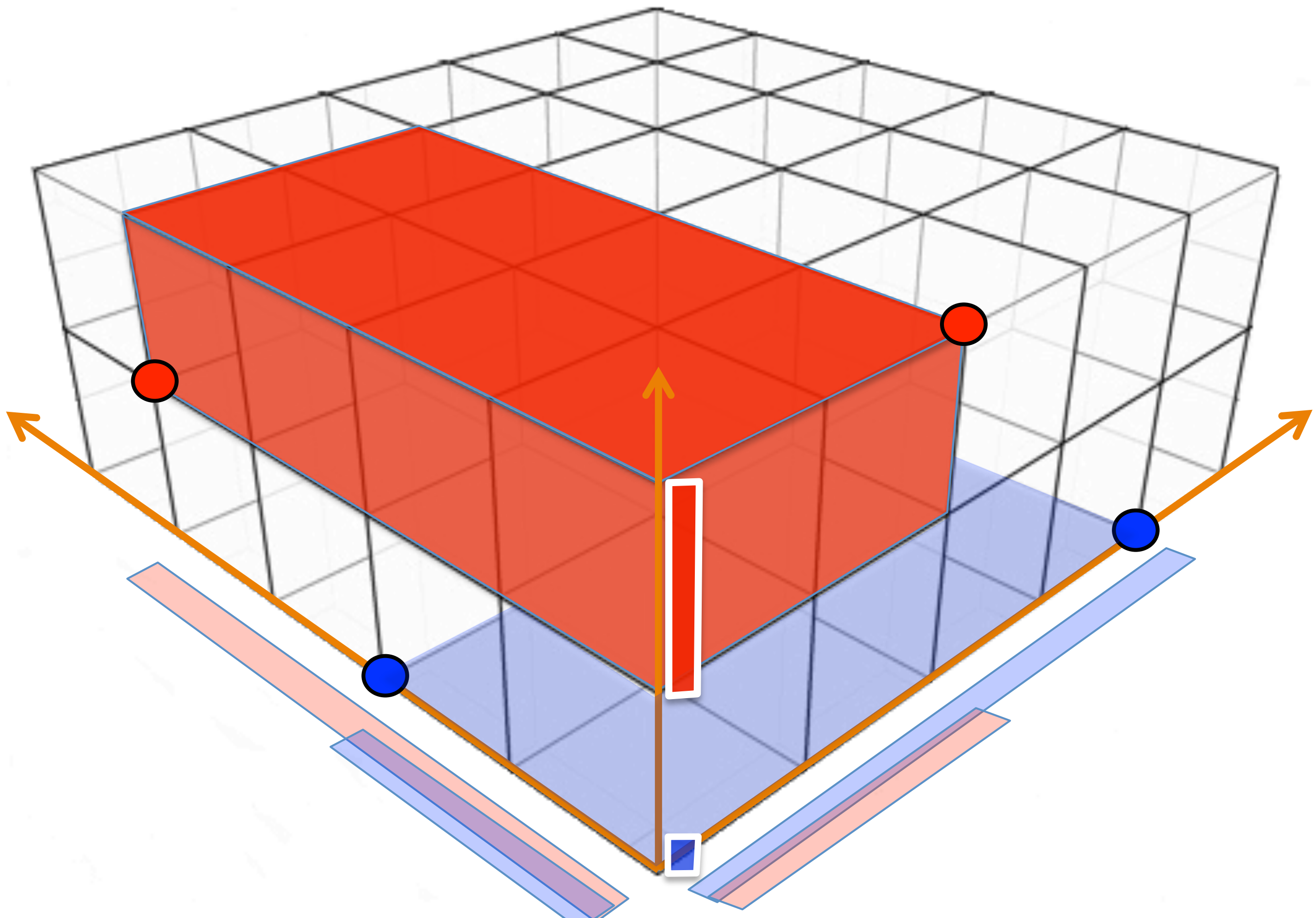

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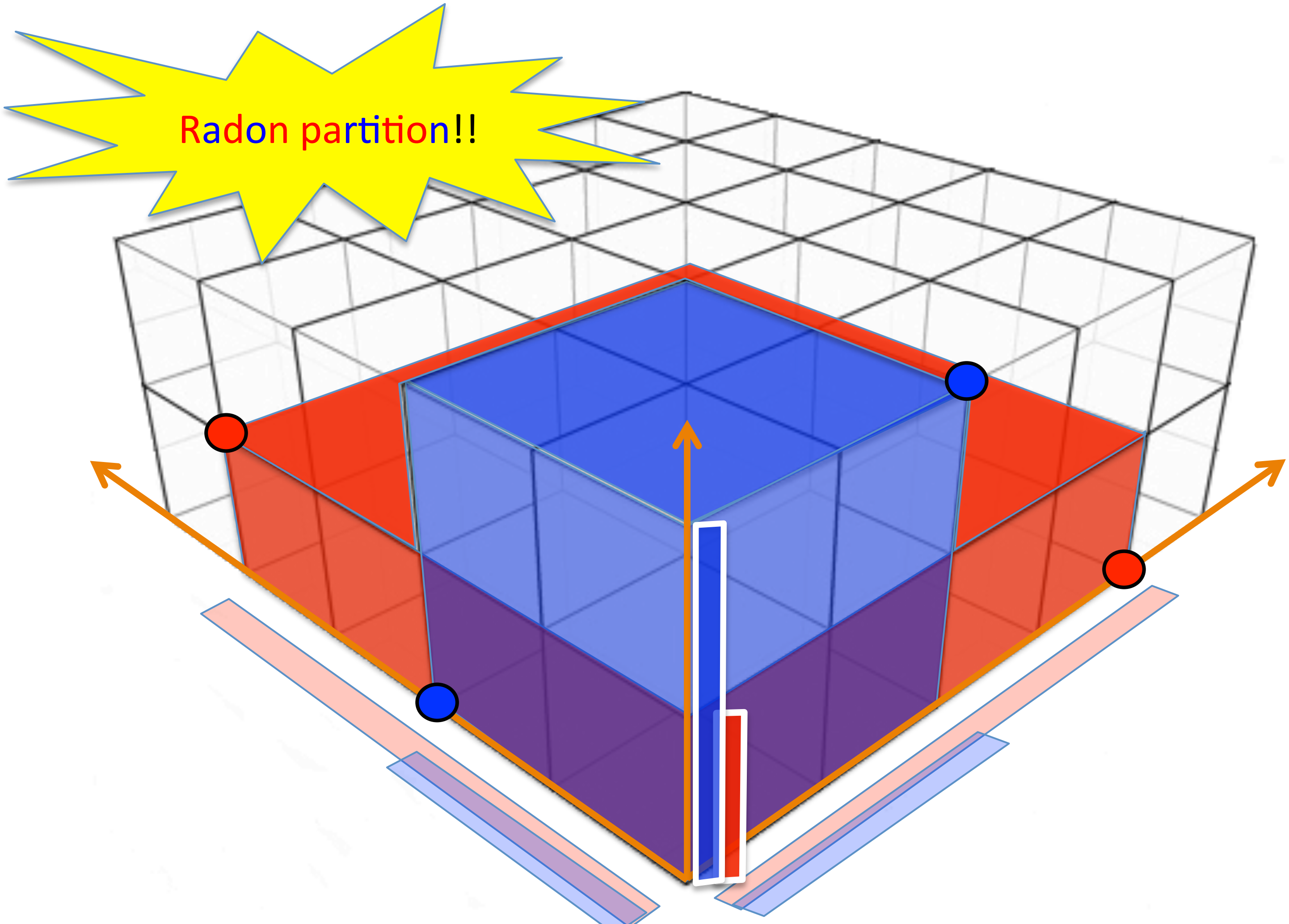

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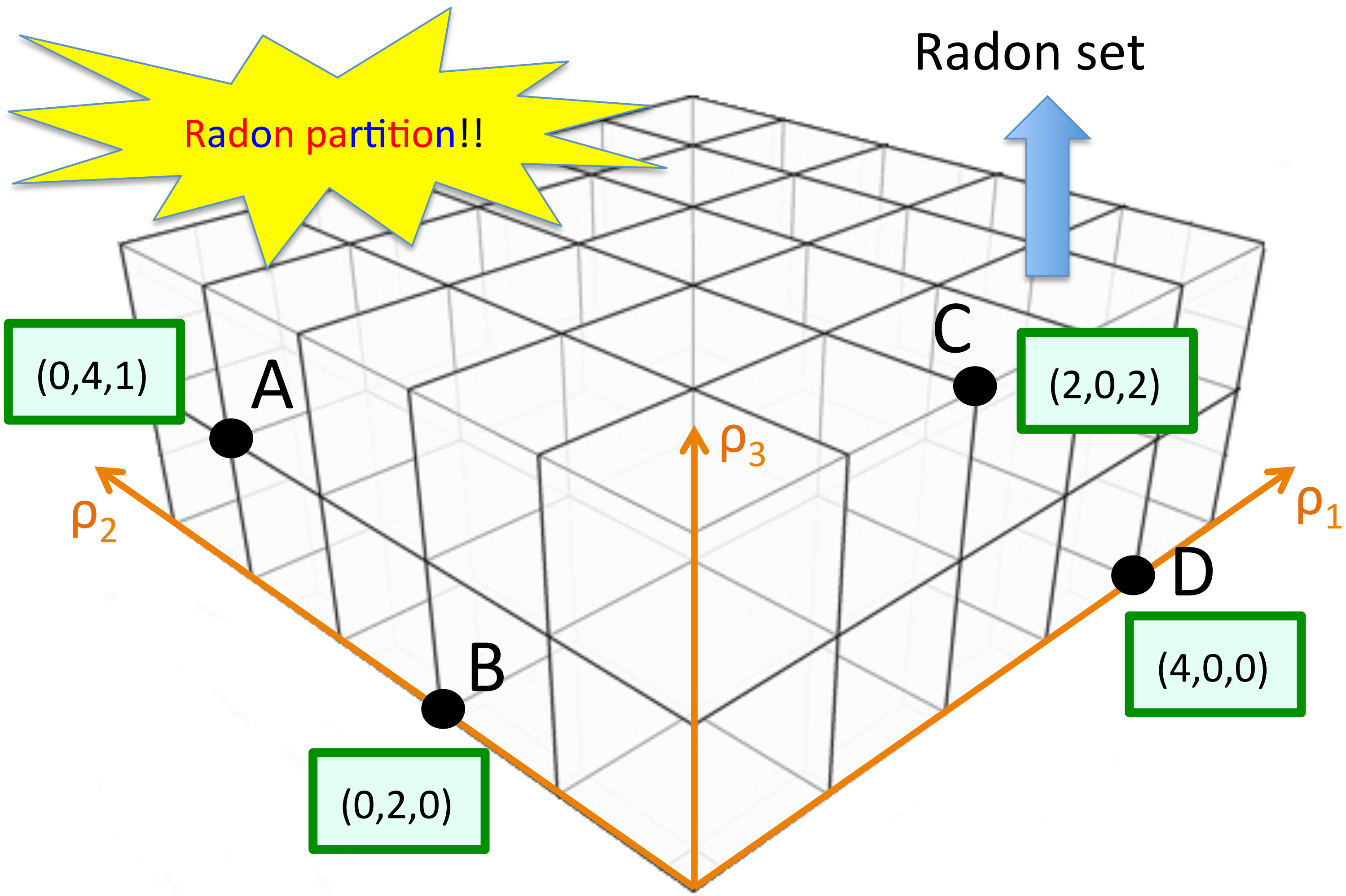

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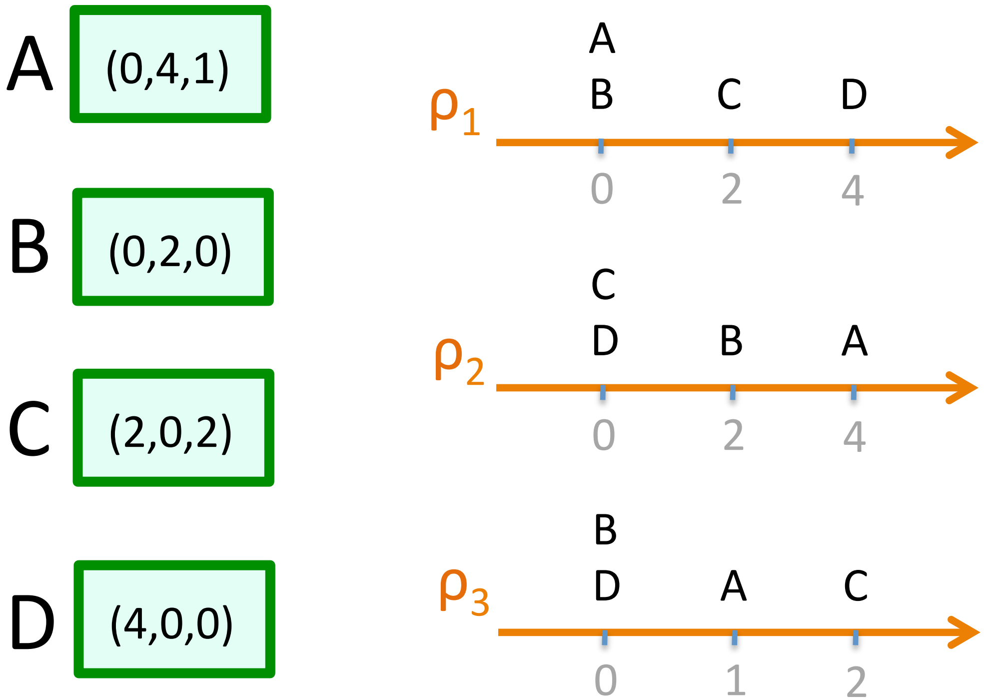

Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

{A,B}, {C,D}

{A,C}, {B,D}

{A,D}, {B,C}

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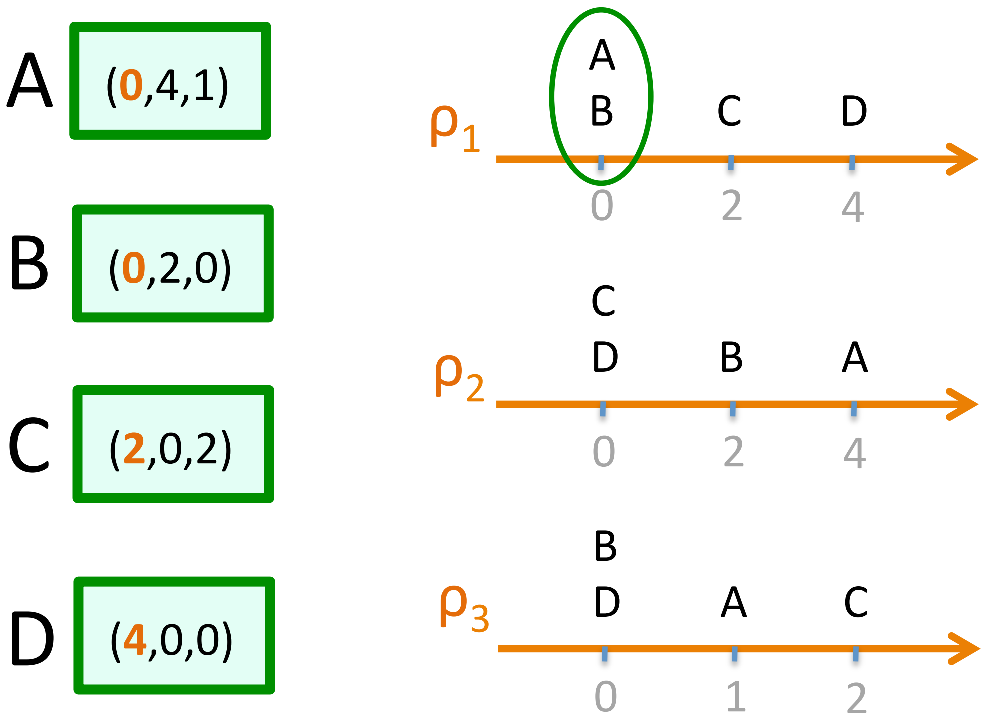

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~~{A,B}, {C,D}~~

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

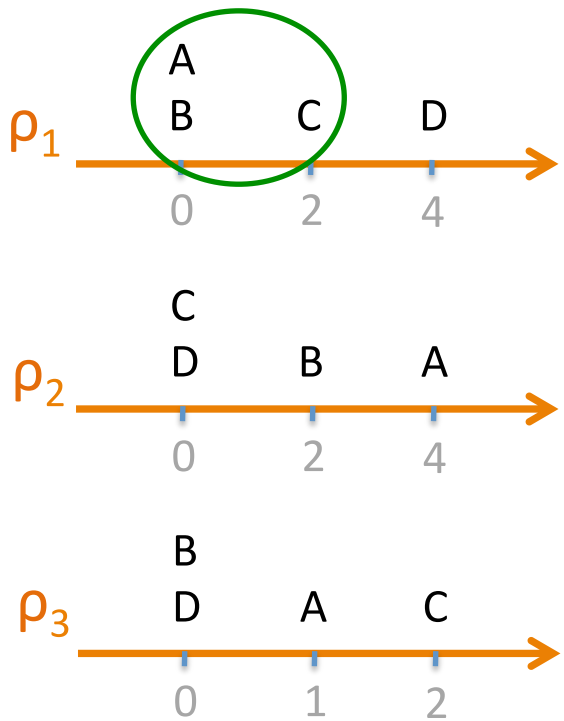
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



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Geodetic Radon number of grids

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A (0,**4**,1)

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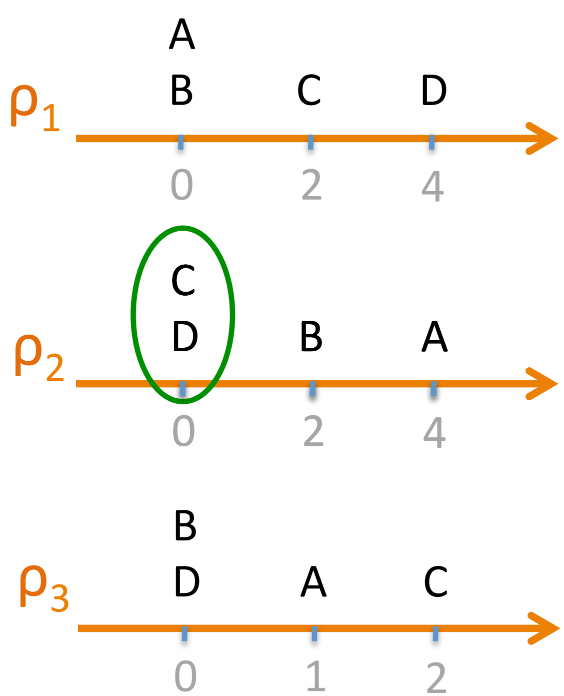

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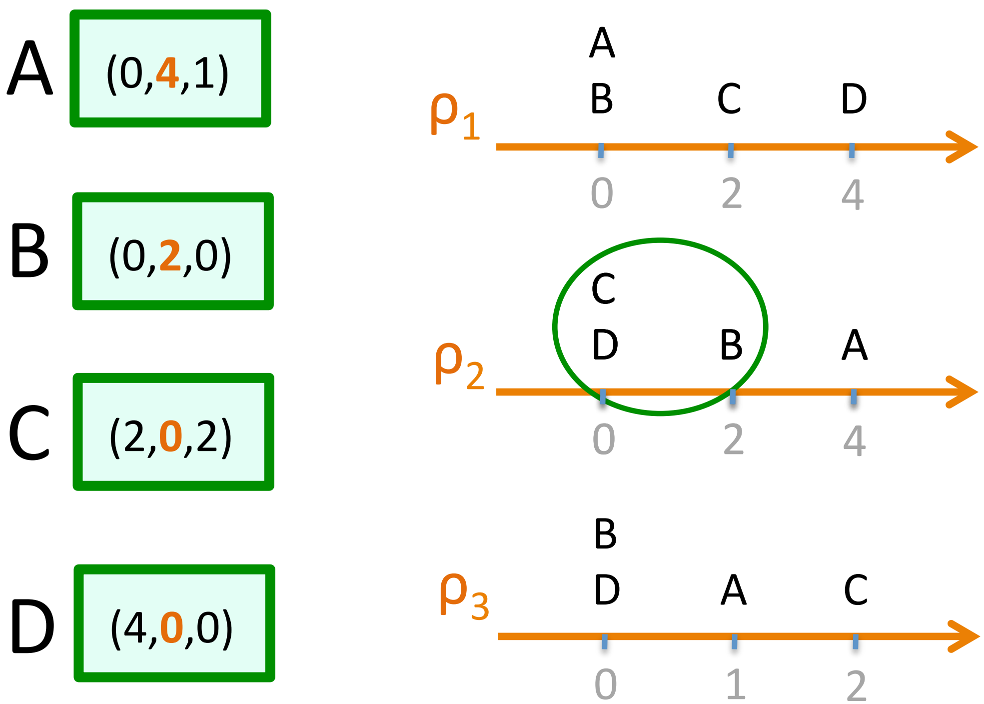

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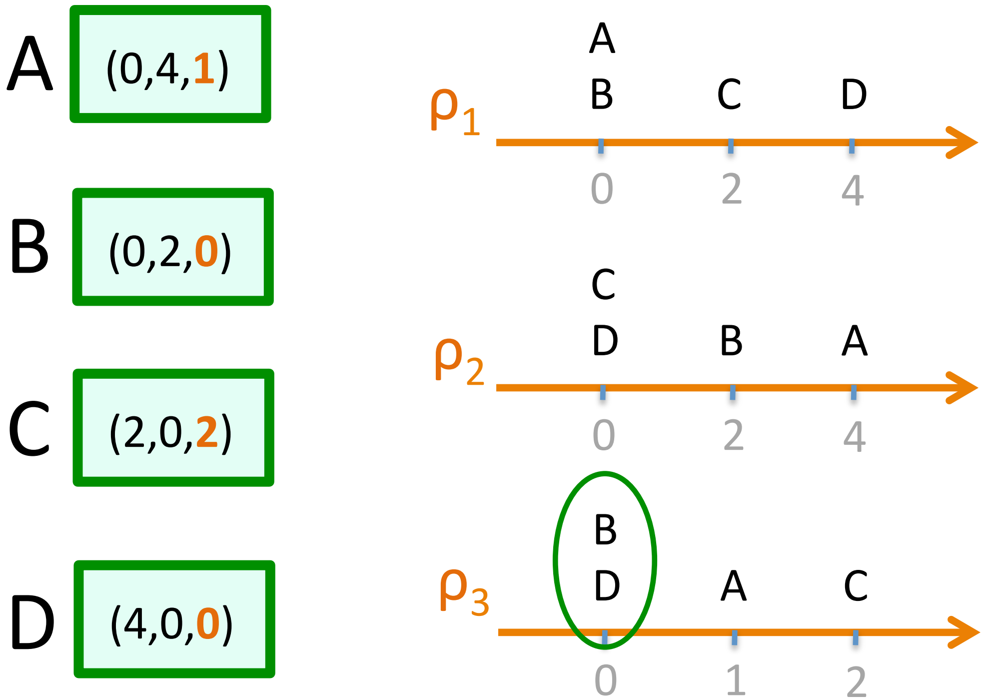

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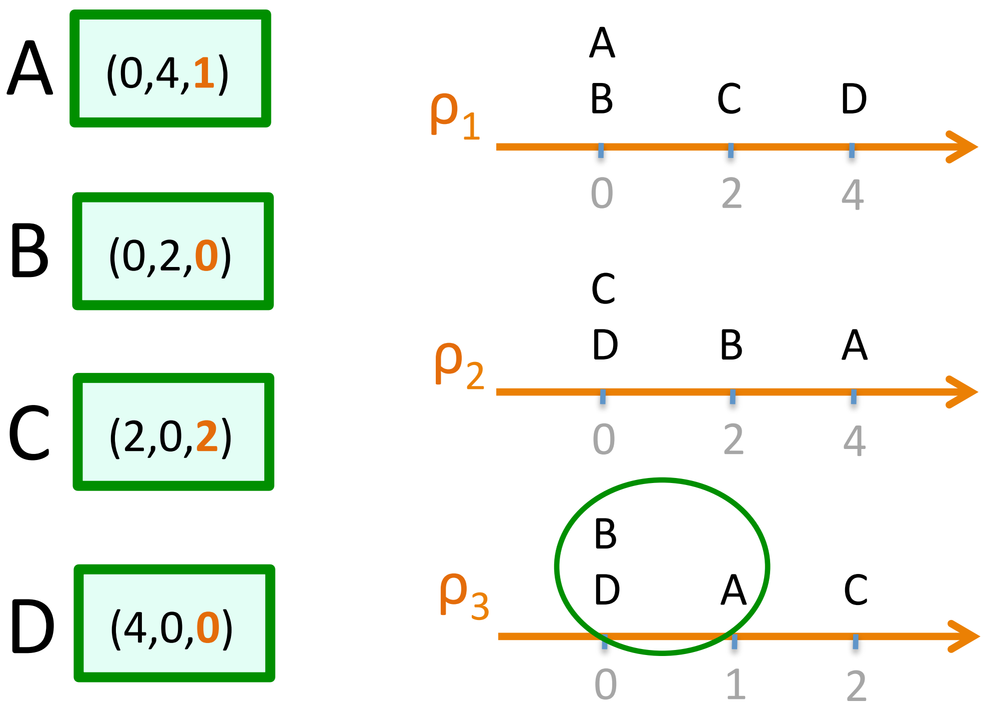

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A	(0,4,1)
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C	(2,0,2)
D	(4,0,0)

Radon set

Radon partition candidates:

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{B}, {A,C,D}

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

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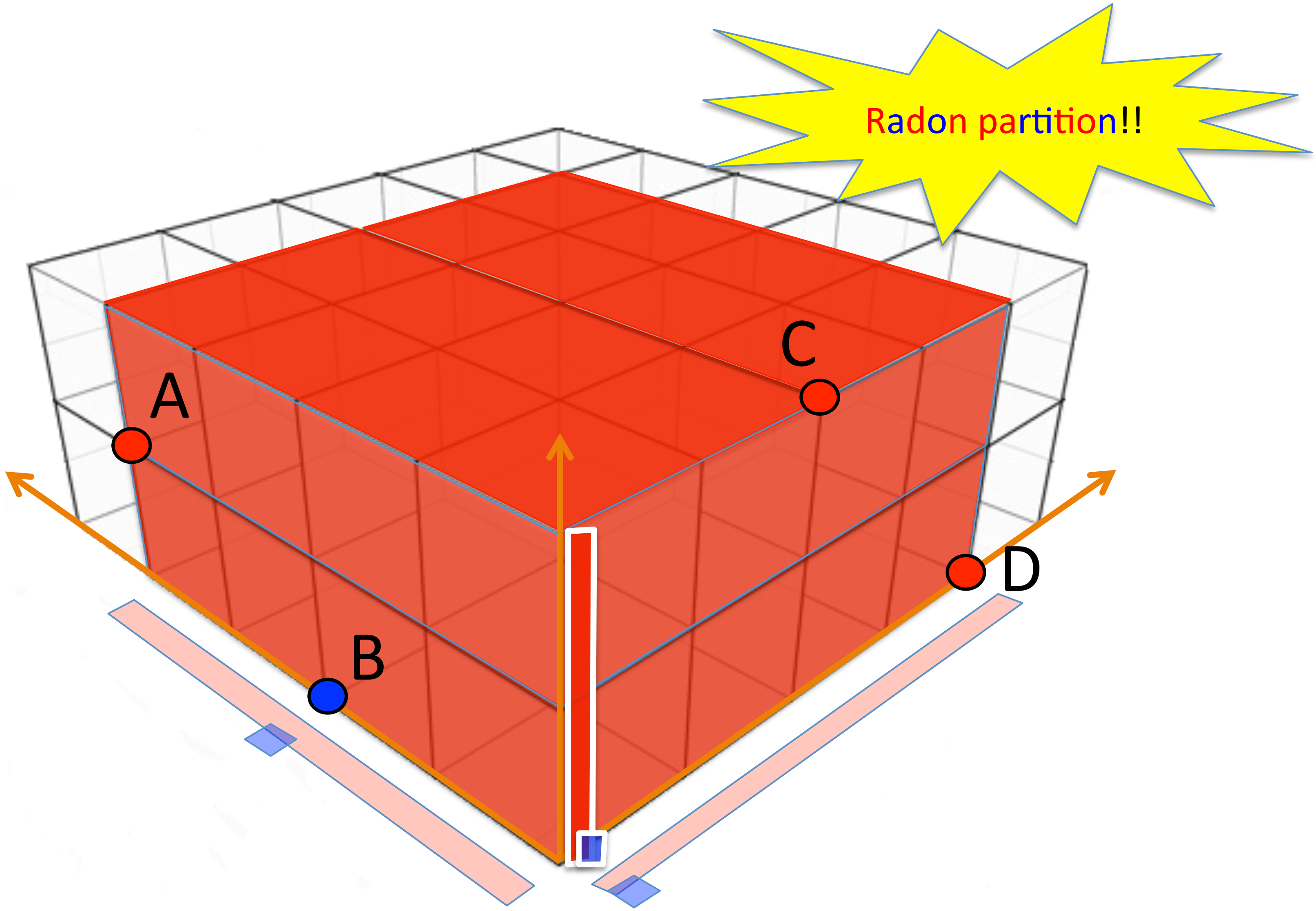

