The Growing Cliques algorithm for the Homogeneous Set Sandwich Problem

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A homogeneous set is a non-trivial module of a graph, i.e. a non-empty, non-unitary, proper subset of a graph's vertices such that all its elements present exactly the same outer neighborhood. Given two graphs $G_1(V, E_1)$, $G_2(V, E_2)$, the Homogeneous Set Sandwich Problem (HSSP) asks whether there exists a sandwich graph $G_S(V, E_S)$, $E_1 \subseteq E_S \subseteq E_2$, which has a homogeneous set.

In 2001, Tang *et al.* published an all-fast $O(n^2 \triangle_2)$ algorithm which was afterwards proven wrong, so that the HSSP's known upper bound would have been reset at former $O(n^4)$ determined by Cerioli *et al.* in 1998. However, an $O(n^{3.5})$ algorithm, based on the balancing technique, and an $O(n^3)$ randomized Monte Carlo algorithm were devised shortly thereafter.

We present the concept of *enemy* vertices, which establishes a relationship between sandwich homogeneous sets and independent sets. It is the underlying principle of the approach in which clique covers are employed to bound the size of the maximum independent set in an auxiliary graph, therefore imposing the best possible upper bound to the size of sandwich homogeneous sets we shall search for at each iteration of a new $O(n^3 \log n)$ deterministic algorithm.