A geometric trigraph model for unit disk graph recognition

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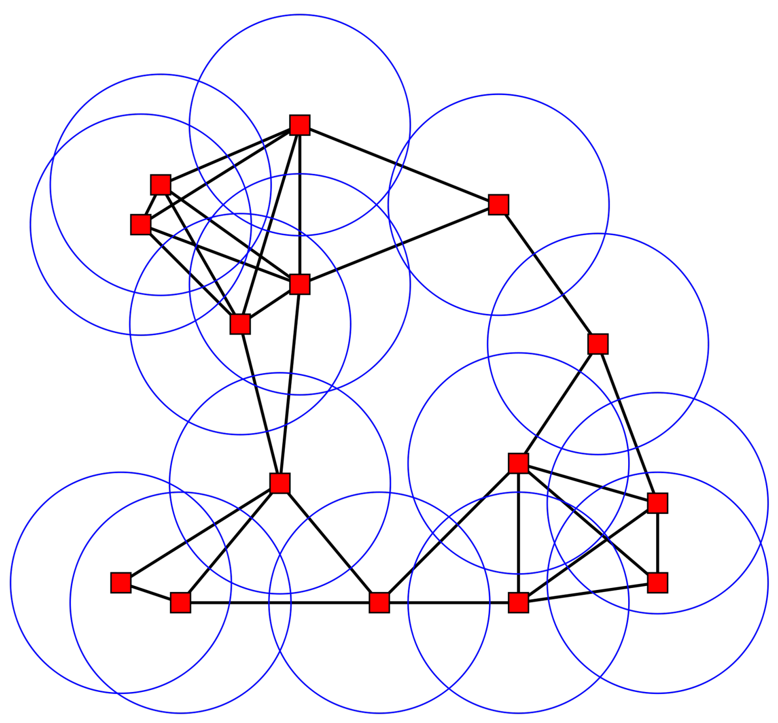
Unit Disk Graphs



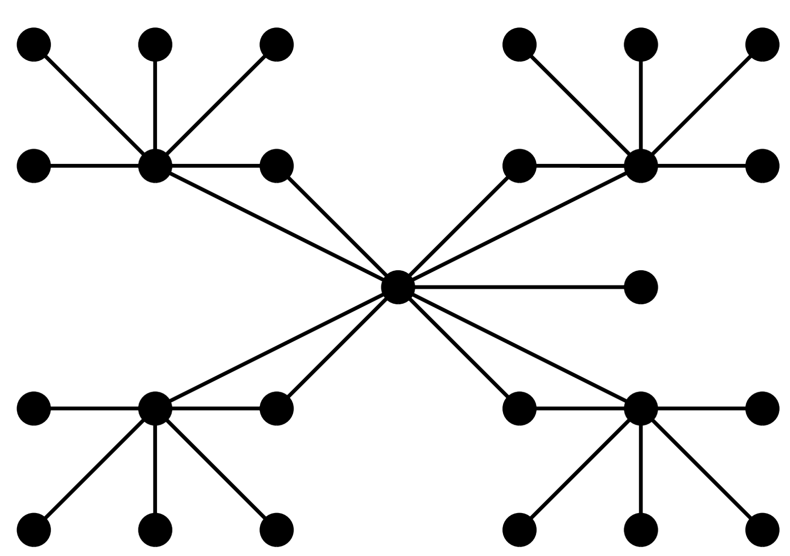
A unit disk graph (UDG) is a graph whose n vertices can be mapped to points on the plane and whose m edges are defined by pairs of points within Euclidean distance at most 1 from one another. Alternatively, one can regard the vertices of a UDG as mapped to coplanar, congruent disks, so that two vertices are adjacent whenever the corresponding disks intersect. Unit disk graphs have been widely studied in recent times due to their applications on wireless ad-hoc networks [1].