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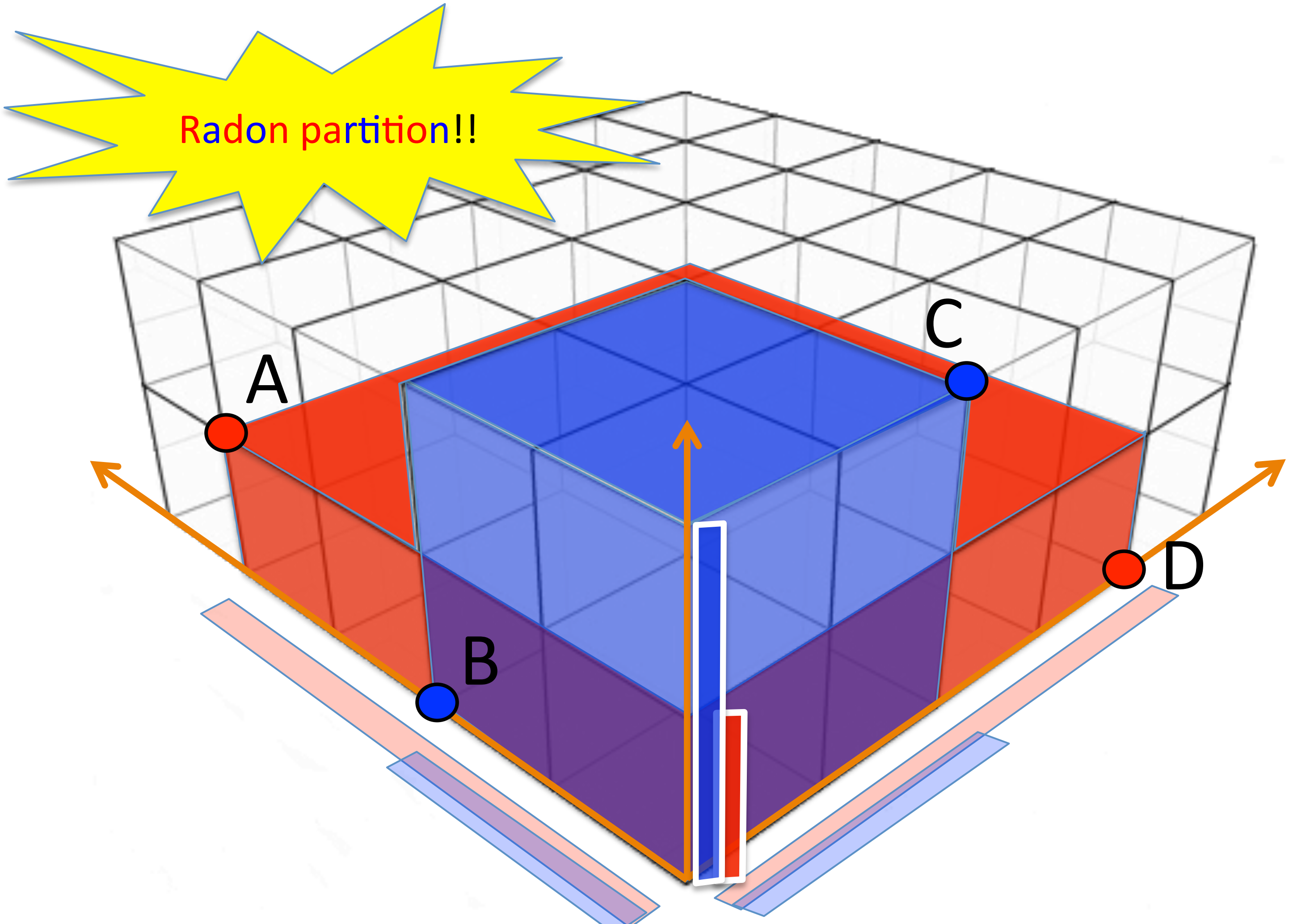


Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Ex.: Grid (9, 9, 9, 9)

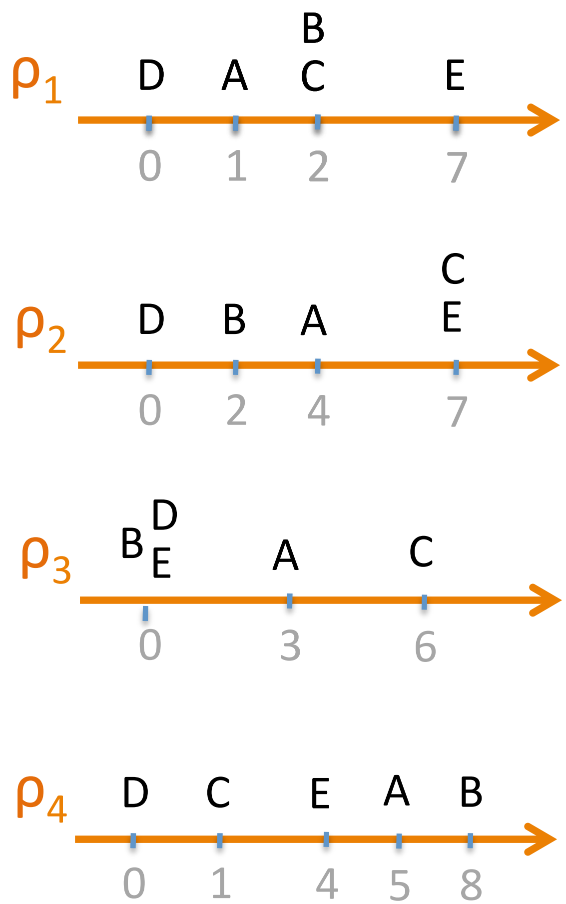
A (1,4,3,5)

B (2,2,0,8)

C (2,7,6,1)

D (0,0,0,0)

E (7,7,0,4)



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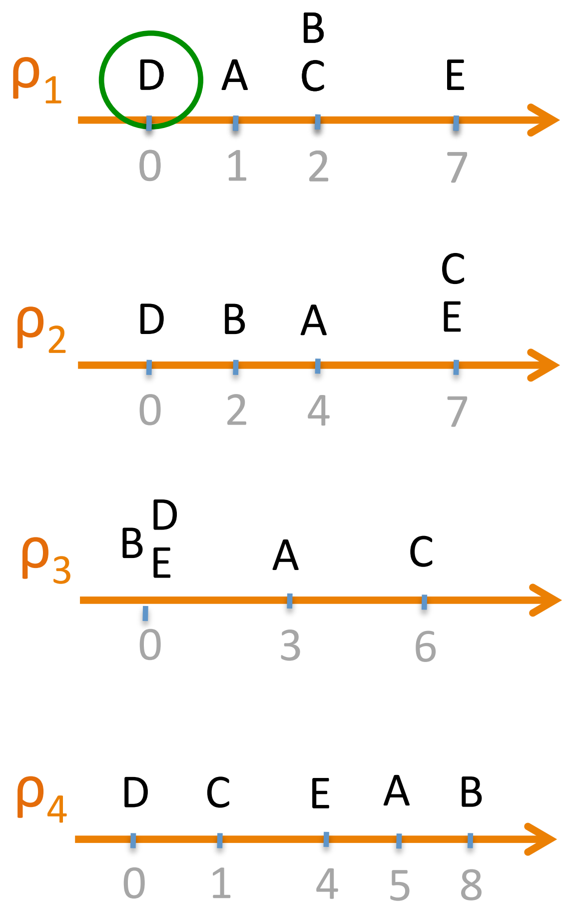

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{C}, {A,B,D,E}
~~{D}, {A,B,C,E}~~
{E}, {A,B,C,D}
{A,B}, {C,D,E}
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{A,E}, {B,C,E}
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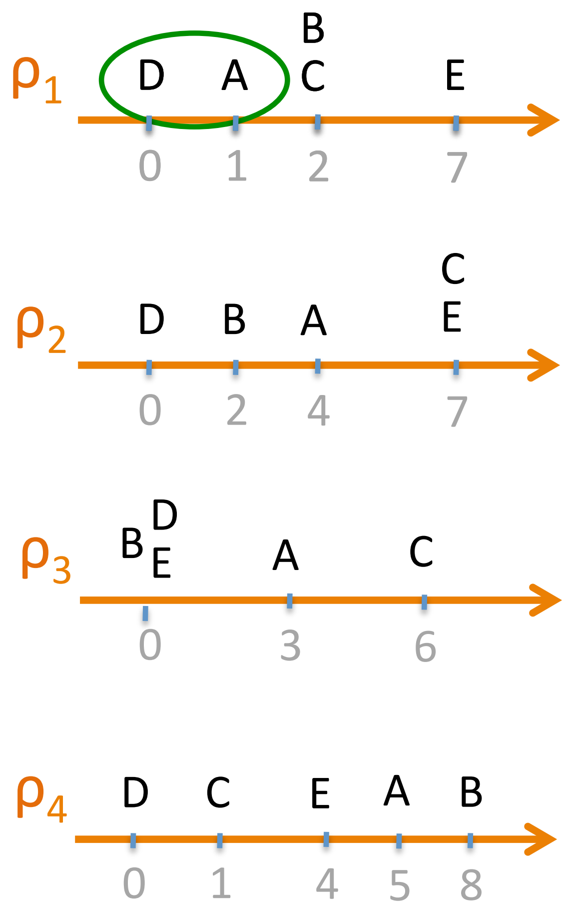

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{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

{E}, {A,B,C,D}

{A,B}, {C,D,E}

{A,C}, {B,D,E}

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

{B,D}, {A,C,E}

{B,E}, {A,C,D}

{C,D}, {A,B,E}

{C,E}, {A,B,D}

{D,E}, {A,B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Ex.: Grid (9, 9, 9, 9)

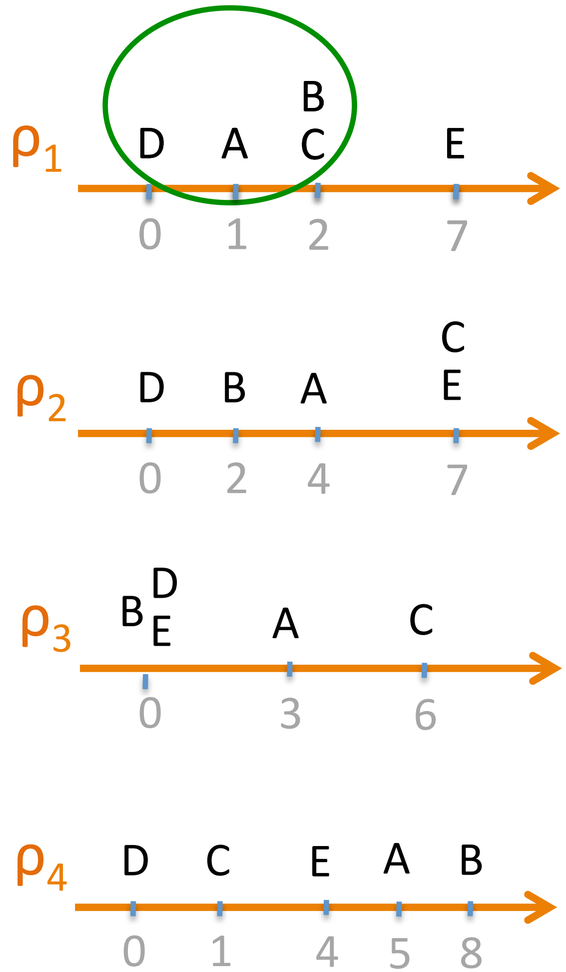
A (1,4,3,5)

B (2,2,0,8)

C (2,7,6,1)

D (0,0,0,0)

E (7,7,0,4)



Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

{A,C}, {B,D,E}

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{A,E}, {B,C,E}

{B,C}, {A,D,E}

{B,D}, {A,C,E}

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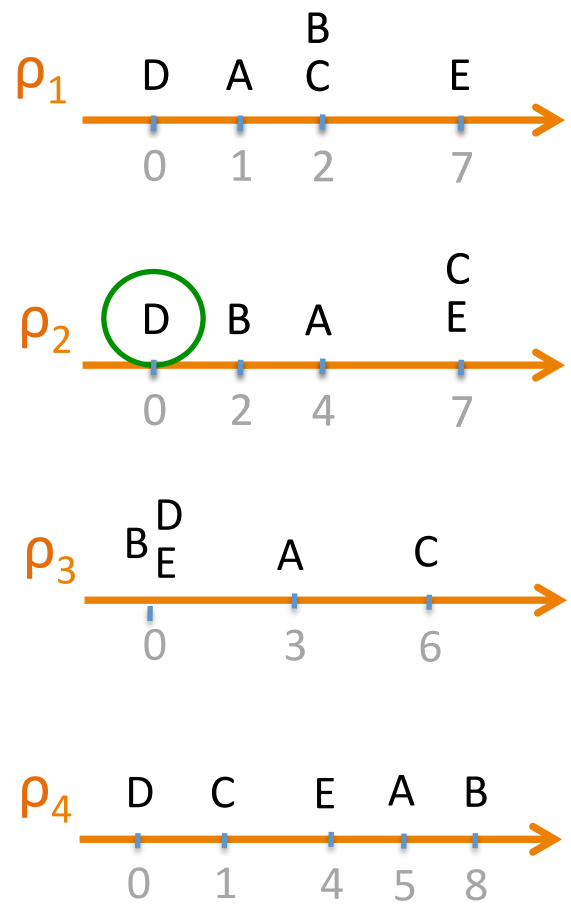

Radon partition candidates:

{A}, {B,C,D,E}
{B}, {A,C,D,E}
{C}, {A,B,D,E}
~~{D}, {A,B,C,E}~~
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{A,B}, {C,D,E}
{A,C}, {B,D,E}
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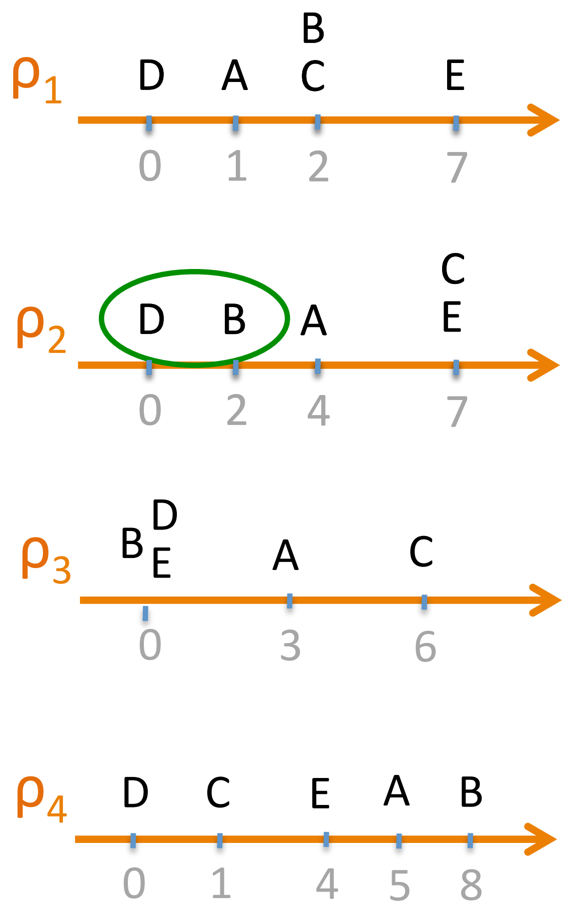

Radon partition candidates:

{A}, {B,C,D,E}
{B}, {A,C,D,E}
{C}, {A,B,D,E}
~~{D}, {A,B,C,E}~~
~~{E}, {A,B,C,D}~~
{A,B}, {C,D,E}
{A,C}, {B,D,E}
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{C,D}, {A,B,E}
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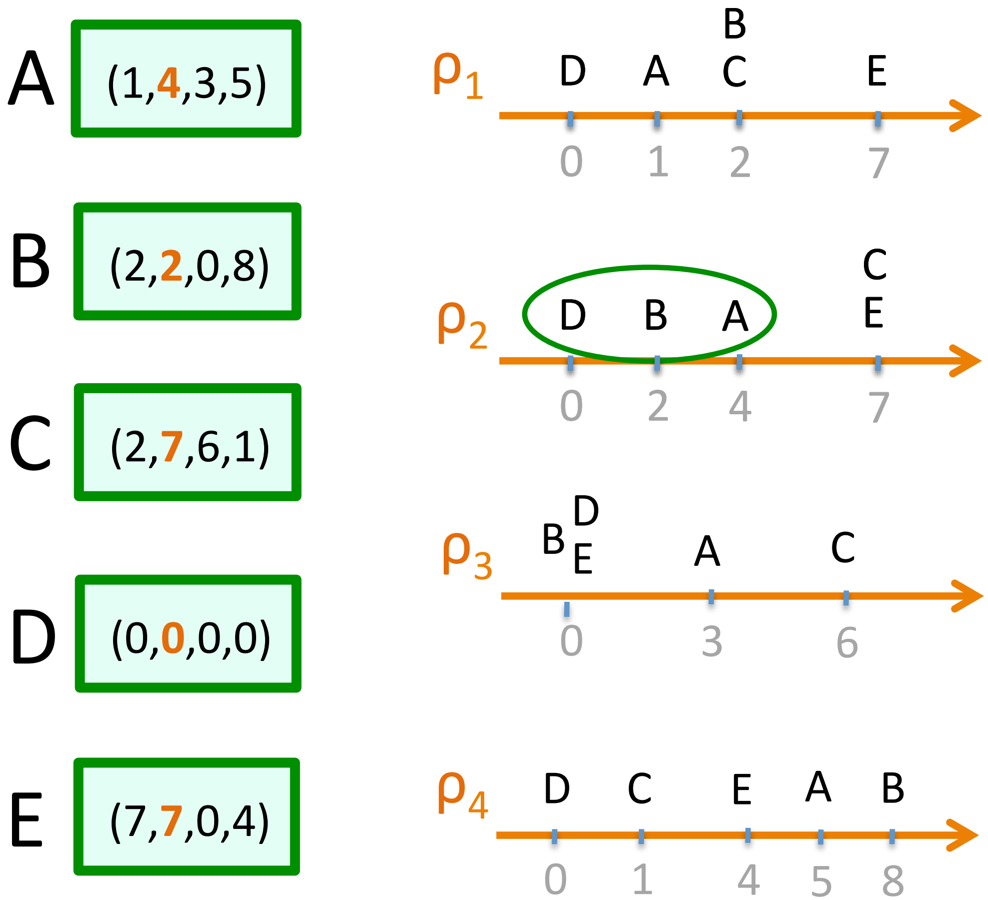
{D,E}, {A,B,C}

Geodetic Radon number of grids

Grid (n_1, n_2, ..., n_d) := P_{n_1} x P_{n_2} x ... x P_{n_d}

Ex.: Grid (9, 9, 9, 9)



<u>Radon partition candidates</u>:

{A}, {B,C,D,E}
{B}, {A,C,D,E}
{C}, {A,B,D,E}
~~{D}, {A,B,C,E}~~
~~{E}, {A,B,C,D}~~
{A,B}, {C,D,E}
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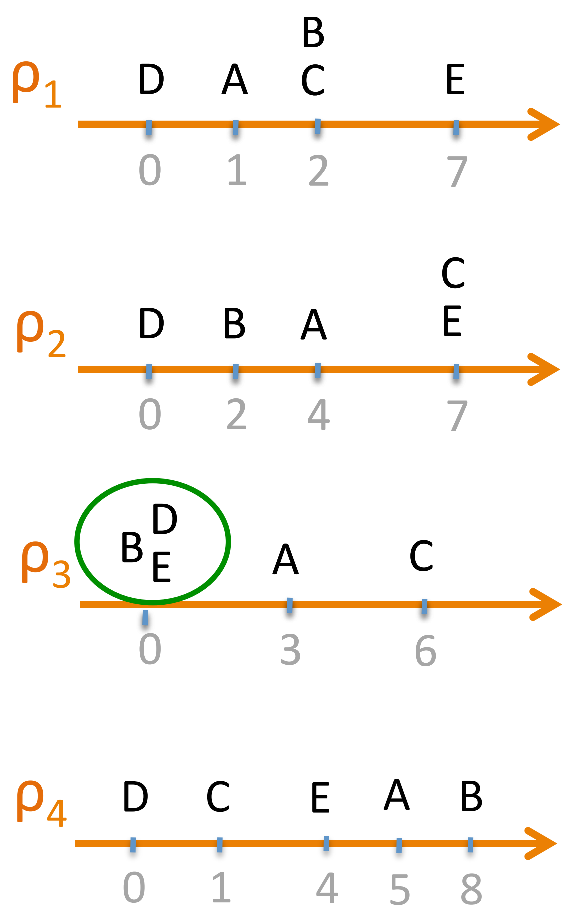

Radon partition candidates:

{A}, {B,C,D,E}
{B}, {A,C,D,E}
{C}, {A,B,D,E}
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- D (0,0,**0**,0)
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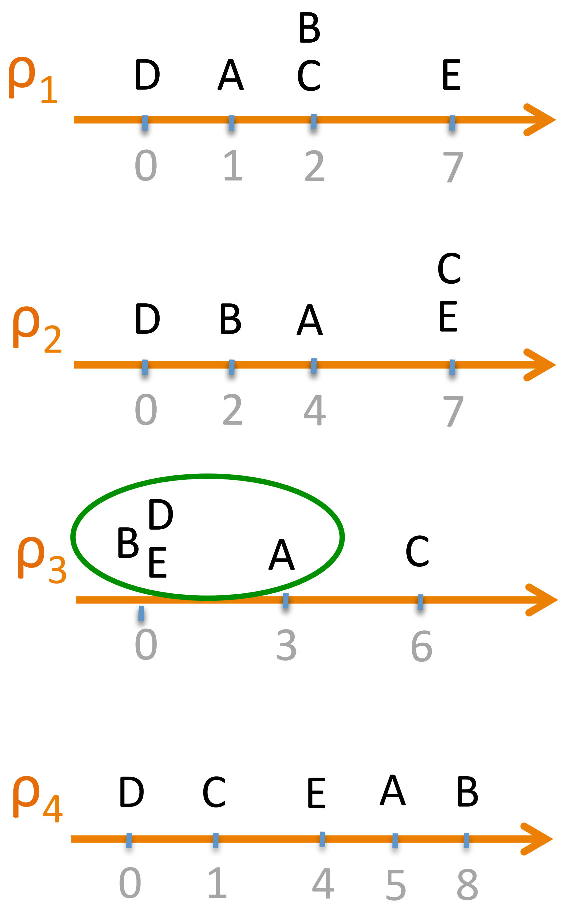

Radon partition candidates:

- {A}, {B,C,D,E}
- {B}, {A,C,D,E}
- ~~{C}, {A,B,D,E}~~
- ~~{D}, {A,B,C,E}~~
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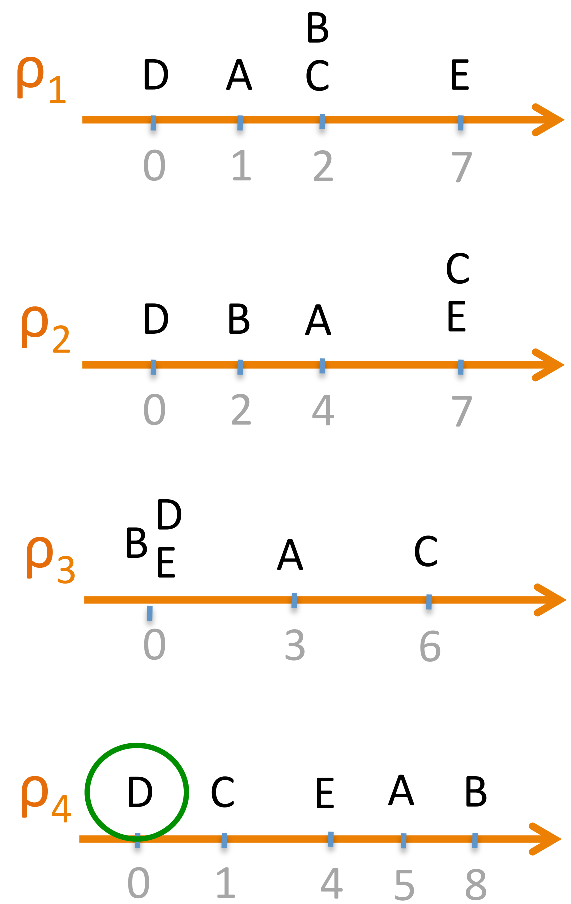

<u>Radon partition candidates</u>:

{A}, {B,C,D,E}
{B}, {A,C,D,E}
~~{C}, {A,B,D,E}~~
~~{D}, {A,B,C,E}~~
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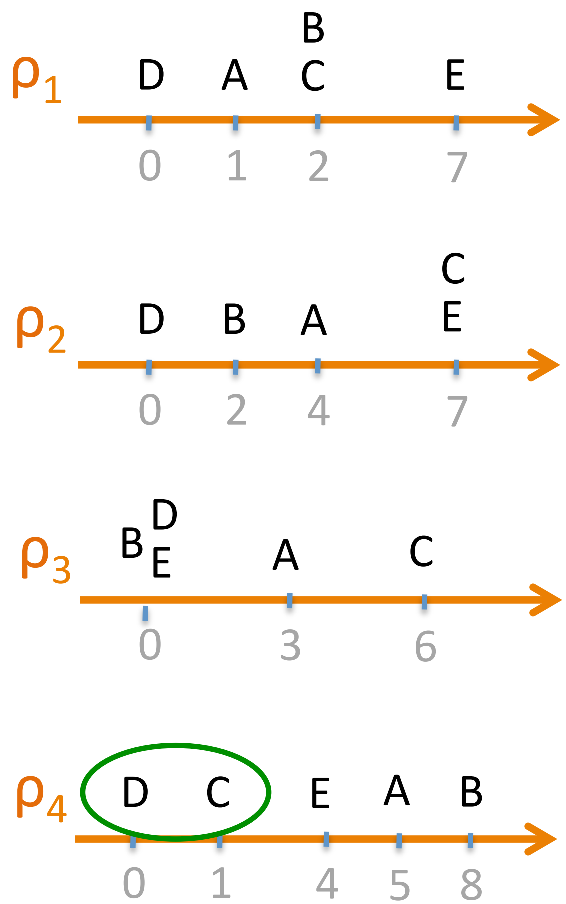

Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

~~{C}, {A,B,D,E}~~

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

~~{A,C}, {B,D,E}~~

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{B,C}, {A,D,E}

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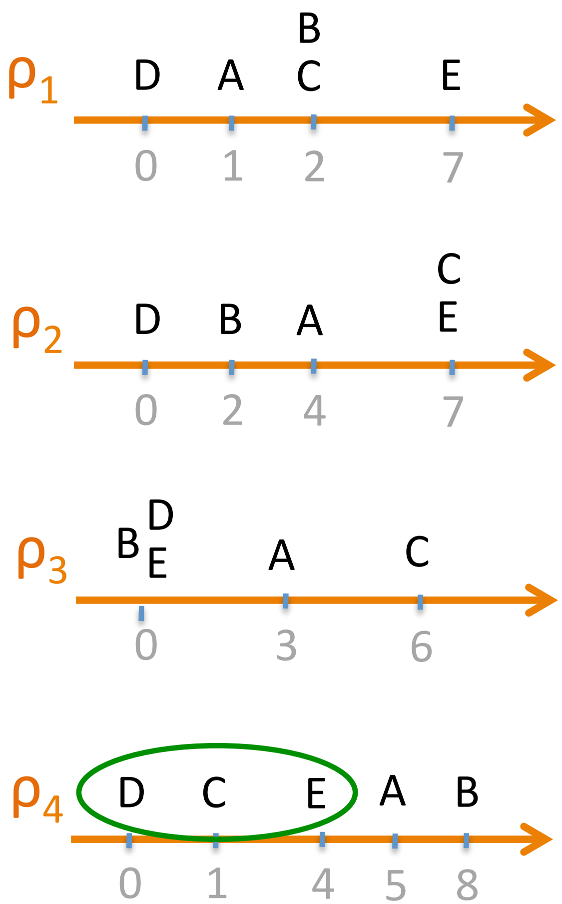

Radon partition candidates:

{A}, {B,C,D,E}
{B}, {A,C,D,E}
~~{C}, {A,B,D,E}~~
~~{D}, {A,B,C,E}~~
~~{E}, {A,B,C,D}~~
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{A,E}, {B,C,E}
{B,C}, {A,D,E}
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{B,E}, {A,C,D}
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Ex.: Grid (9, 9, 9, 9)

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B (2,2,0,**8**)

C (2,7,6,**1**)

D (0,0,0,**0**)

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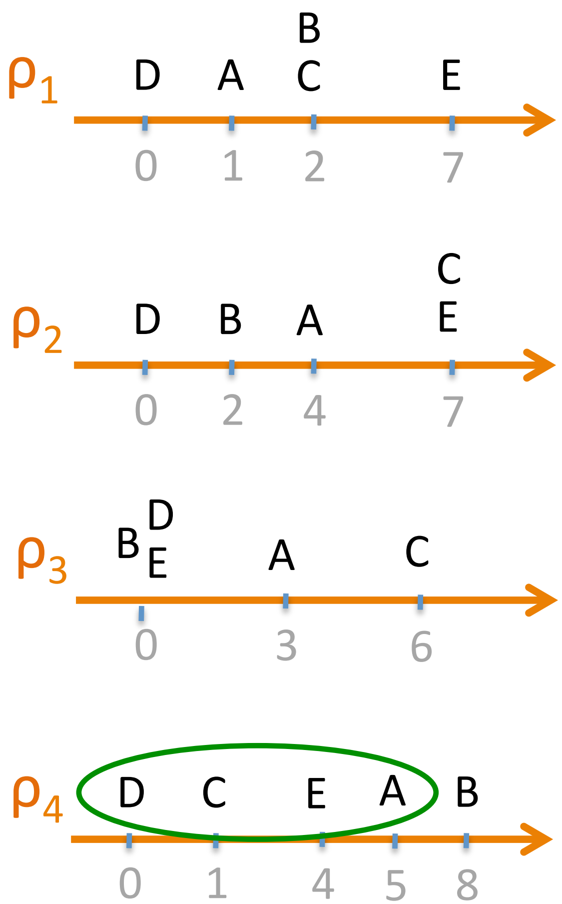

Radon partition candidates:

{A}, {B,C,D,E}

~~{B}, {A,C,D,E}~~

~~{C}, {A,B,D,E}~~

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

~~{A,B}, {C,D,E}~~

~~{A,C}, {B,D,E}~~

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{A,E}, {B,C,E}

{B,C}, {A,D,E}

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{B,E}, {A,C,D}

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Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Ex.: Grid (9, 9, 9, 9)

A	(1,4,3,5)
B	(2,2,0,8)
C	(2,7,6,1)
D	(0,0,0,0)
E	(7,7,0,4)

Radon set

Radon partition candidates:

{A}, {B,C,D,E}
~~{B}, {A,C,D,E}~~
~~{C}, {A,B,D,E}~~
~~{D}, {A,B,C,E}~~
~~{E}, {A,B,C,D}~~
~~{A,B}, {C,D,E}~~
~~{A,C}, {B,D,E}~~
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Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d) := P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

anti-Radon set of size r

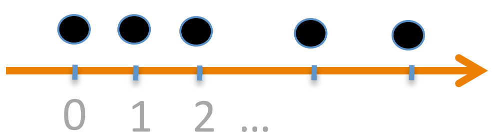
A_1 (?,?,...,?)

A_2 (?,?,...,?)

.
.
.

A_r (?,?,...,?)

ρ_j



$j = 1, \ldots, d$

Radon partition candidates:

HOW MANY ??

$(2^r/2) - 1$

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= P_{n_1} x P_{n_2} x … x P_{n_d}

Radon partition candidates:

HOW MANY ??

$$2^{r-1} - 1$$

anti-Radon set of size r

A_1 (?,?,…,?)

A_2 (?,?,…,?)

.
.
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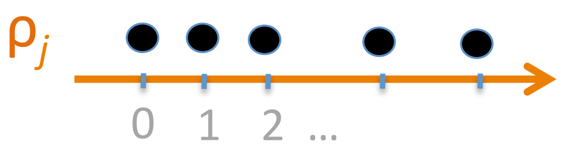
A_r (?,?,…,?)



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⋮

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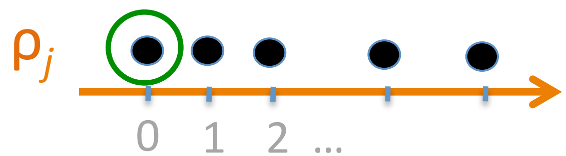

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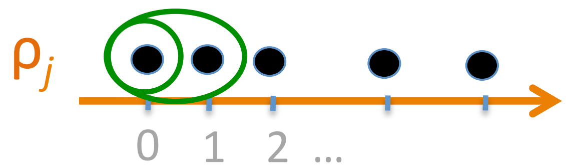

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HOW MANY ??

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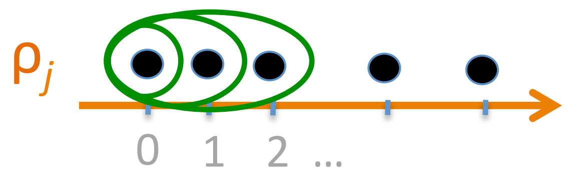

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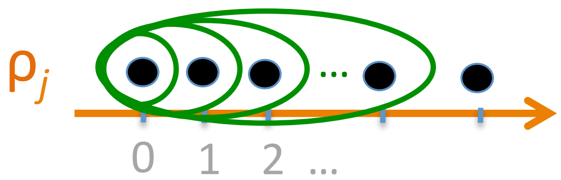

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Radon partition candidates:

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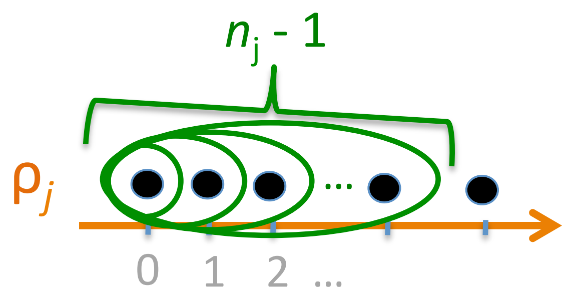

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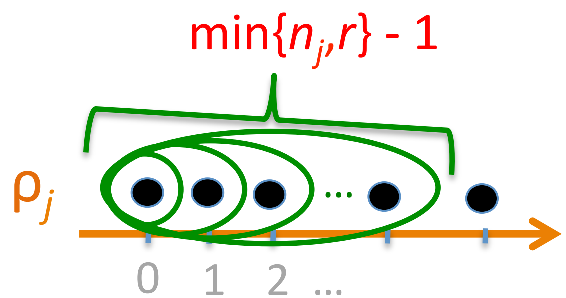


$j = 1, \ldots, d$

Radon partition candidates:

HOW MANY ??

$$2^{r-1} - 1$$

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Necessary condition for anti-Radon of size r :

anti-Radon set of size r ⇒

$$\sum_{j=1}^{d} [\min\{n_j , r\} - 1] \geq 2^{r-1} - 1 \qquad (1)$$

each dimension ρ_j may eliminate up to $\min\{n_j , r\} - 1$ candidate partitions

potential(j)

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d) := P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Necessary condition for anti-Radon of size r :

anti-Radon set of size r $\Rightarrow$ $$\sum_{j=1}^{d} [\min\{n_j, r\} - 1] \geq 2^{r-1} - 1 \qquad (1)$$

Not sufficient, though.

Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Another (tighter) necessary condition…

anti-Radon set of size r ⇒ $2d \geq \binom{r}{k}$, $1 \leq k \leq r/2$

each dimension may eliminate up to
2 candidate partitions **having a partite set of size *k***

k-quota(*j*)

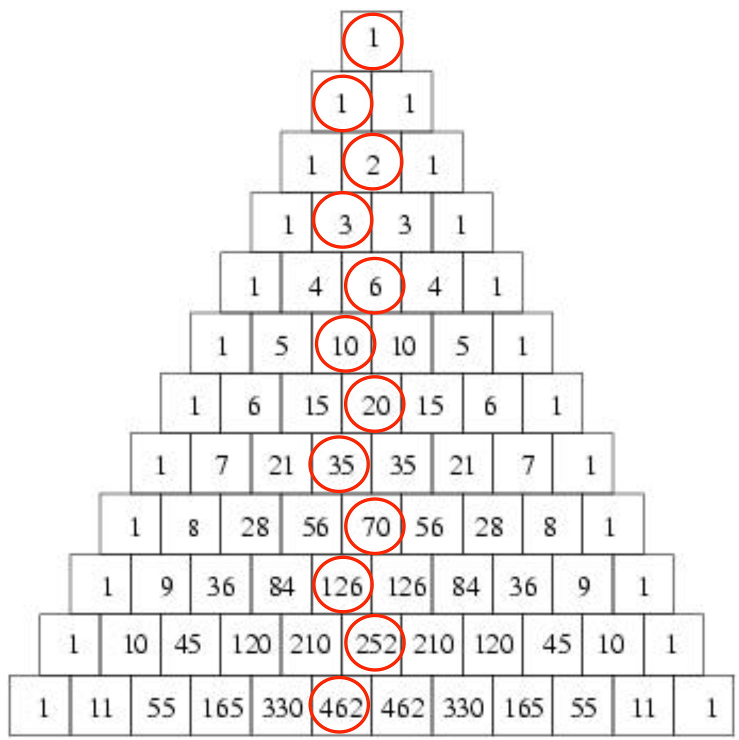
Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Another (tighter) necessary condition…

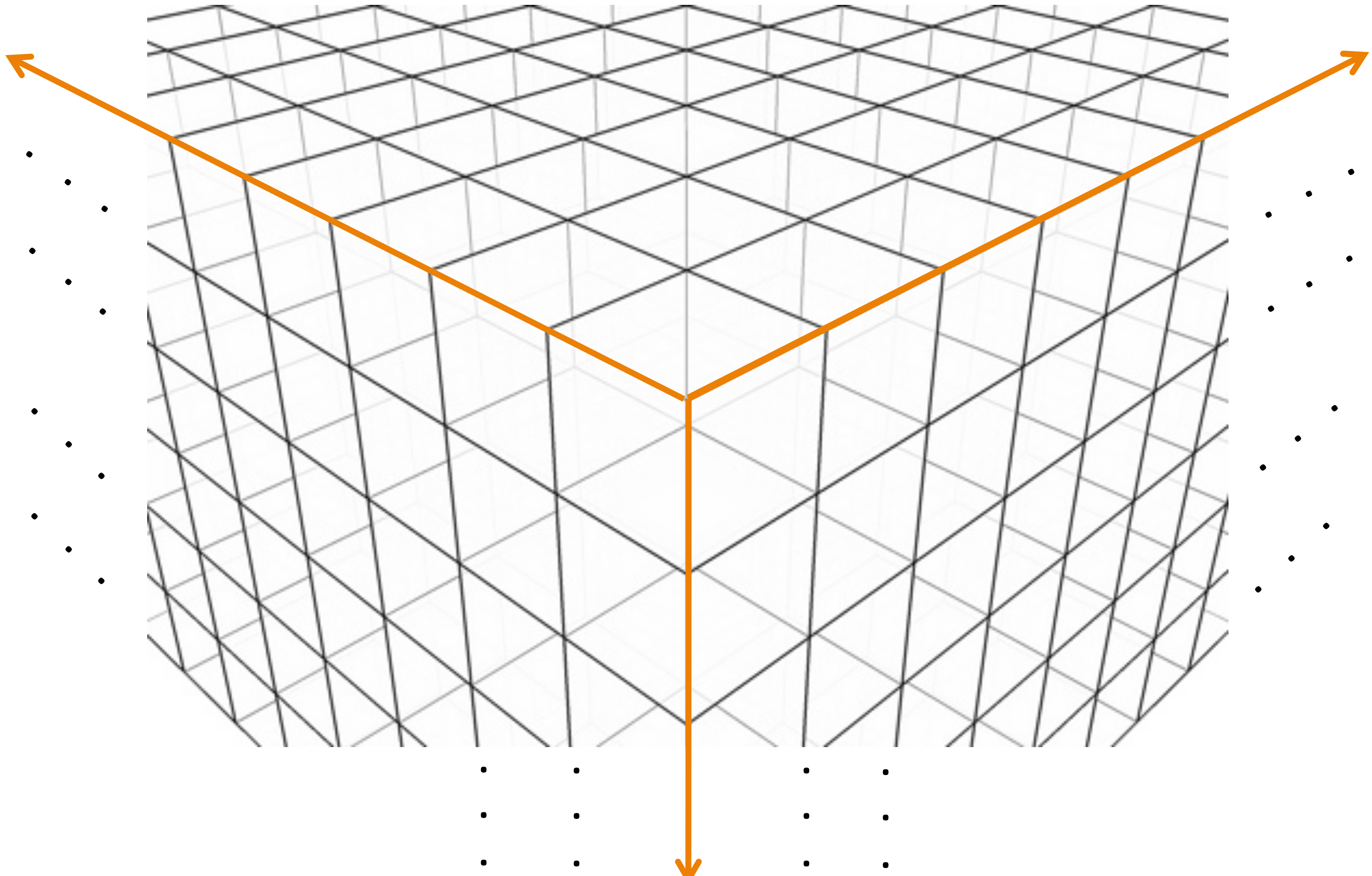
anti-Radon set of size r ⇒ $$2d \geq \binom{r}{\lfloor r/2 \rfloor}$$ (2)

(Eckhoff 1969)



Geodetic Radon number of grids

“Large enough” *d*-dimensional grids



Geodetic Radon number of grids

Grid $(n_1, n_2, \ldots, n_d)$:= $P_{n_1} \times P_{n_2} \times \ldots \times P_{n_d}$

Another (tighter) necessary condition…

anti-Radon set of size r ⇒ $2d \geq \binom{r}{\lfloor r/2 \rfloor}$ (2)

(Eckhoff 1969)

…which is also sufficient for "large enough" grids!