Recognition

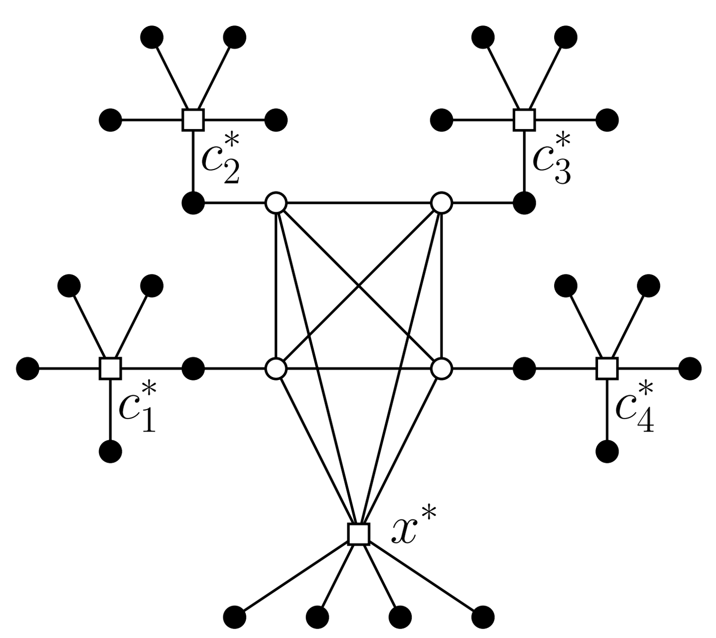
The recognition of unit disk graphs is NP-hard [2]. Moreover, the most natural certificate (the disk coordinates) does not work, since there are unit disk graphs with n vertices such that, in every possible realization (with integer coordinates), at least one coordinate is at least $2^{2^{\Omega(n)}}$ (doubly exponential on the input) [3].

Optimization problems

Several optimization problems have been considered for unit disk graphs. Many of them are difficult, hence motivating the search for approximation algorithms. In this sense, certifying that certain graphs are not UDG would allow for proofs of tighter approximation factors for many such algorithms.



If **not** UDG, upper bound for UDG maximal stable set would decrease



Worst (known) case for approx. alg. for UDG dominating sets.

Practical algorithms and computational proofs

Proving that a certain graph is not UDG is no easy feet. Classical proofs are strongly based on *ad hoc* discrete geometry methods. Moreover, the general recognition algorithm is doubly exponencial, too heavy even for very small graphs. On the other hand, the proposed method has thus far succeeded in obtaining both YES and NO certificates for a number of small graphs.

Discrete trigraph embodiments. Consider a 2-dimensional square mesh of granularity g (the smallest distance between two of its points) and call a *discrete model* for a graph G any embedding of its vertices on the mesh. A *trigraph embodiment* for G is a discrete model such that, for every pair of vertices x,y of G, it holds that:
(1) if x,y are adjacent, then the distance between their corresponding points on the mesh are no greater than $1 + 2^{0.5}g$;
(2) if x,y are not adjacent, then the distance between their corresponding points on the mesh is greater than $1 - 2^{0.5}g$;

Proving YES and proving NO. If no trigraph embodiment is found for G considering a mesh of a certain granularity g, then we prove that G is not a UDG. On the other hand, whenever a trigraph embodiment is found, the discrete model is further refined by considering a mesh of higher granularity (smaller g). A YES answer arises when a model is found such that all pairs of vertices whose distances between one another belong to $(1 - 2^{0.5}g, 1 + 2^{0.5}g]$ correspond to vertices that are adjacent in G (or, analogously, to vertices that are non-adjacent in G).

Reducing the number of candidate embodiments. In order to reduce the number of realizations that must be checked, we use the following techniques:
- backtracking-based approach, positioning one vertex at a time, following an insertion order that preserves connectivity;
- geometric growth of the considered mesh granularities, refining thicker meshes onto thinner ones;
- enforcement of several geometric properties at each step in order to trim the search subtrees as early as possible (e.g. distances and angles between particular points).

[1] Marathe et al. (2005), Simple heuristics for unit disk graphs.
[2] Breu and Kirkpartrick (1998), Unit disk recognition is NP-hard.
[3] McDiarmid and Müller (2013), Integer realizations of disks and segment graphs.