Theorem: *Let r^* be the maximum integer satisfying (2).*
If $n_j \geq r^*$ for all j,
then Max anti-Radon set size **=** r^*
Radon number **=** $r^* + 1$

(Jamison-Waldner 1981)

Geodetic Radon number of grids

Example: $d = 10$

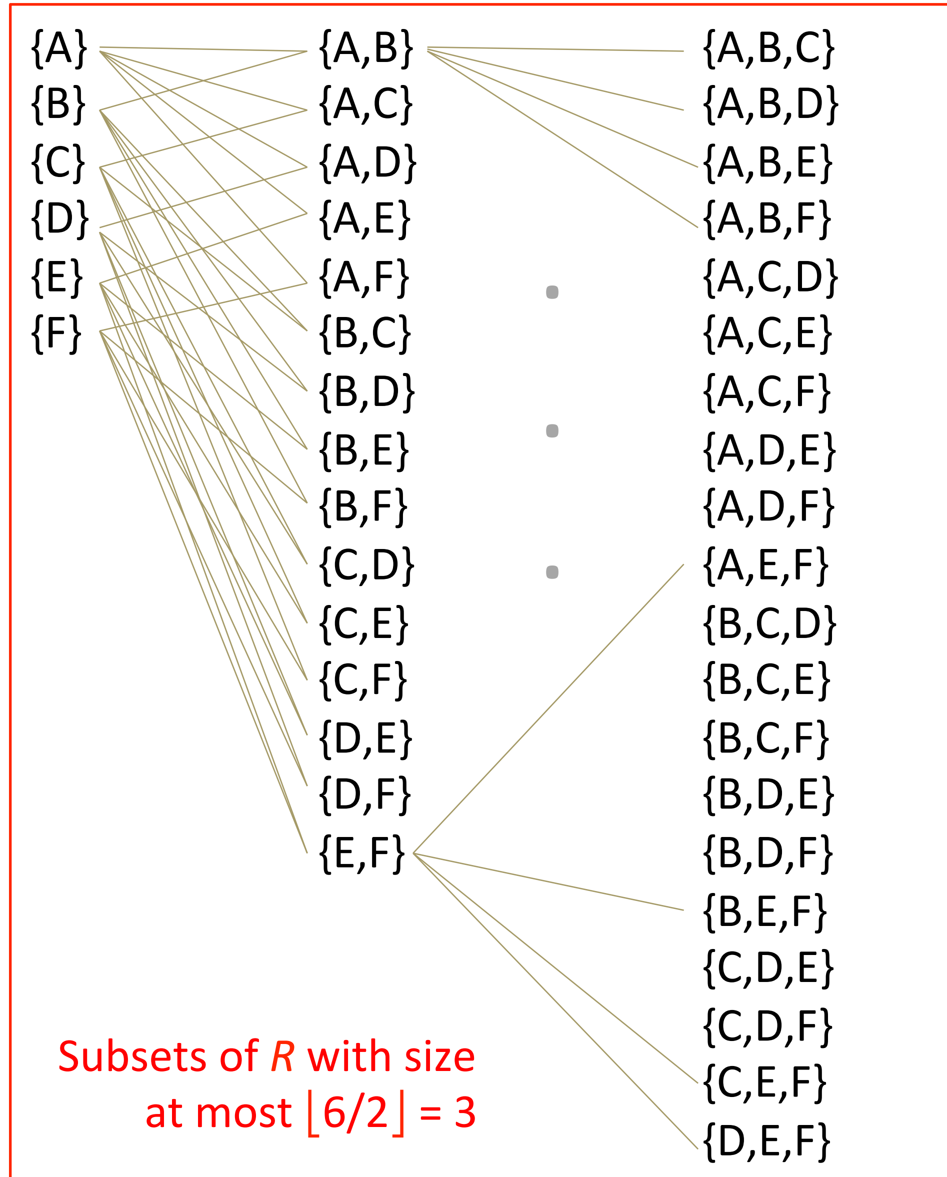
$$\binom{5}{2} = 10 \le 2d$$

$$\binom{6}{3} = 20 \le 2d$$

$$\binom{7}{3} = 35 > 2d$$

$r^* = 6$

anti-Radon set R = {A,B,C,D,E,F}



Geodetic Radon number of grids

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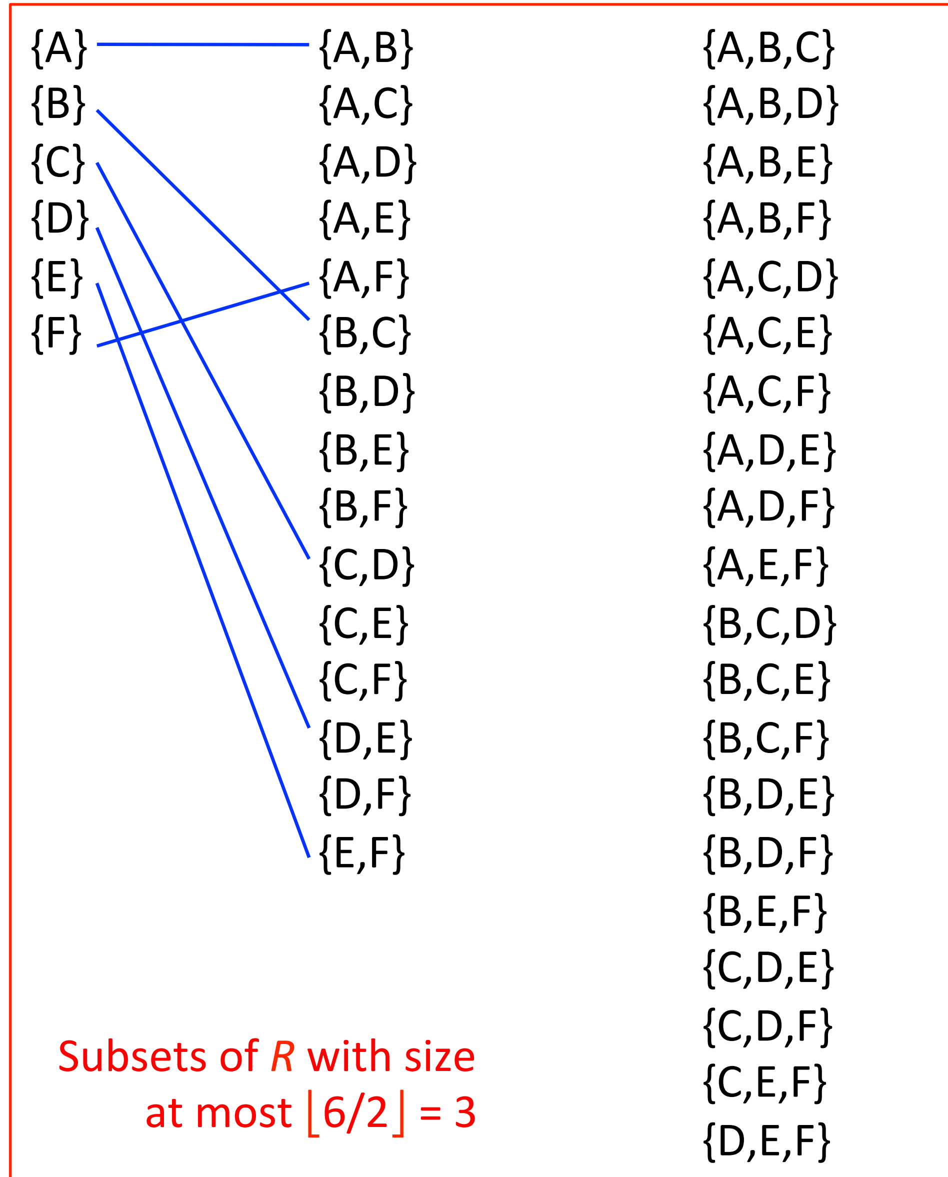

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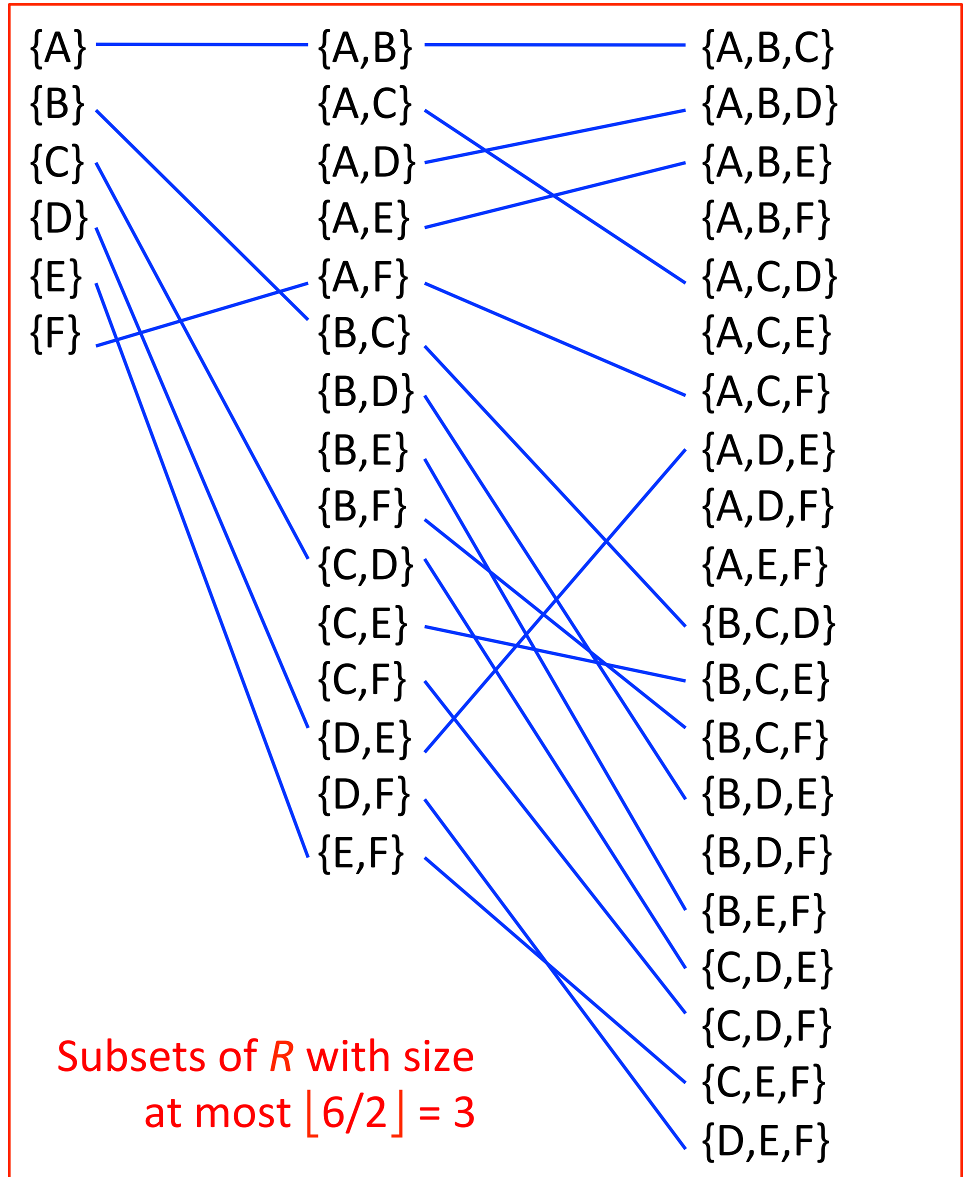

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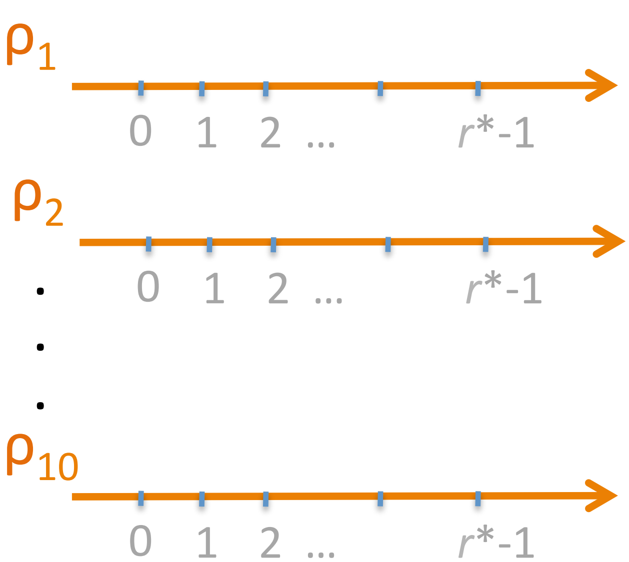

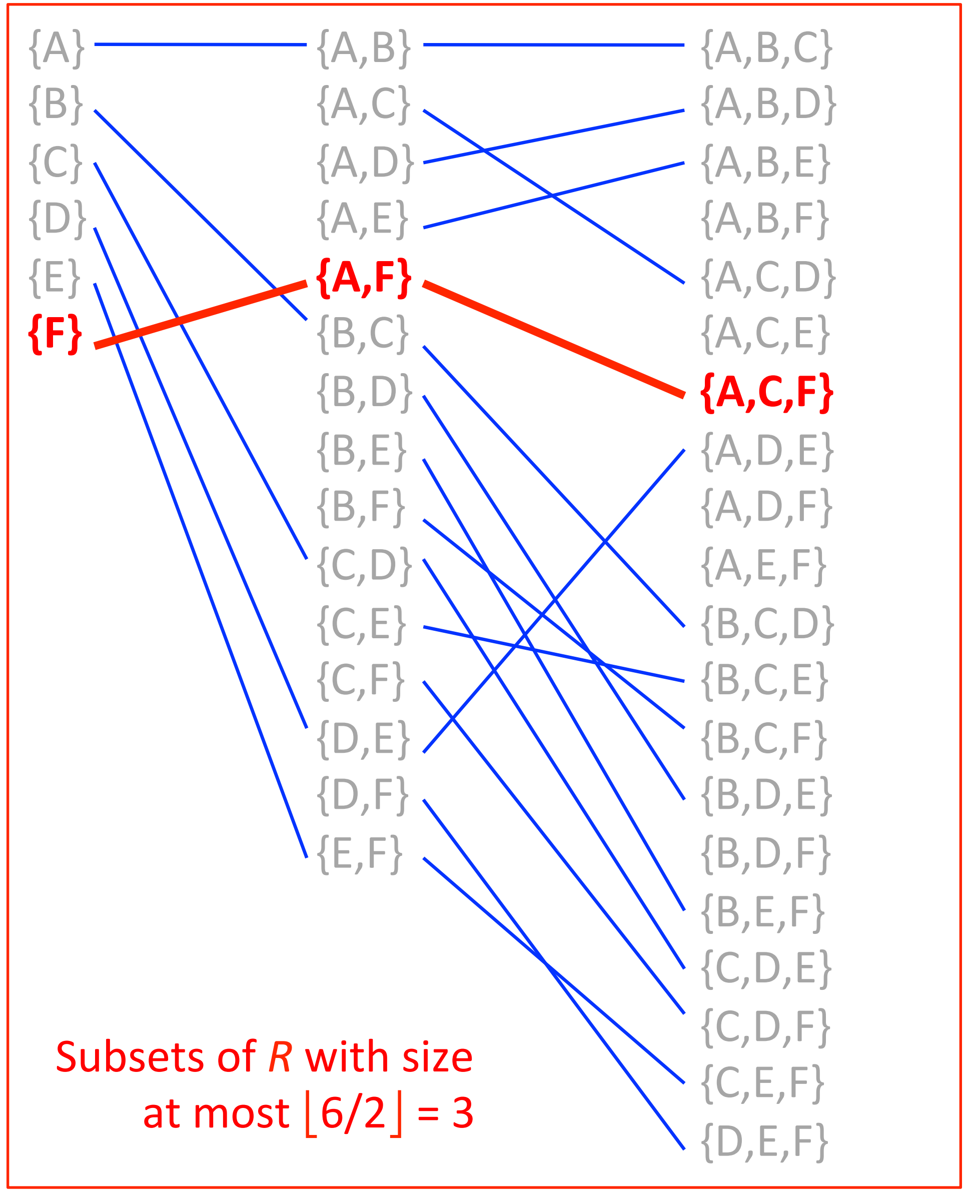

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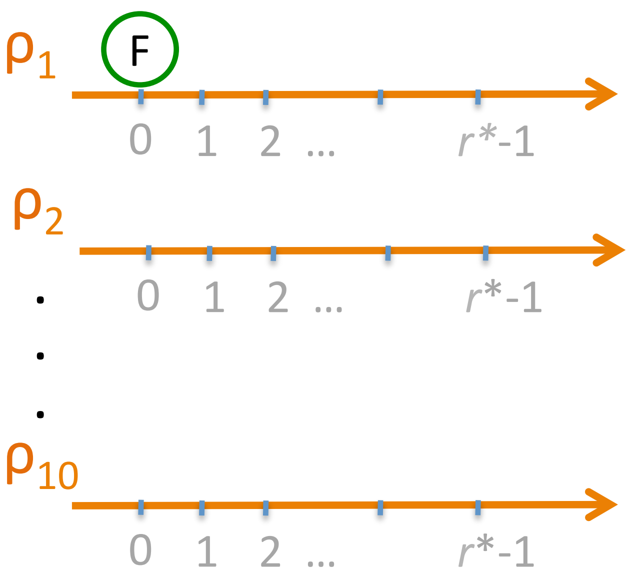

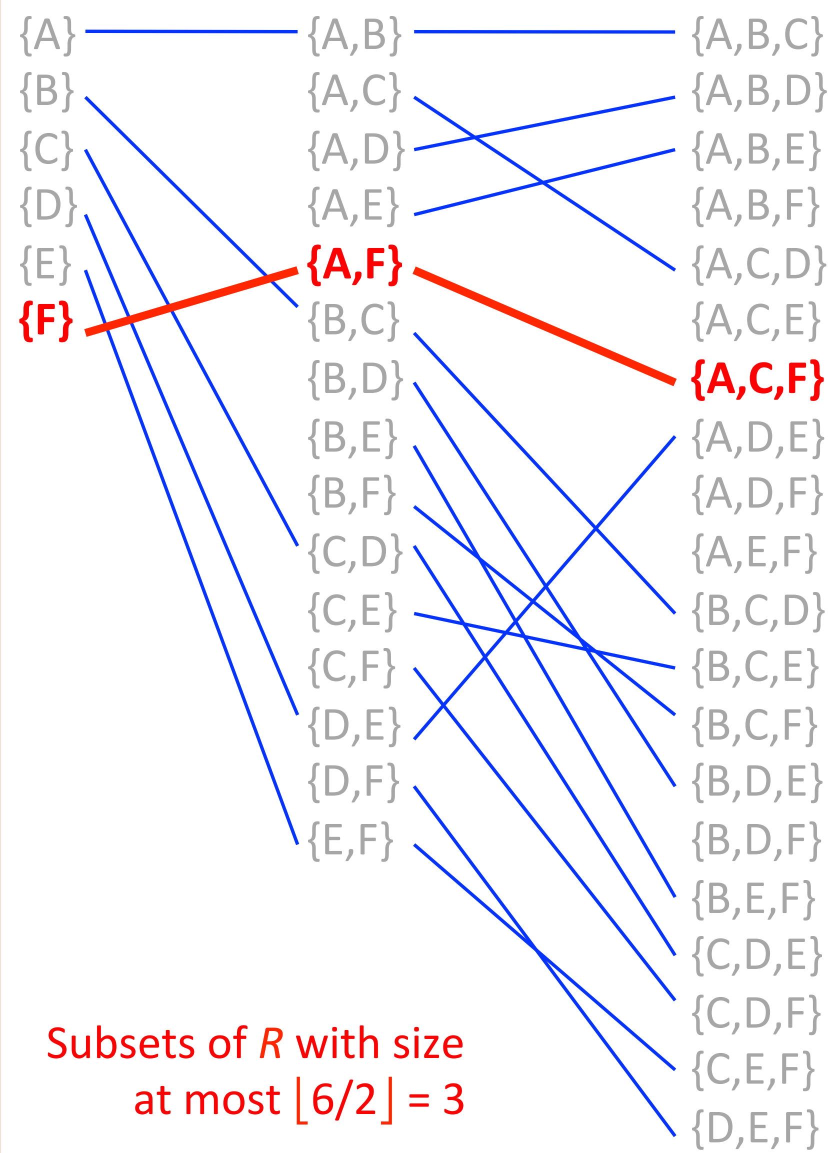

Subsets of R with size at most $\lfloor 6/2 \rfloor = 3$

Geodetic Radon number of grids

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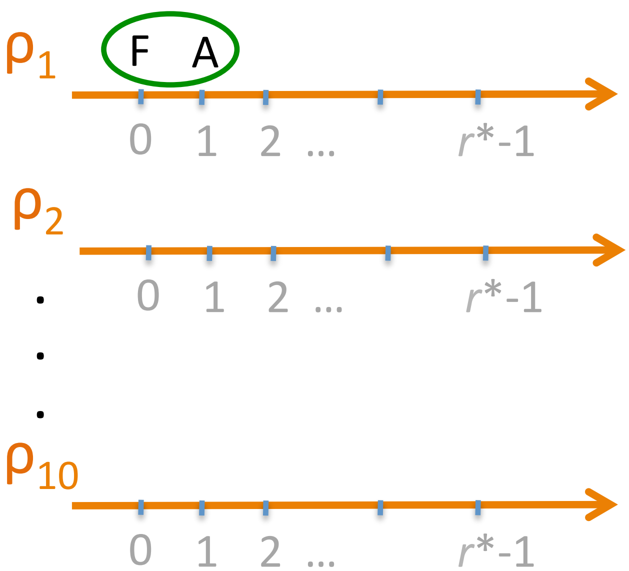

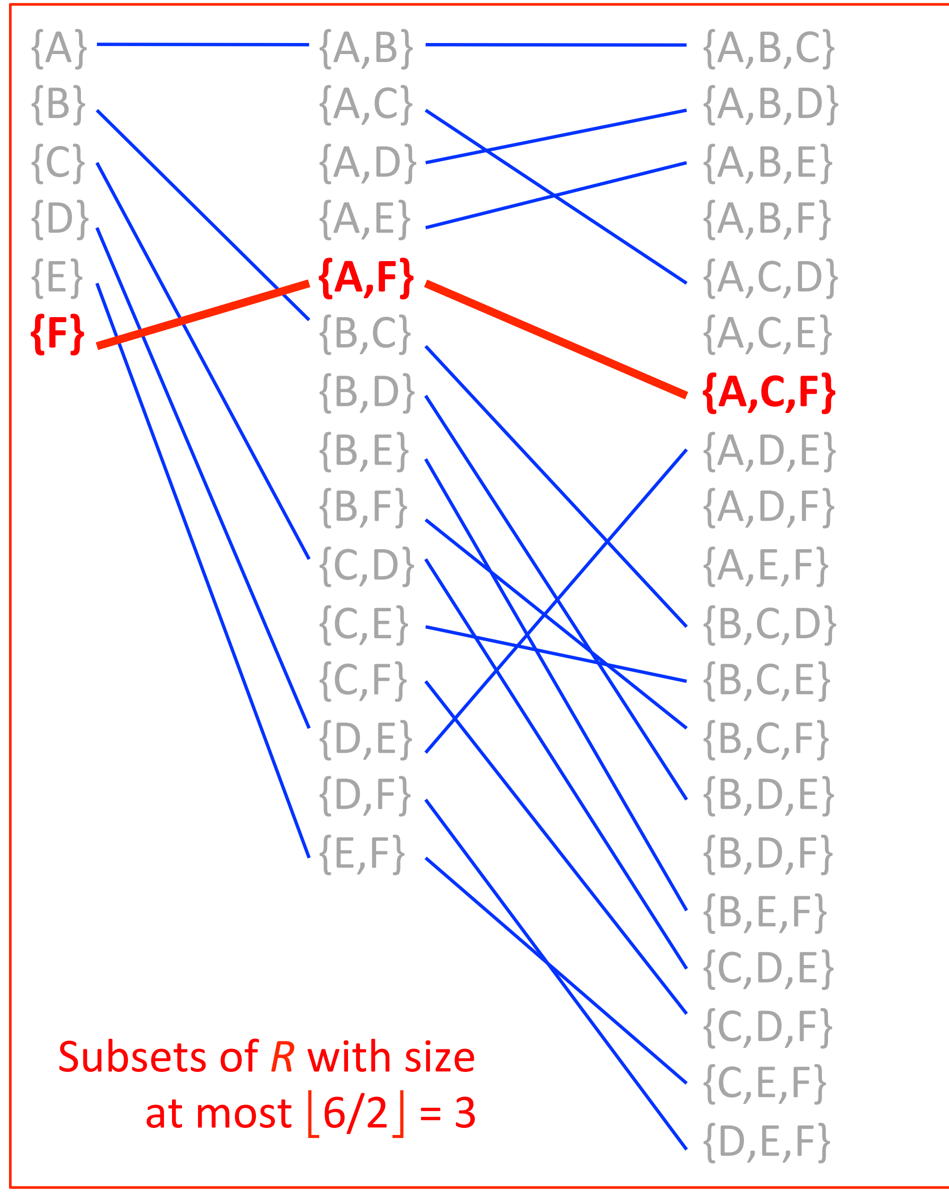

Geodetic Radon number of grids

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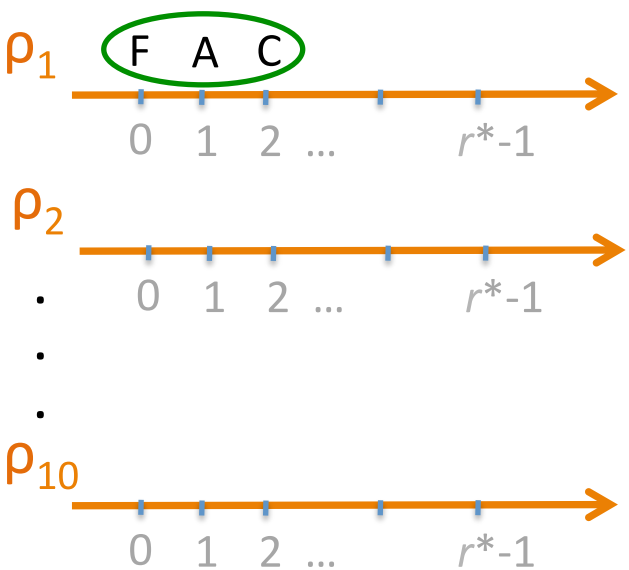
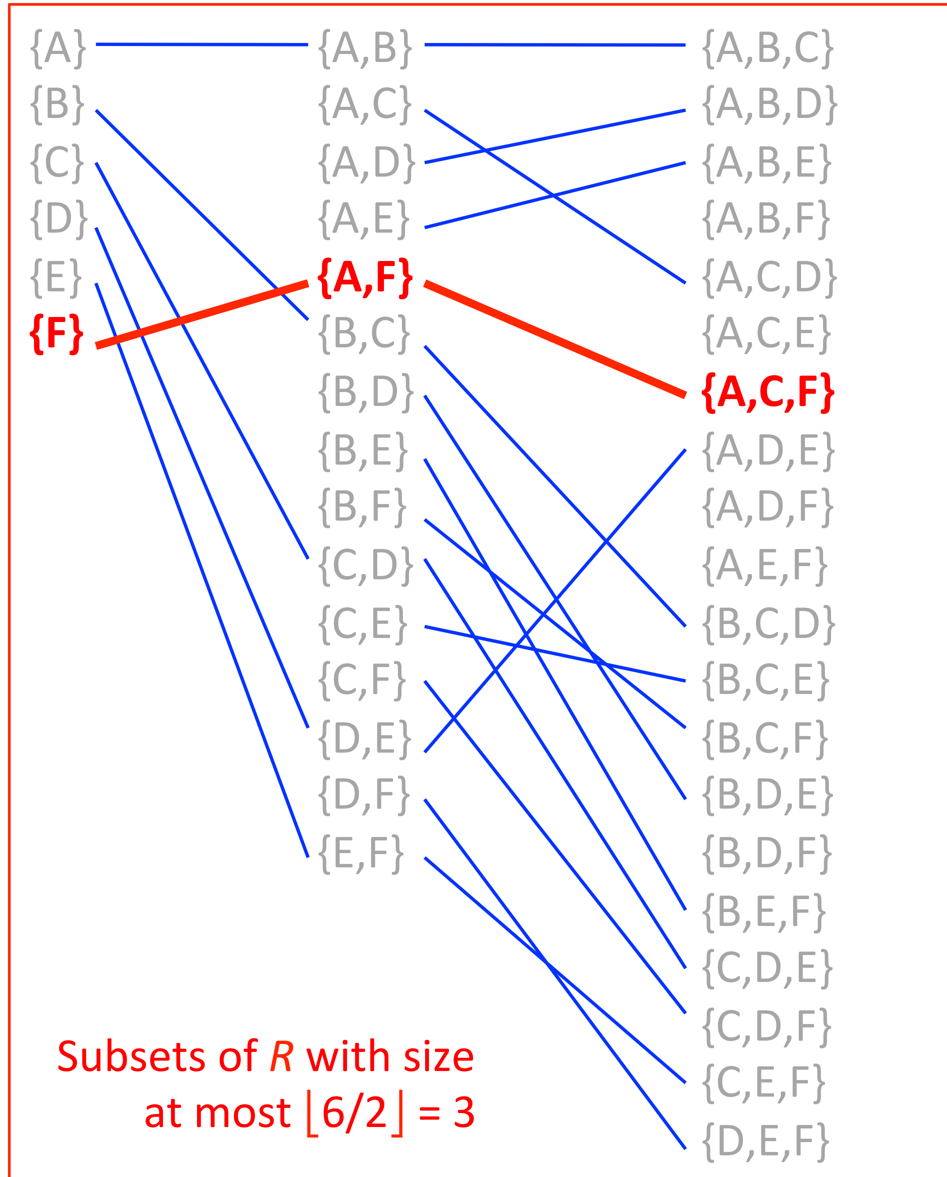
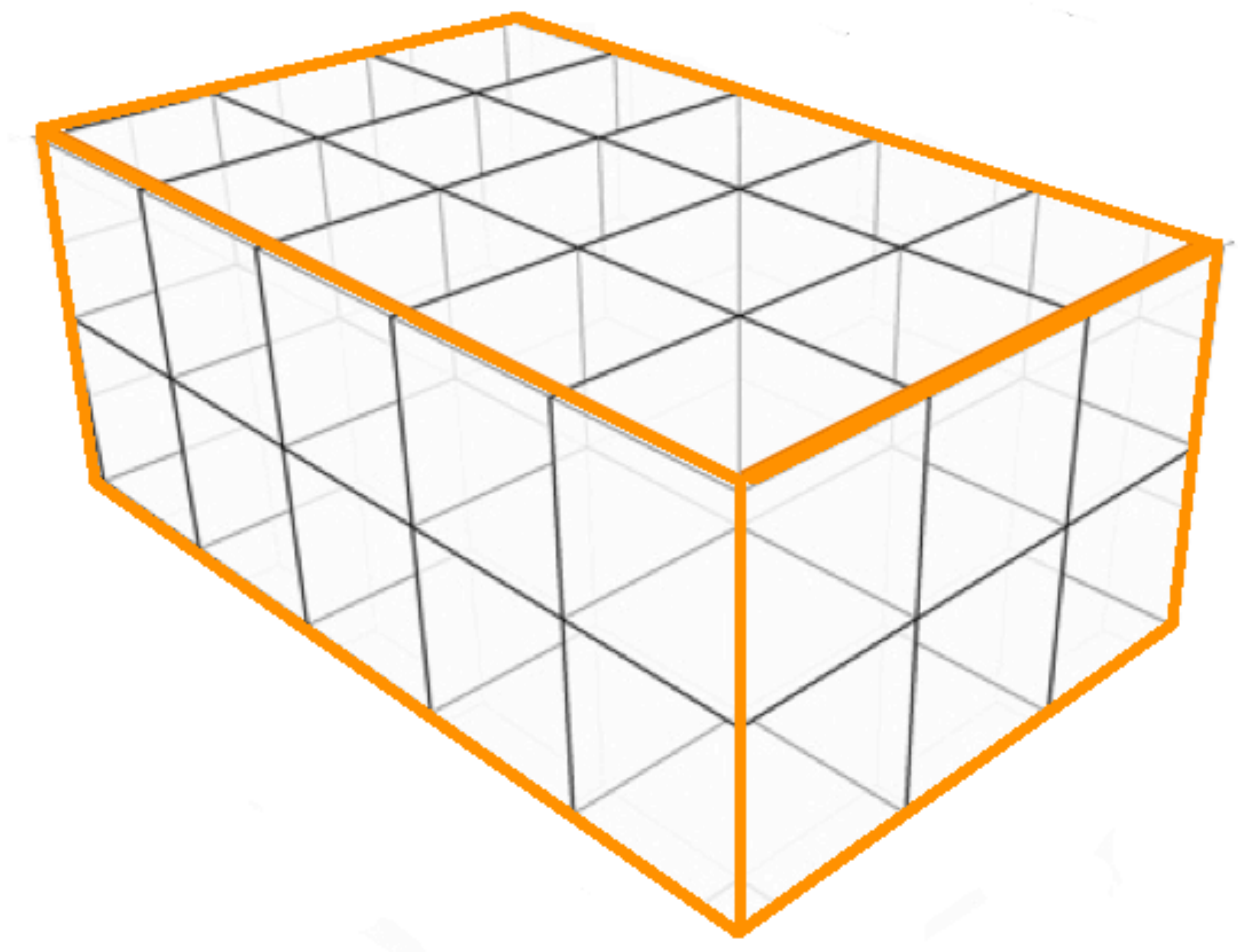




Geodetic Radon number of grids

What about grids that are not "large enough"??



An O(d log d) algorithm

1. Let r^* be the maximum integer that satisfies the necessary condition (2).

anti-Radon set of size r $\Rightarrow$ $$2d \geq \binom{r}{\lfloor r/2 \rfloor} \qquad (2)$$

2. For $r = r^*, r^* - 1, \ldots, 2$ do:
3. For $k = r/2, r/2 - 1, \ldots, 1$ do:
4. Greedily assign a dimension j to each one of the binomial(r, k) permutations having a partite set with k elements. Criteria:
   potential(j) > 0 is maximum;
   k-quota(j) not exceeded.
5. If no dimension j can be chosen, proceed to the next value of r (line 2).
6. Decrement potential(j).
7. Return r.

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1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

Σ 1 10 45 120 210 252 210 120 45 10 1 $= 2^r$

1 11 55 165 330 462 462 330 165 55 11 1

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O(2^r)

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anti-Radon set of size r $\Rightarrow$ $$2d \geq \binom{r}{\lfloor r/2 \rfloor} \quad (2)$$

Overall time complexity:

$O(r^* \cdot 2^{r^*})$

An O(d log d) algorithm

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$$\binom{r}{\lfloor \frac{r}{2} \rfloor} \approx \frac{2^r}{\sqrt{r+1}} \cdot \sqrt{\frac{2}{\pi}}$$

Overall time complexity:

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Overall time complexity:

$O(r^* . 2^{r^*})$

=

$O(d \log d)$

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   O(2^r)

   ...

$$\sum_{r=2}^{r^*} O(2^r)$$

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O(2^r)

...

$$\sum_{r=2}^{r^*} O(2^r) = O(2^{r^*+1} - 3) = O(2^{r^*})$$

An O(d ~~log d~~) algorithm

An O(d) linear-time algorithm !!!

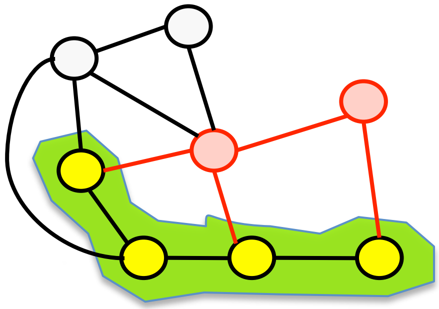
2. For $r = r^*, r^* - 1, \ldots, 2$ do: **O(r^*)**

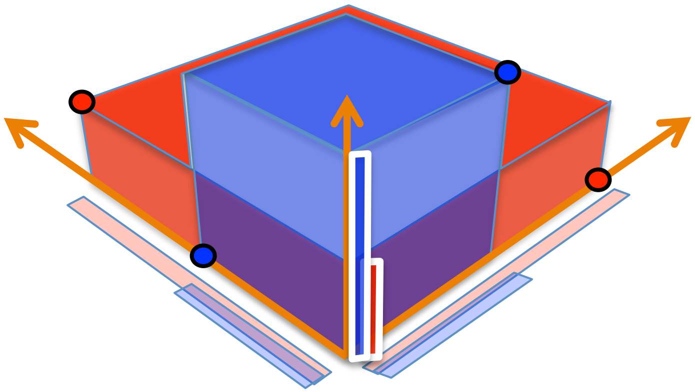
O(2^r)

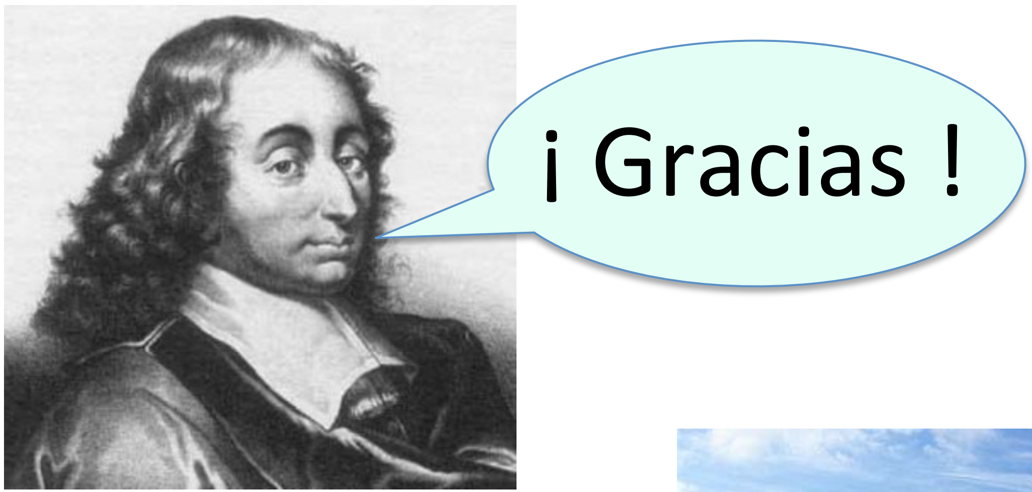
…

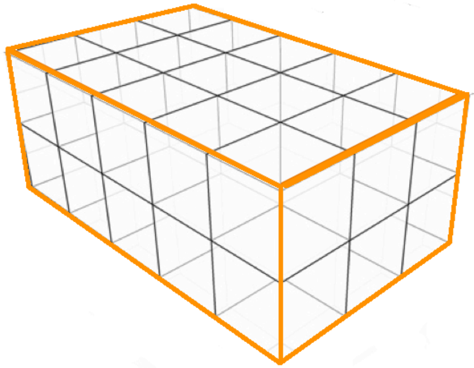
$$\sum_{r=2}^{r^*} O(2^r) = O(2^{r^*+1} - 3) = O(2^{r^*})$$

$$= \boxed{O(d)}$$






¡ Gracias !



Polynomial time algorithm for the Radon number of grids in the geodetic convexity

Mitre Costa Dourado
Dieter Rautenbach
Vinícius Gusmão Pereira de Sá
Jayme Luiz Szwarcfiter

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